Dear Future University Student:

Thank you for purchasing The MATH SAT® Companion©. We know how daunting the SAT test is, and we hope you find our companion guide to *The Official SAT Study Guide, 2nd EditionTM* useful as you prepare for your test day. To use this book effectively to help you perform well on the SAT, we suggest the following steps:

1. Buy ***The Official SAT Study Guide, 2nd EditionTM***. The authors of this book are the creators of the test, and you will get no better practice than by doing these problems.
2. Review/Skim the subject chapters in ***The Official SAT Study Guide, 2nd EditionTM***. The information in here should not be new, but it will be a good refresher.
3. Do problems, problems, and more problems!! Time yourself and take it seriously. Don’t look at the answers until you have completed the section. Don’t cheat yourself! Did we mention do problems?
4. Use The MATH SAT® Companion© as a reference. Again, don’t cheat yourself by reading a problem in the book and then going right to the Companion explanation! Try these problems first, give yourself a real shot at completing them, and then while reviewing your score refer to the Companion to provide a detailed explanation.
5. Study frequently, in smaller chunks of time as opposed to one big block of time. Let’s say you have 2.5 hours per week to study for the SAT. If you commit to 30 minutes, 5 times a week, your resulting progress will be much better than if you spent 2.5 hours in one session. SAT practice is all about frequent exposure to problems. ***Note: 2.5 hours IS NOT our guidance on how much you should study per week. If you can study 5, 10, or 20+ hours a week, do it.***

We’ve written explanations in an easy-to-understand voice, and laid out the steps for each problem. We have not cut corners; the “easy” problems (is there really such a thing?) have explanations that are just as thorough as the hard problems.

The MATH SAT® Companion© is meant to be a comprehensive reference guide. Whether you are reviewing this material with a Commonwealth Education tutor, in an SAT class, or by yourself, you will always be a click away from a complete explanation of the solution to a problem.

There are many ways to complete math problems; our method is surely not the only one! We have chosen our explanations as they make the most sense to us, but if you get to the same answer as we do by another method, please feel free to share it with us! We may not be students anymore, but we are always learning and looking for better ways to answer problems and provide a great product.

We wish you the best of luck, and please tell us how you did when your results come in! If you have any questions or comments, please do not hesitate to contact us at SAT@commonwealthedu.com.

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**Commonwealth Education**

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# Test 1 Section 3

1. Algebra to start with.
   1. Let’s write our given: x = 4
   2. What are we looking for? The answer that has the greatest value.
   3. There are two ways to do this, the first being:

Work every possible answer by inputting 4 for x:

* + - (4+1)(4+2) = (5)\*(6)=30 (answer)
    - (4+1)(4-1) = (5)\*(3)=15
    - (4-2)(4+2) = (2)\*(6)=12
    - (4-2)(4+1) = (2)\*(5)=10
    - (4-4)(4+4) = (0)\*(8)=0

iv) Or logic it out:

* C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfCross out (E) because (4-4) = 0. Any number times 0 is 0, so that’s out.
* Now look at (C) and (D). We are subtracting 2 from **x**. If **x** were really large, this might not matter, but **x** is 4, so taking 2 away from it reduces it by 50%. (C) and (D) are gone.
* We’re left with (A) and (B). They both have (x+1) in them, so just look where they differ. Is there any way (x+2) can be smaller than (x-1)? No way. (B) is gone.

A is the correct answer.

1. 3 Variable Problem
   1. What are we looking for? What speed C is traveling when B’s speed is 7 mph (so B=7 is a given).

The table reflects the speeds we will find.

|  |  |  |
| --- | --- | --- |
| Train A | Train B | Train C |
| **A** | **B** | **C** |

* 1. We have 3 variables here, so this usually means a two-stepper. Let’s use our variables and givens to translate the “math speak” into equations.
     + - Train A’s speed ***is*** 3 ***times*** Train B’s speed.

A ***=*** 3 ***X*** *B*

**A=3B Equation 1**

* + - * Train C’s speed ***is*** ***twice*** Train A’s speed.

C ***= 2×*** *A*

**C=2A Equation 2**

iii) Now we’ve got two equations, so let’s use ‘em. What are we looking for again? C’s speed, that’s right.

B=7 (Given)

A=3B

A=3(7) = 21

C=2A

C=2(21) = 42

E is the correct answer.

3) Average Problem

i) Know that “average” and “mean” are the same. Average = Sum of the terms / # of terms.

ii) Let’s write our givens:

* + - * Terms: x, 5x, 6x
      * Number of Terms: 3
      * Average of terms: 8
  1. What are we looking for? The value of x
  2. AVERAGE= Term 1 + Term 2+…+ Term *n\**

*n\**

\* *n* just means total number of terms. If there were 5 terms in the sequence, *n* = 5. Don’t let variables scare you. Make them your friends (we refer to them as such through this text)!

v) Now plug in given information.

* + - * x + 5x + 6x = 8

3

* 12x = 8

3

* Multiply both sides by 3:

12x ×3 = 8×3

3

* Simplify fraction:

12x ×3 = 24 The 3’s cancel each other out.

3

* Divide by 12:

12x = 24

12 12

* Simplify fraction:

12x = 24 The 12’s on the left side cancel out and we’re left with 24/12.

12 12

* Finish up!

x = 2

B is the correct answer.

4) Graphing Question. There will be many. Own them.

i) What are we looking for? The graph that agrees with the statement, “No two points on the graph have the same x- coordinate.”

ii) Know the rules of a function: Every “x” value of a function has only ONE UNIQUE “y” value. Isn’t that the same thing as the statement in the question? Yes.

* 1. Use the vertical line test. If we can draw a vertical line anywhere on the graph, and it intersects the alleged function at more than ONE point, it is NOT A FUNCTION.

(A)

Answer A fails the vertical line test.

(B)

B also fails.

(C)

C fails.

(D)

D looks like it passes the vertical line test, but since we’ve come this far, let’s check (E).

(E)

E fails.

D is the only graph that passes the vertical line test.

Answer D is correct.

5) Vahndahful Venn Diagrams

i) Know the rules of Venn Diagrams. The total number of items (in this case, students) is everything within the rectangle. The intersection of the circles represents students who study both subjects. All of the areas must add to the total number of students. Don’t forget the space outside the circles.

ii) Let’s write our givens:

Butterfly Students =9 Grasshopper Students=15 Students Studying Both=3 Students Studying Neither=3

* 1. What are we looking for? Percentage of butterfly students only.
  2. What is the percentage? We have to find it. This is a two-stepper. First, find the total number of students:

B + G + B and G + Other = Total

1. + 15 + 3 + 3 = 30
   1. Since we only need the percentage of butterfly students, just use the butterfly and total numbers.
      * C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfButterfly students = 9
      * Total students = 30
      * Know percentage formula: Part × 100 = %

Whole

* + - Input numbers and Finish up.

9 × 100 =30%

30

Answer C is correct.

6) Coordinate Plane Problem. There will be many of these.

* 1. Let’s write our given: AB=CD
  2. Remember that in (x, y) the x means how far right (positive) or left (negative) the point is. The y means how far up (positive) or down (negative) it is. That’s it. Be very comfortable with this.
  3. What are we looking for? The value of ***t***. Where is ***t***? It’s the y-value for point B (-2,***t***). How do we get there?
  4. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfThe key to this problem is AB=CD. It means that the two lines have the same length.
     + A vertical line (like AB) always has the same x-coordinates (-2 in this case). The length of vertical lines ONLY comes from the y-values of its endpoints. To get the length, subtract the smaller one from the larger one, in this case:

3 - ***t*** = length of AB\*

*\*Remember, make variables your friends. Just because 3-t doesn’t give you a number, don’t give up.*

* + - A horizontal line (like CD) always has the same y-coordinates (-3 in this case). The length of horizontal lines ONLY comes from the x-values of its endpoints. To get the length, subtract the smaller one from the larger one, in this case:

6 - *(-4)* = length of CD

6 + *(4)* = length of CD *Subtracting a negative is just like adding the number*.

10 = length of CD

* 1. Bring it all together. What are we looking for? ***t****.* What do we now know?
     + - AB=CD
       - AB=3-***t***
       - CD=10
       - Plug and chug.
       - **AB** = **CD**
       - **3-*t***= **10**
       - 3-***t*****-3** = 10 **-3**
       - -***t*** *= 7*
       - **-**(-***t****) =* ***-7***
       - ***t = -7***

This makes sense, because point B is below the x-axis, so it must be negative.

Answer C is correct.

7) Dreaded Algebra.

i) Know the Order of Operations: Please Excuse My Dear Aunt Sally or PEMDAS. If you don’t know it, Google it, and write it down 10 times every night until you know it.

ii) Let’s write our given: 3***x2*** = 4***y*** = 12

* 1. What are we looking for? The value of ***x2*** ***y.***
  2. This problem looks tricky, but it’s not. Separate the problem into two equations.
     + 3***x2*** = 12 AND 4***y*** = 12
     + Simplify:.
       - 3***x2*** = 12 AND 4***y*** = 12

**3** **3** **4** **4**

3***x2*** = 12 AND 4***y*** = 12 The 3’s on the left cancel out, as do the 4’s. We’re left with:

**3** **3** **4** **4**

* + - * ***x2*** = 4 AND ***y*** = 3
  1. What’s the problem asking? The value of x2y. Don’t we know both of those variables now? Yes. Plug ‘em in. **x2y = ?**

**4×3 = 12**

D is the correct answer.

1. Geometry Problem. MARK UP THE DRAWING.
   1. Let’s write our givens:
      * Radius of Circle A is **2**.
      * Radii of Circle B and C are both **4**.
   2. What are looking for? The radius of the big circle.

**B**  **C**

A

**4 4 2 2 4 4**

* 1. Find the radius of the largest circle:

**2 + 4 + 4 = 10**

D is the correct answer.

9) Number Line Problem

* 1. Let’s write our given: Tick marks are equally spaced.
  2. What are we looking for? The value of ***x***.
  3. Use the spaces.
     + There are 4 ticks between 2 and 42. Another way to say this is that there are 5 equal spaces between 2 and 42, or 5 ticks to go from 2 to 42. Let’s use “spaces.”
       - ***x*** is 2 “spaces” from 2. Find the value of one “space.”
       - 42 - 2 = 40
       - Divide this by the number of “spaces”:

40 = 8 (So, each “space” is 8)

5

* + - Find ***x*** and finish up.
      * Start at 2.
      * Add the value of two “spaces.” Remember, ***x*** is two spaces from 2, and each “space” is 8. So 2(8)=16.
      * 2 + 16 = 18

D is the correct answer.

1. Tricky Geometry. Remember: MARK UP THE DRAWING!!
   1. What are we looking for? The value of ***x***.
   2. Know how many degrees are in a circle (360°).
   3. Look at the graph carefully.
   4. Label the right angle as 90° ***on the graph***.
   5. Write an equation for ***x***.

***x*** + 110° + 30° + 90° = 360°

* 1. Simplify and finish up.

***x*** = 360° - 110° - 30° - 90°

***x*** = 130°

* 1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfTricky Part: Don’t get caught up with the 70°. Look at the picture again. ∠SOV (***x***°) includes ∠ROV (70°) so we don’t even need to use the 70°. They just put it there to mess with us.

C is the correct answer.

1. Algebra Problem. Again, make the variable our friend.
   1. Let’s write our given: ***k*** divided by 7 gives a remainder of 6.
   2. What are we looking for? The remainder when ***k*** + 2 is divided by 7.
   3. First, what is a remainder? It’s the amount left over in division.

Here’s an example: What’s the remainder when we divide 5 by 2?

2 r ***1*** ***1*** is the remainder in this example

2√5

* 1. Back to our problem. Write the skeleton of the problem, in this case just the division sign.

√

* 1. Now let’s fill it in with the given information.

r ***6*** From the statement: “when ***k*** is divided by ***7*** the

***7***√***k*** remainder is ***6***.”

That looks a little more manageable.

* 1. What numbers work for ***k***? Let’s try 7. Input 7 for ***k***.

1 r ***0***

***7***√***7***

Remainder is ***0***. ***k*** can’t be 7. Let’s try ***k*** = 8.

1 r ***1***

***7***√***8***

8 is better, but we’re not there yet. What about 9?

1 r ***2***

***7***√***9***

See the pattern here? Every time we increase the dividend***\**** by 1, the remainder increases by 1. We can continue going all the way up, but hopefully we see that 13 will give us a remainder of 6 as below.

1 r ***6***

***7***√***13***

C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\717RERNY\MC900229885[1].wmfBingo. ***k*** must be 13 (or any multiple of 13).

**\***Not the most exciting topic, but know your terminology. A dividend is the number under the division sign (in this case **13**), and the divisor is what you are dividing by (in this case **7**).

* 1. Use ***k*** in the second equation, ***k*** + 2.

***k*** + 2 = ***?***

***13*** + 2 = ***15***

* 1. Finish up. What’s the question asking again? The remainder when ***k*** +2 is divided by ***7***.

2 r ***1*** **That’s the answer!**

***7***√***15***

14

***1***

B is the correct answer.

1. Graphing Data Problem. Take a second to absorb what’s shown in the table. Forget about the answers for a second.
   1. Look for a pattern in the table of data. See that when ***Depth*** goes up, ***Pressure*** goes up? Up and Up, that’s a “positive” relationship.
   2. Now look at the graphs.
      * ***Depth*** is on the ***x***-axis.
      * ***Pressure*** is on the ***y***-axis.
   3. So Depth is an x-coordinate, and Pressure is a y-coordinate. LABEL ***x*** AND ***y*** AT THE TOP OF THE DATA TABLE.
   4. We love zeros and ones (if we don’t, start), so always start there. The first point of (***Depth***, ***Pressure***), or (***x***, ***y***), is (***0***, ***14.7***). Right off the bat, answers C & E are gone because those lines start at (***0***, ***0***) and (***?***, ***0***). We’re down to A, B & D.
   5. Look at the remaining answers. A & B both have a ***negative*** slope, meaning they slope down from left to right. That indicates that when the ***Depth*** number goes **higher**, the ***Pressure*** number goes **lower**. Is that what the table shows? Not at all! When ***Depth*** goes up, ***Pressure*** goes up. So A & B are out. We’re left with D as the only possible answer. Take a last look at D, but it’s the only one that works.

D is the correct answer.

1. Sequencing Problem
   1. Let’s write our givens:
      * First term is 1.
      * Every term after that is (***-2***) × (***the number before it***). Remember this equation.
   2. What are we looking for? The 6th term.
   3. Draw it out. The skeleton looks like this:

*1st 2nd 3rd 4th 5th* ***6th***

1 □ □ □ □ □

* 1. What’s next? Use the equation in i) above to find the 2nd term.

(***-2***) × ***1*** = ***-2***

* 1. So, plugging -2 in we have:

*1st 2nd 3rd 4th 5th* ***6th***

1 ***-2*** □ □ □ □

* 1. Continue the pattern.

3rd term: (***-2***) × ***-2*** = ***4***

4th term: (***-2***) × ***4*** = ***-8***

5th term: (***-2***) × ***-8*** = ***16***

***6th term***: (***-2***) × ***16*** = ***-32***

E is the correct answer.

1. Factoring and exponents rear their ugly heads. Where to begin?
   1. Let’s write our given: (2x-5)(2x+5) = 5
   2. What are we looking for? The value of ***4x2***.
   3. We can’t factor because the equation = 5, not 0. Sometimes it’s best to just start working and manipulating the problem somehow. What if we foil***\**** the left side of the equation?

(2x+5)(2x-5) = 5

***4x2*** + -10x + 10x + -25 = 5 -10x + 10x = 0. Get rid of those two terms.

***4x2*** + -25 = 5

+ 25 +25

***4x2*** = 30 Aren’t we looking for ***4x2***? We fell into it!

E is the correct answer.

* 1. Shortcut! Recognize the formula for the difference of perfect squares:

C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmf(***x***-***a***)(***x***+***a***) = (***x2***-***a2***)

* + - **The *x*** term ***does not have to be by itself***, it can be 2x, 5001x, or 42x2, as long as it is the SAME in both expressions. This also goes for the ***a*** term; it can be just a number or sometimes another variable (2y, 63z4, or anything else), as long as it’s the SAME in both expressions.
    - Back to our problem. The ***x*** terms (***2x***) are the same, and so are the ***a*** terms (***5***). It doesn’t matter that the ***5*** is subtracted in the second expression; it’s the same number 5.
      * Square both terms, and use the difference of perfect squares formula:
      * *(****2x****)*2- *(****5****)*2 = 5
      * ***4x2*** - 25 = 5

+25 = +25

* + - * ***4x2*** = 30
    - A long explanation, but recognizing the difference of perfect squares makes this problem take 15 seconds at most to solve.

*\*FOILing is when we take 2 expressions like (x+2) and (x-2) and multiply them out.*

*First: x × x + Outside: x × (-2) + Inside: 2 × (x) + Last: 2×(-2)*

*First: x2 + Outside: -2x + Inside: 2x + Last: -4*

*Equation: x2 + -2x + 2x + -4*

*Simplify: x2 + -2x + 2x + -4 -2x + 2x = 0. Get rid of those terms.*

*= x2 -4*

1. Geometry and Algebra Problem. Lots of variables ( friends).
   1. There are a ton of variables in this problem, or at least letters, A, O, B, x, y, ***p***, ***r***. Let’s write our givens:
      * + (***p***, ***r***) are the coordinates of point A.
        + |***p***| > |***r***|. The absolute value (no signs) of ***p*** is greater than ***r***.
        + Now put those two nuggets aside for a second.
   2. What are we looking for? The ***slope*** of ***AB***.
   3. Before doing anything further, look at the line and the answers. What are those answers? They are potential slopes of that line; that’s what we’re looking for, the ***slope*** of ***AB***. Does the slope look positive or negative?

*Remember:*

Positive Slope Negative Slope Zero Slope Undefined Slope

* 1. Which graph looks like the one in the problem? It looks like a negative slope. So answers C, D & E are out. If we get no further, guess, because we’ve got a 50/50 shot of being correct.
  2. We are left with two options answers A (***-2***) and B (**½**). Let’s try both.
  3. Now what is the equation to find the slope of a line given two points? It’s:

***m*** = ***y***2 - ***y1***

***x2*** - ***x1***

* 1. Remember the first given we wrote down: (***p***, ***r***) are the coordinates of point A. The line segment ***AB*** goes through the origin (0,0), so let that be point 1 (***x1***, ***y1***). We’ll be subtracting zero, so it’s easier. Now our equation looks like:

***m*** = ***y2*** - (***0***) = ***y2***  *so here*  ***m*** = ***r***

***x2*** - (***0***) ***x2 p***

* 1. Remember the second given we wrote down: |***p***| > |***r***|. Does answer A (***-2***) or B (**½**) work?
     + ***m*** =-2

That’s the same as ***m*** =***-2*** = ***r*** Is |***1***| > |***-2***|? Nope.

***1 p***

By elimination, the answer must be B, but let’s make sure.

* + - ***m*** =-½

***m*** =***-1***  = ***r*** Is |***2***| > |***-1***|? We have a winner.

***2 p***

B is the correct answer.

1. Algebra Problem. Again, lots of friends here.
   1. Let’s write our given: *3a + 4b = b*
   2. What are we looking for? What is the answer that equals *6a + 6b?*
   3. This is another one where we should just start working with and manipulating the problem. In problems where we’re not sure where to start, it’s always good to combine like terms.
      * + 3***a*** + 4***b*** = ***b***

-4***b*** -4***b***

3***a*** + 0 = -3***b***

3***a*** = -3***b***

Does this do anything for us? Kind of. But it doesn’t look quite like 6***a*** + 6***b***.

* + - * What if we get the ***a***’s and ***b***’s on the same side?

3***a*** = -3***b***

+ 3***b*** = +3***b***

3***a*** + 3***b*** = 0

* + - * Now compare that to 6***a*** + 6***b***. Much better. How do we get from 3***a*** + 3***b*** to 6***a*** + 6***b***?

Multiply 3***a*** + 3***b*** by 2.

2×(3***a*** + 3***b***) = 6***a*** + 6***b***

* 1. What was the question again? What must equal 6***a*** + 6***b?***
     + - Look back at our work. Doesn’t 3***a*** + 3***b*** = 0? Yes, we figured that out above. So what happens when we multiply anything by 0? We get zero. Since 3***a*** + 3***b*** = 0:

2×(3***a*** + 3***b***) = 6***a*** + 6***b***

2×( 0 ) = 6***a*** + 6***b***

0 = 6***a*** + 6***b***

A is the correct answer.

1. Geometry problem with special triangles.
   1. Know your formulas. (They are in the beginning of each math section; memorize them BEFORE test day.)
   2. Write the formula and also the givens:
      * + s s s: 2 short equal legs s√2 : long side

s√2

* + - * EF ∥ AC
      * F is the midpoint of BC.
      * BF = FC
  1. What are we looking for? The area of the rectangle. Find the length of a short side of the rectangle and the long side and multiply them. We’ll get to that.
  2. Back to the triangles. The triangles in this problem are a 45°/45°/90° triangles. When the 2 legs of a triangle are equal, the 2 base angles have to be equal. We know that the angle joining the legs is 90°, so the other two must be 45°.
  3. Now the tricky part. In our problem s = 10√2, ***NOT*** ***s√2*** .
     + - Side AB is our **s**; let’s find ***s√2*** (side AC).
       - ***s*** = 10√2
       - ***s√2*** = 10√2 × ***√2***
       - ***s√2*** = ***20***
  4. F is the midpoint of BC. That means:

BF = FC = BC = 10√2 = 5√2. This is important. Remember it.

2 2

* 1. EF ∥ AC, and F is the midpoint of BC, so EF must be ½ of AC.

EF = ½(AC)

EF = ½(***20***)

EF = ***10***

* + - GH is the opposite side of the rectangle from EF so it’s equal to EF.
    - All the rules about 45°/45°/90° triangles apply to all of the little triangles, too, so look at the little triangle with CD as one of its legs. Pick a letter for the other point; we’ve chosen H. Pick whatever. Just LABEL IT ON THE GRAPH.

F

***s*** s√2

H ***s*** C

* + - * Let’s figure out what our ***s*** is for this little triangle (Both CH and HF).
      * Remember the midpoint: FC = 5√2
      * So…s√2 = 5√2

s = 5

HF = ***5***

* 1. Back to the rectangles. We found two sides, right?
     + E ***10*** F

***5***

G H (We made up names for points G & H)

* 1. Area of a Rectangle = Length × Width

Area of Rectangle EFHG = ***10*** × ***5*** = 50

C is the correct answer.

1. Exponents Problem
   1. Let’s write our given:
      * + 3 variables: ***k***, ***a*** and ***x***
   2. Remember, o’s and 1’s are great places to start when we are unsure of how to start. Look at the table.

When ***x*** = ***0***, ***f(x)*** = ½ .

* 1. Now let’s put that in our formula:

***f(x)*** = ***kax***

**½** = ***ka0***

Still too many variables, right? Look again. What is any number (or variable) raised to the o (zero) power? That’s right, it’s 1. Keep going.

* 1. Now put that in the formula:

**½** = ***ka0***

**½** = ***k***(***1***)

**½** = ***k*** Just like that, we have our ***k***.

* 1. Now pick a different pair of numbers from the table. Zero is our favorite to start with, but 1 is a very close second. Let’s use ***x*** = ***1*** as our next input in the formula. When ***x*** = ***1***, ***f(x)*** = 2. Put it in the formula and plug and chug!
     + - ***f(x)*** = ***kax***
       - ***2***  =  ***½a1*** A number to the first power is just the base. The one drops out.
       - ***2***  = **½*a*** Get ***a*** by itself by multiplying both sides by 2/1.
       - ***2*** × 2/1 = **½*a*** × 2/1
       - ***2*** × 2/1 = **½*a*** × 2/1 **½** × 2 = 1. We’re left with ***a*** by itself.
       - ***2*** × 2/1 = ***a***
       - 4 = ***a***

D is the correct answer.

1. A nasty geometry problem with a pyramid and variables. MARK UP THE PICTURE.
   1. Read the info. What are our givens? This is a pyramid with a ***square base****.* Each edge has length e. Look at the picture. e = ***m***, so label every edge as ***m*** on the figure.
   2. What are we looking for? ***h*** in terms of ***m***. There are no numbers here, so we will be carrying the variables through the entire problem. The answer *is* a variable.Label the unknown points in the base of the pyramid***.*** Let’s*s*ay the edges are A, B, C and D, and the center is E. Our picture should now look like:

***m***

***m***

***m***

***m***

***h***

V

A

B

C

D

E

***m***

***m***

***m***

X

* 1. How can we get to ***h***? Well, ***h*** is the length of VE. Doesn’t the line from the center of the base of a pyramid up to the vertex (“math speak” for top) form a right angle with the base? Of course. So we can form a triangle, and more specifically a right triangle, and use the Pythagorean Theorem: a2 + b2 = c2.
  2. Now draw a line from point B to point E.

***h***

E

***m***

***m***

***m***

***m***

A

C

D

V

B

***m***

***m***

***m***

* 1. Now we have ∆VEB, with a right angle (90°) at ∠VEB.

***h***

V

B

E

***m***

We have one side, VB, but we need to find BE before we can get to VE using the Pythagorean Theorem.

* 1. Find BE. Back to the big pyramid.

B

C

***m***

***X***

* Label midpoint of BC as ***X***.
* BC = ***m***, so BX = ***m***/2.
  + - Draw ∆EXB. This must be a right triangle because the line EX from the center of the base of the pyramid (E) to the midpoint (X) forms a right angle at ∠EXB. Since the base is a square, EX must have half the length of a base edge.

B

E

***X***

***m***/2

***m***/2

This is a 45°/45°/90° triangle, with BX and EX being **m**/2 or “s” (remember the formula for triangles). This means that BE would be s√2. Plug ***m***/2 in for “s” to get s√2 .

* + - * 1. BX = ***m***/2
        2. BE = (***m***/2) × √2
        3. BE = ***m***√2

2

* 1. We’ve got EB ***and*** *VB in terms of* ***m*** *and VE in terms of* ***h***. Use the Pythagorean Theorem and finish up.
     + - a2 + b2 = c2. a and b are legs, and c is the hypotenuse (long side).

a = VE = ***h***

b = BE = ***m***√2

2

c = VB = ***m***

* + - * a2 + b2 = c2

***h***2 + ***m***√2 2 = ***m***2 Now distribute the 2 exponent through the 2nd term.

2

***h***2 + (***m***)2(√2)2 = ***m***2

(2)2

***h***2 + 2***m***2 = ***m***2 Simplify the fraction.

4

***h***2 + ***m***2 = ***m***2 Combine like terms.

2

***h***2 + ***m***2 - ***m***2 = ***m***2 - ***m***2

2 2 2

***h***2 = ***m***2 - ***m***2 If ***m***2 was 1, wouldn’t this be 1 - ½? Same thing here.

2

***h***2 = ***m***2 Take square root of both sides to find ***h***.

2

√***h***2 = ***m***2

2

***h*** = ***m***

√2

A is the correct answer.

1. Word problem with friends.
   1. Write down givens:
      * + Commission is ***k%***
        + 2 cars sold
        + Each car sold for $14,000
   2. What are we looking for? The ***expression*** that represents the commission. There are no numbers here, so we will be carrying the variables through the entire problem. The answer *is* a variable.
   3. Let’s get started. Find the total dollar amount of sales.
      * + 2 cars sold at $14,000 each is the total sale amount.

2 × 14,000 = ***28,000***

* 1. What is the calculation for commission?
     + - C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfEasy. ***k*** × 28,000? Nope. It’s ***k%*** × 28,000.
       - But that’s not one of our answer choices. Can we simplify this? Yes. What does ***%*** mean? It means ***percentage***, which is literally ***per hundred***. That is written as ***/100***.
       - Simplify the expression:
       - ***k%*** × 28,000
       - ***k/100*** × 28,000 Now cross out the last two zeros.
       - ***k/1*** × 280 What’s any variable over 1? Just the variable.
       - ***k*** × 280
       - 280***k*** That’s our expression.

A is the correct answer.

# Test 1 Section 7

1. Chart Problem
   1. Look at the answers. Two are small; three are very large. Why the difference? Make sure to read the chart carefully.
   2. Let’s write our given: Each house in the picture represents 2,000 homes.
   3. What are we looking for? The number of ***new homes*** that were built from ’61 through ’90.
   4. Add up the houses. 1961-1970: ***2***

1971-1980: ***4***

1981-1990: ***8***

***14*** houses in the drawing

* 1. ***14*** houses × 2,000 homes = ***28,000*** built during those three periods.

E is the correct answer.

1. Geometry Problem. Let’s say it together, “MARK UP THE DRAWING!”
   1. What are we looking for? The value of ***w***.
   2. Go ahead and give names to the two other angles in the triangle. Let’s use ***x*** and ***y***. Our picture should be:

35° 45°

***x***  ***y***

***w***

* 1. Remember vertical angles? When we draw two lines that cross each other, 4 angles are formed. The angles across the intersection point are equal. So:
     + - ***x***° = 35°
       - ***y°*** = ***45°***
  2. The 3 angles of a triangle add up to 180°.
     + - ***x***° + ***y***° + ***w***° = 180°
       - ***35°*** + ***45°*** + ***w***° = 180°
       - ***35°*** + ***45°*** + ***w***° = 180°
       - -35° - 45° -35° -45°
       - ***0*** + ***0*** + ***w***° = 180° -35° -45°

***w***° = 180° -80°

***w***° = ***100°***

B is the correct answer.

1. Generic Word Problem
   1. Let’s write our givens:
      * + 19 tables
        + 84 people
        + Tables seat 4 or 5 people
   2. What are we looking for? The number of 5-person tables.
   3. We need some friends (variables) here. This looks like a system of equations because it doesn’t seem like one equation will give us our answer.
      * + Let ***x*** = Number of 4-person tables
        + Let ***y*** = Number of 5-person tables That’s what we’re looking for.
   4. Let’s set up our equations:
      * + Tables:***x*** + ***y*** = 19
        + People: 4***x*** + 5***y*** = 84
        + Now we’ve got our equations. We can either use substitution or adding down. Substitution is easier here because in the “Tables” equation, the variables don’t have any numbers in front of them. Let’s isolate the ***x*** variable so we can get rid of it, since we’re looking for ***y***.
          1. ***x*** + ***y*** = 19

+ ***-y*** ***-y***

* + - * 1. ***x*** = 19 ***-y***
      * Now plug the (19 ***- y***) expression for the ***x*** in the “People” equation.
        1. 4***x*** + 5***y*** = 84
        2. 4(19 ***- y***) + 5***y*** = 84 Next distribute the 4 by multiplying it out.
        3. 76 - 4***y*** + 5***y*** = 84 Now combine like terms.
        4. 76 + ***y*** = 84

-76 -76

***y*** = 8

E is the correct answer.

1. Exponents Problem
   1. Given that ***a*** = 4, substitute ***4*** for ***a*** in the ***a***m2 + ***a***m2 + ***a*** expression.
      * + ***a***m2 + ***a***m2 + ***a***
        + ***4***m2 + ***4***m + ***4***
   2. Look at the answers. Is there a way to get to m3 from our expression? No. So cross out A. We’re down to four potentials.
   3. Back to our equation. See anything similar in each term? That’s right, there’s a ***4*** in all of them. Let’s factor out that ***4***.
      * + ***4***m2 + ***4***m + ***4***
        + ***4***(m2+ m + 1)
        + What does that look like? Answer D. Double check the others, but there is absolutely no way any of them can be the correct result with the given information.

D is the correct answer.

1. Geometry Problem. You know what’s next. Mark…Up…The…DRAWING!
   1. Label the figure.

B 2 C

2 O 2

A 2 D

* 1. What are we looking for? The area of the shaded portion of the circle.
  2. What kind of triangle is ∆OCD? Because ABCD is a square, and O is the center of the inscribed circle, it must be a 45°/45°/90°.
  3. ∠COD is 90° (the biggest ∠ in a 45°/45°/90° triangle).
  4. How many degrees are there in a circle? 360°.
  5. If ∠COD is 90°, then that is how much of 360°?

90° = ***1***

360° = ***4*** That’s how much of the circle is shaded. Remember this.

* 1. The formula for the area of a circle is:
     + - What’s ***r***? The radius of the circle. Let’s mark it up. Add another point on CD, call it ***X,*** and connect it to the center.

B 2 C

2 O ***X*** 2

A 2 D

Is OX a radius of the circle? Yes, so ***r*** = ***OX***. The length must be ***1*** because:

* BC is 2 (as are AD, AB, CD),
* the line (not fully drawn) that goes through the center (O), the diameter, is also 2, AND
* the formula for the radius of a circle is ½ the diameter.
* So, the radius, ***OX,*** has to be ***1***.
  + - * We have our radius. Plug and chug.
      * 1 raised to any power is 1.
      * + We’re not done yet. That’s the area of the ENTIRE circle. We need the AREA of the SHADED PART of the circle. Remember that we calculated how much of the circle is shaded. Now let’s use that information:
      * Entire Circle Area:
      * Shaded Portion Area:

That’s the same thing as

A is the correct answer.

1. Coordinate plane problem with an equation of a line.
   1. Let’s write our given: Equation for line l is:
   2. What are we looking for? The ***slope*** of the line that is perpendicular to line l.
   3. First, notice that we can’t find the slope of the line in that form. Let’s get it into slope-intercept form (), where m = slope and b = y-intercept {y-coordinate of the point (x,y) where the line crosses the y-axis (vertical axis)}.

-x -x

* + - * Divide the entire equation by 3.

3y = -x + 12

3 3 3

* + - * Now that’s something that will give us slope.
      * Remember, is the ***slope*** of l, and we’re looking for the ***slope*** of the line that is perpendicular to line l.
      * What is the relationship between the slopes of two perpendicular lines? They are ***negative reciprocals*** of each other. That’s a fancy way of saying that if we have the ***slope*** of one line, we can change the sign (for example, negative to positive), flip the top and bottom numbers in the fraction, and we’ll have the ***slope*** of the perpendicular line.

= ***slope*** of l

= ***3*** = ***slope*** of the perpendicular line

* + - * Now what is that ***3*** in the form of the equation of the perpendicular line? It’s ***m***, the ***slope*** that we’re looking for.
      * Look at the answers. Do we see any with an ***m*** of ***3***? There’s only 1.

C is the correct answer.

* + - * C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfTripping point. Don’t get caught up because none of the answers had the y-intercepts (*b*’s) equal to 4. Look at the question again. Does it say anywhere that the y-intercept of the perpendicular line is 4? No. There could be perpendicular lines crossing line *l* at any point, and they all would have different y-intercepts. There is a line perpendicular to *l* that crosses the y-axis at (0,4) but again, that’s only one of many possible perpendicular lines.

1. Geometry Problem. DRAW THE PICTURE.
   1. Draw the triangle and givens, and label the 3rd side that we don’t know as ***x***.

***5*** ***5***

***x***

* 1. What are we looking for? The answer that could NOT represent the length of the side ***x***.
  2. Know the basic rule of triangles: The sum of any two sides of a triangle must be ***greater*** than the third side.
     + Try one out, let’s say . So we have a triangle with sides of ***5***, ***5*** and ***1***. Do these numbers satisfy the rule? Let’s see.
       - Is that true? Yes.
       - Also true.
       - Also true.
       - So works.
     + Looking at the answers, when , the rule is also satisfied. Forget about the .o7 on the they’re just trying to confuse us with the radical. If works, 7.07 will work.
     + That leaves only one potential ***x***, 10, that does not work.

E is the correct answer.

1. Word problem with lots of information. Probably a system of equations.
   1. Let’s write our givens:
      * + 2.8 million voters
        + Two candidates, ***I*** or ***II***
        + ***I*** received 28,000 more votes than ***II***.
   2. What are we looking for? The ***percentage*** (***%***) of all votes that are for ***I***.
   3. We need to find out ***I*** and ***II*** before finding any percentages.
   4. This is definitely a system of equations. So let’s make some.
      * + All the votes go to ***I*** or ***II***, and there are 2.8 million total votes, so:

***I*** + ***II*** = 2.8 million *Equation 1*

* + - * ***I*** **received** 28,000 **more** votes than ***II***.

***I*** **=** 28,000 **+** ***II***

***I*** = 28,000 + ***II*** *Equation 2*

* 1. Solve the system. Look at Equation 1. Let’s isolate ***II*** so we can get rid of it and be left with ***I***.
     + - ***I*** + ***II*** = 2.8 million Now subtract ***I*** from both sides.

***-I*** ***-I***

* + - * ***II*** = 2.8 million - ***I***
  1. Go to Equation 2 and plug in our new value of ***II***.
     + - ***I*** = 28,000 + ***II***

***I*** = 28,000 + 2.8 million - ***I***

***+I +I***

2***I*** =2,828,000 -0

* + - * 2***I*** =2,828,000

2 2

* + - * ***I*** = ***1,414,000***
  1. Not done yet. We need the ***%***. A percentage is just Part/Whole × 100.
     + - Part × 100 = ***I*** \_\_ x 100

Whole Total Number of Votes

* + - * ***1,414,000*** × 100 = ***%***

2,800,000

* + - * 0.505 × 100 = ***50.5%***

C is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. This is a weird radical problem.
   1. Let’s write our given: 2***p*** = 18
   2. What are we looking for? The value of ***p***.
   3. If both sides are equal, and both have a sign, can’t we just focus on what’s underneath the sign? Indeed. Here’s a simple example:
      * + If 4 = x, what does x have to be?
        + The 4 = 2, so 2 = x.
        + What’s the only number that works for x? 4. Isn’t that the same thing as what’s on the left side of the equation? Yes. It doesn’t matter that in the problem, we have a friend under the sign. Back to our problem.
        + 2***p*** = 18

2 2

***p*** = ***9***

9 is the correct answer.

1. Rounding Problem. Just write down the information to start.
   1. What’s 1.783 rounded to the nearest ***whole number***? It’s ***2***.
   2. What’s 1.*7*83 rounded to the nearest ***tenth***? It’s ***1.8***.
   3. How much bigger is ***2*** than ***1.8***?
      * + ***2*** - ***1.8*** = ***0.2***

.2 is the correct answer.

1. Probability Word Problem
   1. Let’s write our givens:
      * + 6 brown towels
        + 2/5 probability of picking a brown towel
   2. What are we looking for? ***Total*** towels.
   3. Now let’s write down our probability/proportion equation.
      * + 2 = Brown

5 ***Total***

* 1. Plug in our given.
     + - 2 = 6

5 ***Total***

* 1. Now cross multiply.
     + - 2(***Total***) = 6 × 5
       - 2(***Total***) = 30
       - ***Total*** = ***15***

***15*** Is the correct answer.

1. Geometry problem with friends.
   1. DRAW THE PICTURE, and write down the givens:

A B C D E

* + - * AD = 4.5
      * BE = 3.5
      * CD = 2
  1. What are we looking for? A possible value for the length of ***BC***.
  2. Now label the givens in our picture.

A B C D E

***BC*** 2

4.5

3.5

* 1. Can we get straight to BC? Nope. But we can start filling in the pieces and find ***AB*** through first finding AC.

A B C D E

***BC*** 2

4.5

* + - * AC + CD = AD

AC + 2 = 4.5

-2 -2

AC + 0 = 2.5

AC = 2.5

* 1. Now back to the full line.

A B C D E

2.5 2

4.5

3.5

***BC***

* 1. ***BC*** is constrained by 2 things, AC and BE. Let’s look at AC first.
     + - AC = 2.5 so AB + ***BC*** must equal 2.5.

AB + ***BC*** = 2.5

* 1. Pick a number for ***BC***, say 2. Seems like a good number.
     + - ***BC*** = ***2***

AB + ***BC*** = 2.5

AB + ***2*** = 2.5

-2 -2

AB = 0.5 So ***BC*** = ***2*** could work.

* + - * Now check it with BE.

A B C D E

2

3.5

***BC***

* + - * Looking at the line, ***BC*** + CD + DE must add up to 3.5. Put in our chosen value of ***2*** for ***BC*** to see if it works.

***BC*** + CD + DE = 3.5

***2*** + 2 + DE = 3.5

4 + DE = 3.5

See the problem here? DE would have to be ***negative*** 0.5. This can’t be right, at least not in our number line world. So ***BC*** = ***2*** is out.

* 1. Let’s go smaller for a potential ***BC***. How about ***1***.
     + - AB + ***BC*** = 2.5

AB + ***1*** = 2.5

-1 -1

AB = 1.5 So ***BC*** = ***1*** could work.

* + - * What about the BE constraint?

***BC*** + CD + DE = 3.5

***1*** + 2 + DE = 3.5

3 + DE = 3.5

-3 -3

DE = 0.5

Okay, ***BC*** = ***1*** DOES work. That’s one possible answer.

* 1. Basically, ***BC*** could be anything greater than 0 (it has be positive) and less than 1.5 (because BE is 3.5).

1. Word problem with no friends. Let’s make some friends.
   1. Write our givens and define some friends.
      * + 30 days in April
        + ***R*** : Rainy Days
        + ***S*** : Non Rainy Days (sunny)
        + 3 days of rain (***R***) for every 2 days of no rain (***S***)
        + We should be thinking “system of equations,” since we’ve got two friends (variables).
   2. What are we looking for? How many ***more*** rainy days there are than sunny days.
   3. Let’s find one equation. Use the April info: 30 days. Some days it rained (***R***), and some it didn’t (***S***).
      * + Rainy Days + Non-Rainy Days = Total Days in April

***R*** + ***S*** = 30

* + - * ***R*** +  ***S*** = 30 Equation 1
  1. Can we figure out ***R*** and ***S***? Of course.
     + - For every 3 days of rain, there are 2 days of no rain. Sounds like a ratio. 3:2 = ***R***:***S***

That’s not user-friendly. Let’s write it in fraction form.

* + - * ***R*** = 3 Much better. Now cross-multiply to get rid of the fractions.

***S*** 2

* + - * 2***R*** = 3***S*** Equation 2
  1. Now set up our system of equations.
     + - ***R*** +  ***S*** = 30 Equation 1
       - 2***R*** = 3***S*** Equation 2
  2. Isolate R in Equation 1.

***R*** +  ***S*** = 30

-***S*** -***S***

***R*** = 30 - ***S***

* 1. Substitute our new value of ***R*** into Equation 2.
     + - 2***R*** = 3***S***
       - 2(30 - ***S***) = 3***S***
       - 60 - 2***S*** = 3***S***

+ 2***S*** +2***S***

* + - * 60 = 5***S***
      * 60 = 5***S***

5 5

* + - * ***12*** = ***S*** (***Non rainy days***)
  1. Now let’s figure out our rainy days.
     + - ***R*** +  ***S*** = 30
       - ***R*** +  ***12*** = 30

-12 -12

* + - * ***R***  = 18 (***Rainy days***)
  1. So R is ***18***, and S is ***12***. What are we looking for again? How much greater the number of rainy days is than the number of days it didn’t rain. Sounds like a difference (subtraction). Let’s convert the “math speak” into an equation.
     + - ***# of rainy days*** was ***how much*** greater than ***# of non-rainy days***

***R*** = ***x*** + ***S***

We’re looking for that **x** value.

* + - * ***R*** = ***x*** ***+*** ***S***

-***S*** -***S***

* + - * ***x***  = ***R*** - ***S***
      * Plug in our ***R*** and ***S*** and finish up.
      * ***x***  = ***18*** - ***12***

***x***  = ***6***

***6*** is the correct answer.

* 1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfWe could save loads of time in solving this problem by making a factor-like tree:

30 Total Days in April

5 5 5 5 5 5 (2 non-rainy +3 rainy = 5)

***2*** ***3*** ***2*** ***3*** ***2*** ***3*** ***2*** ***3*** ***2*** ***3*** ***2*** ***3***  5 days split into rain and no rain days

* + - * The ***2***’s are all the non-rainy days, the ***3***’s are all the rainy days. Add up the ***2***’s and ***3***’s and subtract them.
      * ***2*** + ***2*** + ***2*** + ***2*** + ***2*** + ***2*** = 12
      * ***3*** + ***3*** + ***3*** + ***3*** + ***3*** + ***3*** = 18
      * 18 - 12 = ***6*** That’s the difference.

1. Sequencing Problem
   1. Let’s write our givens:
      * + Each term is greater than the one before it.
        + 3rd term is 17.
        + 6th term is 77.
   2. What are we looking for? The ***8th*** term.
   3. Start by drawing a number line and label it.
      * + How high should it go? How about 8, since we’re looking for the ***8th*** term.

*1st 2nd 3rd 4th 5th 6th 7th* ***8th***

* + - * Let’s write in our givens.

*1st 2nd 3rd 4th 5th 6th 7th* ***8th***

17 77

* + - * Remember, the spaces between terms are equal. Make a friend, ***x*** , and let ***x*** be the value of a space.

*1st* ***x*** *2nd* ***x*** *3rd* ***x*** *4th* ***x*** *5th* ***x*** *6th* ***x*** *7th* ***x*** ***8th***

17 77

* + - * How many ***x***’s are there between 17 and 77? There are 3 ***x***’s.
      * How far is it from 17 to 77? Subtract to find out:

77 - 17 = 60

* + - * The 3 ***x***’s are spread out evenly. So what is ***x***?
      * 3***x*** = 60
      * 3***x*** = 60

3 3

* + - * ***x*** = ***20*** That’s the value of our equal spaces.
  1. Now we know ***x***. How many x’s are there between the 6th term and the ***8th term***? Looks like 2 to me. So the ***8th term*** = 77 + 2***x***.
     + - ***8th term*** = 77 + 2***x*** Now sub in our value of ***x*.**
       - ***8th term*** = 77 + 2(***20***)
       - ***8th term*** = 77 + 40
       - ***8th term*** = ***117***

***117*** is the correct answer.

1. Absolute Value Problem
   1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfLet’s write our given: |***x*** 3| =
   2. What are we looking for? The ***least*** value of ***x***.
   3. Looks easy, right? The answer is 3.5. Not so fast… With absolute value equations, there are 2 answers. (When it’s a ≤ or a ≥, the answer is a range of ***x***’s, but we don’t have to worry about that here.)
   4. Solve the absolute value problem.
      * + |***x*** 3| = means ***x*** 3 =
        + Split that into two equations and solve them both for ***x***.
        + ***x*** - 3 = or ***x*** - 3 =

+3 +3 +3 +3

* + - * ***x*** = 3 or ***x*** =
      * Which one of these is the ***least***? . There’s our answer***\****.

is the correct answer.

***C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmf\* When you go to grid in your answer, make sure you convert the mixed fraction (2½) into an improper fraction (5/2) or a decimal (2.5). If you write in 2 ½ in the grid, the computers will read it as 21/2, (11½), which is wrong.***

1. Ugly Algebra Problem. We have a lot of friends but very few numbers.
   1. Let’s write our givens:.
      * + ***W***, ***X***, ***Y*** and ***Z*** are all ***different*** integers.
        + ***W*** ***X*** ***Y*** ***Z*** is a 4-digit number, so we know that ***W***, ***X***, ***Y*** and ***Z*** all have to be somewhere between 0 and 9.
   2. What are we looking for? The 4-digit number ***WXYZ.***
   3. Equation 1 has 4 variables, so this does not look like a nice starting point.
   4. Equations 2 and 3 both have two variables so this will be a better place to start. It looks like if we pick a value for ***Y***, we can figure out ***W*** in Equation 2, then ***Z*** in Equation 3. We can use all of those to figure out ***X*** in Equation 1. So, this is another reason why this is a good a starting point.
   5. Get started with values for ***Y***.
      * + Let ***Y*** = 0 and solve Equations 2 & 3.

***W*** = ***Y*** + 1 ***Z*** = ***W*** -5

***W*** = 0 + 1 ***Z*** = 1 -5

***W*** = 1 ***Z*** = -4

Can one of the integers be negative? Not in this problem. There would have to be another integer (a 5th one) in front, and it would be -1. There are only 4 digits, so Z can’t be negative. Y = 0 didn’t work. Keep going.

* + - * Let ***Y*** = 1 and repeat 2 & 3.

***W*** = ***Y*** + 1 ***Z*** = ***W*** -5

***W*** = 1 + 1 ***Z*** = 2 -5

***W*** = 2 ***Z*** = -3 Still negative. No dice.

* + - * Let y = 2 and repeat.

***W*** = ***Y*** + 1 ***Z*** = ***W*** -5

***W*** = 2 + 1 ***Z*** = 3 -5

***W*** = 3 ***Z*** = -2 Nope.

See a pattern here? When we increase ***Y*** by 1, ***Z*** increases by one. Let’s jump to ***Y*** = 4.

* + - * Let ***Y*** = 4 and repeat.

***W*** = ***Y*** + 1 ***Z*** = ***W*** -5

***W*** = 4 + 1 ***Z*** = 5 -5

***W*** = 5 ***Z*** = 0 That works. Now what about ***X***?

* 1. Now, we can use Equation 1 to find ***X***, since we now know 3 of our friends, ***Y***, ***W*** and ***Z***.

***X*** = ***W*** + ***Y*** + ***Z***

***X*** = 5 + 4 + 0

***X*** = 9 No negative here so we’re looking good.

* 1. What are we looking for? The 4-digit number ***WXYZ***. Plug in our values and find out the answer.

***W*** ***X*** ***Y*** ***Z***

***5*** ***9*** ***4*** ***0***

***5940*** is the correct answer. *(FYI: No comma on the bubble sheet)*

1. Nasty Geometry Problem. Start drawing.
   1. Label the picture and write down the givens. There’s one point on the picture without a letter; label it ***X***.
      * + CD = DE = EF = 10
        + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmf∆ADF and ∆BCE are both equilateral. KNOW WHAT THAT MEANS. It means that all the sides are equal. It also means the angles all have to be the same. The angles of a triangle add up to 180°, so each angle must be 60° (60° + 60° + 60° =180°). This is the key to the problem.

C

D

E

F

A

***X***

B

***10***

***10***

***10***

* 1. What are we looking for? The perimeter of the flag. That’s the sum of all the exterior edges. But we already knew that. In this case the ***Perimeter*** is the sum of AX, XB, BC, CF and FA.
     + - ***P*** = AX + XB + BC + CF + FA
  2. Let’s start filling in the pieces. CF is an easy one. It’s the sum of CD, DE and EF.
     + - CD + DE + EF = CF
       - 10 + 10 + 10 = CF
       - 30 = CF That’s one piece of ***P***. 4 more and we’re good.
  3. Back to the big equilateral triangles. Let’s look at ∆ADF. We said before that since it’s an equilateral triangle, all the sides and angles are the same. Do we know the length of any of the sides of ∆ADF or ∆BCE? Indeed.
     + - AD = DF = FA BC = CE = EB
       - DF = DE + EF CE = CD + DE
       - DF = 10 + 10 CE = 10 + 10
       - DF = 20 CE = 20
       - AD = 20 BC = 20
       - FA = 20 EB = 20
       - MARK ALL OF THOSE ON THE DIAGRAM.

C

D

E

F

A

***X***

B

***20***

***20***

***10***

***10***

***10***

***20***

***20***

***20***

***20***

* + - * Now we know CF, FA and BC. Just need ***AX*** and ***XB***.
  1. We can find ***AX*** and ***XB*** by looking at the small triangle, ∆DE***X***. What are those angles? Well we know that ∆DE***X*** is an equilateral triangle, so ∠ADF is 60°. ∠***X***DE is the same as ∠ADF, so it is also 60°. The same logic applies to ∠CEB (in big triangle) and ∠DE***X*** (little triangle).
  2. So in ∆DE***X***, we have two angles that are 60°. What must the 3rd angle (∠***X***ED) be? That’s right, 60°. So ∆DE***X*** is an equiangular triangle. That means it’s also equilateral.
     + - DE = E***X*** = ***X***D We were given DE = 10.
       - 10 = E***X*** = ***X***D
       - E***X*** = 10
       - ***X***D = 10 Now label that info!

C

D

E

F

A

***X***

B

***20***

***20***

***10***

***10***

***10***

***10***

***10***

* 1. Find A***X*** and ***X***B and finish up.
     + - First ***X***B

***X***B + ***X***E = BE

***X***B + 10 = 20 Subtract 10 from both sides.

***X***B = 10

* + - * Now A***X***

A***X*** + ***X***D = AD

A***X*** + 10 = 20 Subtract 10 from both sides.

A***X*** = 10

* 1. Now we’ve got all the pieces of the ***Perimeter*** (***P***).
     + - ***P*** = A***X*** + ***X***B + BC + CF + FA
       - ***P*** = 10 + 10 + 20 + 30 + 20
       - ***P*** = ***90***

***90*** is the correct answer.

1. What is this problem? Geometry with algebra. Last problem in section, so we know it’s going to be tough. Remember, write down something even if you don’t know the answer for sure.
   1. Look at the graph. We’ve got some definable points here. Let’s label them.

(3,0)

(-3,0)

(0,-4)

* 1. What are we looking for? The value of ***a***. Yuck.
  2. What’s our function?
     + - g(x) = k(x+3)(x-3) This is the parabola shown.
       - g(x) is just the y-coordinate in a point (x,y) that lies on the function.
  3. Check back on the given info
     + - g(***a*** -1.2) = 0
       - ***a*** › 0
       - This info just means that when we substitute ***a*** -1.2 for x in the function, the y-value of the point is zero. Don’t we have two of those? Yes. Good thing we labeled the graph.
       - (-3, 0) and (3,0) both satisfy the requirement. Which one of those has a positive value for the x-coordinate? Only one, (3,0).
  4. Now the tricky part, we’re not done yet.
     + - C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfIn the expression g(***a*** -1.2), the x coordinate equals ***a*** -1.2, NOT JUST ***a***.
       - We said above our point that satisfies the equation is (3,0), so:
       - ***a*** -1.2 is our x, which in this point is 3
       - ***a*** -1.2 = 3

+1.2 +1.2

* + - * ***a*** = ***4.2***

***4.2*** is the correct answer.

# Test 1 Section 8

1. Analyzing a graph.
   1. Let’s write our givens: There are 5 categories and 2 levels of prizes.
   2. Look at the two prize levels: Prize Ribbons and Honorable Mention Ribbons
   3. What are we looking for? ***Total Honorable Mention Ribbons***.
   4. Count the ***Honorable Mention Ribbons*** in each category.
      * + Painting + Pottery + Photography + Metalwork + Silkscreen = ***Total***

4 + 2 + 5 + 1 + 1 = ***Total***

***13*** = ***Total Honorable Mention Ribbons***

D is the correct answer.

1. Geometry fun. MARK UP THE DRAWING.
   1. Let’s write our givens:
      * + BD and CE are diameters. That means that AC, AB, AD and AE are all radii of the circle.
        + ***BC*** = ***4***
        + ***AD*** = ***6***
   2. What are we looking for? A ***true*** statement.
   3. Label the picture. Because BD and CE are diameters, we know that AC, AB, AD and AE are all radii of the circle, which means they are equal. AC = AB = ***AD*** = AE = ***6***

A

B

D

C

E

***6***

***4***

6

6

6

* 1. Remember vertical angles?***\**** ∠CAB = ∠EAD

A

B

D

C

E

***6***

***4***

6

6

6

* 1. The SAS (Side Angle Side) Rule for congruent triangles means that ∆ABC = ∆ADE, so ED = ***BC*** = ***4***

A

B

D

C

E

***6***

***4***

6

6

6

4

* 1. Now that our picture is complete, let’s look at our choices.
     1. Can’t be true because CA = 6.
     2. Can’t be true because ED = 4.
     3. Nope. BA = 6 That’s greater than 4.
     4. Nope. CA = 6
     5. Only one left. And it happens to be true.

E is the correct answer.

\**Vertical Angles are angles formed by 2 straight lines intersecting at a point. The opposite angles are known as vertical angles and are always equal.*

1. Algebra Problem. Friends in a weird situation.
   1. Let’s write our givens: ii) What are we looking for? The value of:

***a***

***b***

***c***

***5***

***2***

***6***

* = ***ab*** - ***ac*** + ***c***
  1. Don’t get thrown by the boxes. If we said x ☺ y = x + y, and x = 1 and y =2, then x ☺ y = 1 + 2 = 3. The symbol could be anything. It just means it’s a function of some sort.
  2. Plug and Chug!

***5***

***2***

***6***

***a***

***b***

***c***

* = ***ab*** - ***ac*** + ***c***
* = ***52*** - ***5***(***6***) + ***6***
* = 25 - 30 + 6
* = ***1***

A is the correct answer.

1. Coordinate plane with a square.
   1. Let’s write our givens:
      * + We have a square.
        + Opposite points are at (-2, -2) and (2,2).
   2. What are we looking for? The ***area*** of that square.
   3. DRAW THE PICTURE. What are the two other ***points***?

***(-2,2)***

***(2,2)***

***(-2,-2)***

***(-2,2)***

* 1. How do we figure area of a square? Side × Side, Length × Width, Edge × Edge, however you’d like to say it. What’s the length of a side?
     + - Use points ***(-2,2)*** and ***(2,2)***. The x values are different, so subtract those for the length.
       - ***2*** - (***-2***) = ***4***. That’s our side.
       - Area = Side × Side

Area = 4 × 4

Area = ***16***

C is the correct answer.

1. Logic problem with family members. If you get this one easily, take the LSAT in a few years.
   1. Let’s write our givens:
      * + 4 kids are ***Owen***, ***Chadd***, ***Steph*** and ***Daria***
        + ***Chadd*** is not the youngest or oldest.
        + ***Daria*** is one of the two older kids.
        + ***Steph*** is the ***youngest.***
        + ***Owen*** is taken care of by his older brother or sister.
   2. What are we looking for? The oldest child.
   3. Draw the skeleton and sketch out the givens.

***Steph***

***Daria?***

***Owen or Chadd?***

***Oldest***

***Youngest***

* 1. Just by looking at this, we know ***Chadd*** isn’t the oldest(given), and neither is ***Owen*** (because he’s taken care of by his ***older*** brother and sister). So, ***Daria*** is the oldest.

B is the correct answer.

1. Parallel Lines Problem. Know the rules!

›

›

***y°***

***x°***

Q

R

P

S

* 1. Mark up the drawing
  2. What are we looking for? The ***value*** of *2(x + y* ).
  3. How many degrees are in a 4-sided figure? Know it. ***360°.***
  4. Let’s assume PQRS is a parallelogram. That means opposite angles are equal. Now we have:

***y°***

***x°***

Q

R

P

S

›

›

***y°***

***x°***

* 1. Write the equation and finish up.
     + - ***x°*** + ***x°*** + ***y°*** + ***y°*** = ***360°*** We can forget about the degrees since it’s all in degrees.
       - ***2x*** + ***2y*** = ***360*** Looks close to what we’re trying to find. Factor out the 2.
       - ***2(x*** + ***y)*** = ***360***

E is the correct answer.

1. Algebra Problem. Word party with friends. This is all about converting “math speak.”
   1. Let’s write our givens:
      * + ***x***, ***y*** and ***z*** are positive.
        + The average of ***x***, ***y*** and ***z*** is ***12***.
        + The greatest number subtracted from the sum of the other two numbers results in 4.
        + ***x*** < ***y*** < ***z***
   2. What are we looking for? The ***system of equations*** that reflects the above information. Translate the “math speak.”
   3. First what’s an average? The sum of the terms, divided by the number (***n***) of terms.
      * + ***x*** + ***y*** + ***z*** = ***12*** What is (***n***)? The number of terms, so ***3*** in this problem.

***n***

* + - * ***x*** + ***y*** + ***z*** = ***12*** That doesn’t look like our answers. What about moving the ***3***?

***3***

* + - * ***x*** + ***y*** + ***z*** × 3 = ***12*** × 3

***3***

* + - * ***x*** + ***y*** + ***z*** = ***36*** That’s **Equation 1**. This means (C) and (D) are out.
  1. Let’s find the next equation. Go to the next given.
     + - Use our condition (***x*** < ***y*** < ***z***) to help here.

***z*** = largest ***y*** = middle ***x*** = smallest

* + - * The ***greatest number*** subtracted from the ***sum*** of the other ***two numbers*** ***results in*** ***4***.

***z*** (-) (+) ***x*** ***y = 4.***

* + - * That’s the same as:

The ***sum*** of the other ***two numbers*** minus the ***greatest number*** ***results in*** ***4***.

***x*** ***+*** ***y*** - ***z***  ***= 4.***

* + - * ***x*** + ***y*** - ***z*** = 4 **Equation 2**
      * Now we have: ***x*** + ***y*** + ***z*** = ***36***

***x*** + ***y*** - ***z*** = ***4***

A is the correct answer.

1. Exponents and friends. Remember our exponent rules.
   1. Let’s write our givens:
      * + ***x*** and ***y*** are positive integers.
        + 32***x*** × 32***y*** = 81
   2. What are we looking for? The ***value*** of ***x*** ***+*** ***y***.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfLook at the second given. It should hit us that 81 is a perfect square that is equal to ***9*** × ***9***. Let’s see if that takes us anywhere.
      * + 32***x*** × 32***y*** = 81
        + 9 × 9 = 81 Set the two terms with exponents equal to 9.
        + 32***x*** = 9 32***y*** = 9 Can we solve that? What’s 9 with a base of 3? It’s 32.

32***x*** = 32 32***y*** = 32 Bases are the same so exponents have to be the same.

* + - * 2***x*** = 2 2***y*** = 2 Divide both equations by 2.
      * ***x***  = 1 ***y*** = 1
      * ***x*** + ***y*** = ***?***
      * ***1*** + ***1*** = ***2***

B is the correct answer.

1. Graphing Question. Watch the words.
   1. What are we looking for? The ***x-value*** that gives the ***function*** its maximum value.
   2. What does maximum value of the function mean? The highest point. Looks like point ***z*** below.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfIt looks like z is 5 tick marks up, so the answer is 5 (C), right? NO! 5 is the ***y-value*** of the point, and we’re looking for the ***x-value*** of the point.

8

-8

8

-8

***z***

* 1. What’s the ***x-value*** of point z? It’s ***4***.

B is the correct answer.

1. Algebra problem with more friends. This section is a party.
   1. Let’s write our givens:
      * + ***k*** = ***x***/3
        + ***x*** ≠ 0
   2. What are we looking for? 3***x*** in terms of ***k***.
   3. Seems like if we find ***x***, we can multiply it by 3 and get our answer. Rearrange the first given.
      * + ***k*** = ***x***/3

***k*** × 3 = ***x***/3 × 3

* + - * 3***k*** = ***x*** We’ve got ***x;*** let’s find 3***x*** in terms of ***k***.
      * ***x*** = 3***k***
      * ***x*** × 3 = 3***k*** × 3
      * 3***x*** = 9***k***

B is the correct answer.

1. Geometry Word Problem
   1. Let’s write our givens:
      * + ***Cube***. This means all edges are the same length, and each face has the same area.
        + 2 faces painted black
        + Total area of white faces is 64 square inches.
   2. Sketch a cube and label the colored sides.
   3. What are we looking for? Volume of the cube.

What’s ***Volume***? ***Length*** × ***Width*** × ***Height***.

* 1. How do we find the edge? Work backwards from white faces. How many white faces are there?

4 (Front, Left, Bottom, Back).

* 1. Find the area of one white face.
     + - Total area = 64 Divide that by 4 white faces.
       - Total area = 64
       - 4 4
       - Area of one white face = ***16*** That’s ***Length*** × ***Width***. We need one edge.
       - The area of a square = edge2. Figure out length of an edge.
       - 16 = edge2
       - 4 = edge 4 is an edge.
       - 4 = ***Length*** = ***Width*** = ***Height*** In a cube all edges are the same. Solve.
       - ***Length*** × ***Width*** × ***Height*** = ***Volume***
       - ***4***  ×  ***4***  × ***4***  = ***Volume***
       - ***64*** = ***Volume***

A is the correct answer.

1. Number line problem with friends and fractions!
   1. Don’t we know that each space is ¼? Yes. Count the spaces between -1 and 0 (also 0 and 1). There are 4. So each hash mark is ¼.
   2. Draw a number line, and give our friends some values.

***-3/4***

***-1/2***

***3/4***

***1/4***

-1

1

0

***y***

***x***

***w***

***v***

* 1. What are we looking for? The expression that gives the ***least*** value. Let’s get working.
  2. Plug in the values from our number line to check the values in the answer choices.
     1. ***v*** + ***y***
        + ***-3/4*** + ***3/4***
        + ***0*** We’ve got one number, but we need more to see which is the least.
     2. ***v*** + ***x***
        + ***-3/4*** + ***1/4***
        + ***-1/2*** Lower than zero, so we have a new leader.
     3. ***w*** + ***x***
        + ***-1/2*** + ***1/4***
        + ***-1/4*** A close second. (B) is still the least.
     4. ***v*** - ***w***
        + ***-3/4*** - ***-1/2*** Negative minus negative is same as adding.
        + ***-3/4*** + ***1/2***

***-1/4*** Same as (C). (B) still lower.

* + 1. ***y*** - ***x***
       - ***3/4*** - ***1/4***
       - ***1/2*** Positive. So (E) is out. ***v*** + ***x, answer*** (B), gives the least value.

B is the correct answer.

1. Median Problem
   1. KNOW THE DEFINITION OF MEDIAN. Median is the middle number in a set of numbers (ordered least to greatest) with an odd number of terms, and the average of the two middle numbers when the set has an even number of terms. It is all about ***position***.
   2. Let’s write our givens:
      * + ***n*** is an integer.
        + The new number of terms will be 7.
   3. What are we looking for? Potential medians.
   4. Let’s look at (I), 6 as a possibility. What if ***n*** = ***2***. Our set would look like:

*1st 2nd 3rd 4th 5th 6th 7th*

* + - * ***2*** , 3 , 4 , 6 , 7 , 10 , 12
      * What’s the median of a 7-term set? It’s the term in the middle, or the 4th term.
      * Median is 6. So (I) is a good answer. That means (B) and (C) are out.
  1. Let’s look at (II), 6½. Is there any way we can have a fraction in a median of a set of integers? Of course, but only when there are an ***even*** number of terms. How many do we have? 7 terms. That’s odd in my book. 6½ is not a possible answer.***\**** So, (E) is also out. Down to (A) and (D). Let’s check (III).
  2. Can 7 be a median? Well, we’d need to get 7 to be the median in our new set. Possible? Indeed. Try ***n*** = ***9***, or any number 8 or greater.

*1st 2nd 3rd 4th 5th 6th 7th*

* + - * 3 , 4 , 6 , 7 , ***9*** , 10 , 12,
      * Looks like a winner. 3 terms below 7, 3 terms above it, 7 is the median. So (III) is a good answer as well. Only answer (D) works here.

D is the correct answer.

*\*If this is hard to grasp, think about a four-term set of the following numbers: 1,2,3,4. What’s the median? It’s not 2; it’s not 3; it’s somewhere in between. So you take the average of the middle two terms and that’s your median. 2+3 = 5; 5/2 or 2½ is the median.*

1. Logic Problem. Coloring Choices.
   1. Let’s write our givens:
      * + A ball with a 2-color design
        + 5 different colors available
   2. What are we looking for? Total number of different designs.
   3. Easiest way to solve this type of problem is to define 5 colors (how about 1-5?) and make a table.

***1 2 3 4 5***

|  |  |
| --- | --- |
| Ball Color 1 | Ball Color 2 |
| ***1*** | ***2*** |
| ***1*** | ***3*** |
| ***1*** | ***4*** |
| ***1*** | ***5*** |

|  |  |
| --- | --- |
| Ball Color 1 | Ball Color 2 |
| ***2*** | ***1*** |
| ***2*** | ***3*** |
| ***2*** | ***4*** |
| ***2*** | ***5*** |

* 1. See a pattern here? Every color can be paired with 4 other colors. If using color ***1*** we get ***4*** different designs, and with color ***2***, we get ***4*** designs, isn’t it safe to say that colors ***3***, ***4***, and ***5*** will also produce ***4*** different designs? We could complete a table for all of the colors, but that would take up more time, and it’s not needed if we recognize the pattern above.
  2. Figure out the total number of different designs.
     + - ***5*** colors of ***4*** designs each.
       - ***5 × 4 = 20***

B is the correct answer.

1. Geometry Word Problem. DRAW A PICTURE.
   1. Let’s write our givens:
      * + Figure is a rectangle.
        + ***Length*** is increased by 30%.
        + ***Width*** is decreased by 30%.
   2. What are we looking for? The ***effect on area*** after satisfying the conditions. What’s area? Length × Width.
   3. Sketch out the rectangle. Let’s also give our length and width some values and call them our “original numbers”. Since we’re working with percentages, let’s use multiples of 100. Let ***Length*** = 200 and ***Width*** = 100, and see what happens.

100

200

* 1. Using our givens and original numbers, let’s find the new ***Length*** and ***Width***.
     + - ***Length*** is increased by 30%
       - ***Length***(New) = ***Length*** + 30%(***Length***)
       - ***Length***(New) = 200 + 30%×(200)
       - ***Length***(New) = 200 + 60
       - ***Length***(New) = 260
       - ***Width*** is decreased by 30%
       - ***Width***(New) = ***Width*** **-** 30%(***Width***)
       - ***Width***(New) = 100 **-** 30%×(100)
       - ***Width***(New) = 100 - 30
       - ***Width***(New) = 70
  2. Now we’ve got an original and new set of Lengths and Widths. Calculate the areas and finish up.
     + - Original: ***Length*** x ***Width*** = Original Area
       - 200 x 100 = 20,000
       - New : ***Length*** x ***Width*** = Original Area
       - 260 x 70 = 18,200
  3. Original area is bigger, so subtract the new area from the original to get the difference.
     + - Original - New = Difference
       - 20,000 - 18,200 = 1,800
  4. But our answers are in percentages. How do we convert that 1,800 into a percentage of the original? Just make a fraction, and put the difference in the numerator, and the original in the denominator:
     + - Difference x 100 = Percentage

Original

* + - * 1,800 x 100 = Percentage

20,000

* + - * .09 x 100 =  ***9%*** Up or down? New area got smaller, so it’s a ***decrease***.

E is the correct answer.

1. Algebra, exponents, friends. Trouble.
   1. Let’s write our givens:
      * + n(***t***) gives the number of bees for a ***day*** (***t***).
        + ***k*** is a constant.
        + 0 ≤ ***t*** ≤ 99
   2. What are we looking for? What ***day*** (or value of ***t***) were there the same number of bees as day ***10***?
   3. Where to begin? First off, look at ***k***. Do we really care what that number is? Not really, because the meat of this function is with the ***t*** values, the k just gets added on at the end. It will have the same effect on any ***t***. Let’s just leave ***k*** dangling for now.
   4. So let’s find the number of bees on day 10. Use our function and find the value for day ***10 (*** ***t*** = ***10)***.
      * + n(***t***) = ***t***2 - 20***t*** + ***k***

2

* + - * n(***10***) = (***10***)2 - 20(***10***) + ***k***

2

* + - * n(***10***) = 100 - 200 + ***k***

2

* + - * n(***10***) = 50 - 200 + ***k***
      * n(***10***) = -150 + ***k*** Is there any way that k can be less than 150? No, because that

would make n(t) negative, which means there would be negative

bees. No can do. Let’s say ***k*** =***200*** for our problem.

* n(***10***) = -150 + ***200***
* n(***10***) = ***50*** So on ***day*** ***10*** (***t*** = ***10***), we have ***50*** bees.
  1. Now we need to find another ***t*** that gives us ***50*** bees. Let’s use our answers to help guide us.
     1. t= 20
        + n(***20***) = (***20***)2 - 20(***20***) + ***200***

2

* + - * n(***20***) = 400 - 400 + ***200***

2

* + - * n(***20***) = 200 - 400 + ***200***
      * n(***20***) = 0 Not equal to 50 bees. (A) is out.
    1. t= 30
       - n(***30***) = (***30***)2 - 20(***30***) + ***200***

2

* + - * n(***30***) = 900 - 600 + ***200***

2

* + - * n(***30***) = 450 - 600 + ***200***
      * n(***30***) = ***50*** Equals ***50*** bees on day ***30***. (B) works. The answer is day

***30***. Check the others, but in doing the math correctly we’re done at answer B.

B is the correct answer.

# Test 2 Section 2

1. Sequencing problem starting with “math speak.”
   1. Let’s write our givens:
      * + First term is 1.
        + Each term is 2 more than twice the previous term.
        + Sequence is ***1, 4, 10***, ***t***, 46.
   2. What are we looking for? The value of ***t***.
   3. Convert the “math speak.”
      * + Each term after the first is ***2 more*** than ***twice*** the previous term.
        + next term = ***2 +*** ***2 ×*** previous term
        + Next term = 2 + 2x(previous term)
   4. Test it out on the second term (***4***) to see if our equation works.
      * + Next term = 2 + 2×(previous term)
        + ***4*** = 2 + 2×(***1***)

***4*** = 2 + 2

***4*** = 4 Looks good. Now figure out ***t*** using 10.

* ***t*** = 2 + 2×(***10***)
* ***t*** = 2 + 20
* ***t*** = 22

D is the correct answer.

1. Word Problem
   1. Let’s write our given: Machine fills 24 cartons in 1 hour.
   2. What are we looking for? The number of ***cartons*** that can be filled in ***5*** minutes.
   3. How many times does 5 minutes go into 1 hour? ***60*** minutes in an hour, right? So:
      * + 1 hour = 60 minutes
        + ***60*** minutes = ***12*** (***5***-minute periods in ***60*** minutes)

***5*** minutes

* + - * So, if in 12 ***5***-minute periods we fill 24 cartons, how do we get to one ***5***-minute period? Divide.
      * 24 cartons (in ***60*** minutes) .

12 (***5***-minute periods in ***60*** minutes)

* + - * 24 = ***2 cartons*** in one ***5***-minute period

12

A is the correct answer.

1. Data analysis with a line graph.
   1. Understand the graph. Cars on the y-axis; months on the x-axis; the line shows the trend of cars sold.
   2. What are we looking for? How many more cars were sold in May than in January and February combined?
   3. Convert the “math speak.”
      * + ***How many more cars*** were sold in ***May*** than in ***Jan and Feb combined***?
        + ***x*** ***+*** ***May*** ***cars*** ***(Jan + Feb)***.
        + How do we make this an equation? Which value is higher? ***May*** cars or (***Jan + Feb***) cars? ***May*** cars, so the ***x*** has to be added to (***Jan + Feb***) to find out how many more were sold in ***May***.
        + ***x*** ***+*** ***( Jan + Feb )*** = ***May*** Now plug and chug.
        + ***x*** ***+*** ***( 20 + 18 )*** = ***48***
        + ***x*** ***+***  ***38***  = ***48***

-38 -38

* + - * ***x***  = ***10***

A is the correct answer.

1. Data analysis with a line graph continued, plus some geometry.
   1. We’re told to put the 6 months of data in a circle graph. DRAW THE PICTURE.
   2. What are we looking for?
      * + The ***central angle*** for the ***April*** sector.
        + What’s a central angle? It’s the angle formed by two radii of a circle.

That’s a central angle

* 1. Draw the picture. We’re looking for ***April***.

***20***

***18***

***22***

***30***

***42***

***48***

***Jan***

***Feb***

***Mar***

***Apr***

***Jun***

***May***

* 1. How do we get from numbers to an angle?
     + - How many total degrees in a circle? 360°.
       - How many total cars do we have? Add ‘em up.

***20*** + ***18*** + ***22*** + ***30*** + ***48*** + ***42*** = 180 Total Cars

* + - * Set up the ratio to find the ***April central angle***.
      * ***April°***  = ***30 April Cars***

360° 180 Total Cars

* + - * ***April°***  × 360 = ***30***  × 360 360/360 = 1 on left, 360/180 = 2 on the right.

360° 180

* + - * ***April°*** = ***30*** × 2
      * ***April °*** = ***60°***

C is the correct answer.

1. Geometry Problem
   1. Know that rotating something counterclockwise by 90° is turning the object left by 90°.
   2. Pick one of the colored areas to focus on when you turn this thing. Let’s focus on the ***green*** area.

Rotate it:

***Counterclockwise***

* + - * Staying with the green, it looks like we have two blocks pointing up on the left side. Answers (A), (B), and (E) are out. We’re down to 2 answers.
      * Look at the ***blue*** areas. Look the same in both (C) and (D). Next color.
      * Look at the ***red*** areas. We’ve found it. How many ***red*** blocks are in the original? 3. (C) has only 2. We’re left with (D), as it has 3 ***red*** blocks.

D is the correct answer.

1. Pure “math speak” problem.
   1. What are we looking for?
      * + ***4 times*** ***a number***. We should underline that in the question. Looks like a long lost friend. Let’s define the ***number*** and call it ***x***.
   2. Now let’s translate the “math speak”. Do it in two parts, as the sentence seems to have two parts.
      * + ***3*** ***more*** than ***twice*** a ***number*** ***is equal*** to ***10***
          - ***3*** ***+*** ***2×*** ***x*** ***=*** ***10***
          - ***3*** ***+*** ***2x*** ***=*** ***10***
          - ***3*** ***+*** ***2x*** ***=*** ***10***

-3 -3

* + - * + ***2x*** ***=*** 7 Divide both sides by 2.
* ***x*** ***=***
  + - * Now do the second part of the sentence and finish up.
* ***4 times*** ***a number***
* ***4 × x***
* ***4x***
* ***4***() = ***28/2*** = ***14***

D is the correct answer.

1. Sequencing problem with friends.
   1. Let’s write our given: ***a*** < 0
   2. What are we looking for? The ***greatest number***.
   3. Define ***a*** so that it satisfies the given condition. Remember, we like simple. Let’s say ***a*** = ***-1***.
   4. Plug and chug.
      * + ***a***, 2***a***, 4***a***, 8***a***
        + ***-1***, 2(***-1***), 4(***-1***), 8(***-1***)
        + ***-1***, -2, -4, -8 Out of those four what is the greatest? ***-1***.
        + ***-1 = a ,*** so ***a*** is the greatest***.***

A is the correct answer.

1. Geometry Problem
   1. Draw the picture. Draw an extra line to help us out.

***2***

***4***

***4***

***6***

***2***

***2***

***4***

***4***

***6***

* 1. What are we looking for? The ***total area*** of the figure Can wefigure outthe length of the ***blue*** line? Of course. Look at all those right angles.
     + - The ***blue*** line is ***6*** - ***4*** = ***2*** Label it.
  2. Because of those right angles, we now have two rectangles: ***2*** by ***2***, and ***6*** by ***4***.
  3. Plug and chug.
     + - ***Total Area*** = Area of Rectangle 1 + Area of Rectangle 2
       - The area of a rectangle is length × width.
       - Rectangle 1 = ***2*** × ***2*** = 4
       - Rectangle 2 = ***6*** × ***4*** = 24
       - ***Total Area*** = 24 + 4
       - ***Total Area*** = ***28***

B is the correct answer.

1. Algebra problem with friends.
   1. Let’s write our givens:
      * + (***x*** - 2)2 = 25
        + ***x*** < 0
   2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfWhat are we looking for? ***x***
   3. We can FOIL this thing out and simplify to get our answer, but let’s take a shortcut.
   4. See a perfect square in the first given? Yes, 25. What’s the square root of 25? There are two, -5 and 5. That means that the (***x*** - 2) has to equal either -5 or 5. Is there any way you can get a ***positive*** 5 by plugging in any of the ***negative*** answers in for ***x***? Nope. So the (***x*** - 2) = -5.
   5. Plug and chug.
      * + ***x*** - 2 = -5

+2 +2

* + - * ***x*** = -3

D is the correct answer.

1. Triangles and some angular friends. Study the picture well.
   1. Let’s write our givens:
      * + ∠QTP = x°, and ∠RSP = x°, so ∠QTP = ∠RSP
        + ***QT*** = 8
        + ***RS*** = 10
   2. What is the relationship between these triangles? They share ∠P, and we were given ∠QTP = ∠RSP. If two of the three angles are the same, don’t the third angles, ∠R and ∠S, also have to be equal? Yes, they do. That means that these triangles are similar in the triangle sense. Corresponding sides of similar triangles are proportional.
   3. What are we looking for? ***PT/PS***.
   4. Let’s make our proportion equation using the given ***QT*** and ***RS***. Make sure the edges of the big ∆PRS (and little ∆PQT) are lined up ***across*** the proportion equation.
      * + ***PT = QT*** (*Little ∆PQT*)

***PS RS*** (*Big ∆PRS*)

* + - * ***PT = 8*** Simplify the fraction. (Divide top and bottom by 2.)

***PS 10***

* + - * ***PT/PS = 4/5***

E is the correct answer.

1. Graphical analysis with friends other than x.
   1. Let’s write our givens:
      * + ***L*** is length, and is plotted on ***y***-axis
        + ***W*** is week, and is plotted on ***x***-axis
   2. What are we looking for? The ***equation*** that best fits the data. Sounds like a function, right? Indeed.
   3. What’s the equation of a line? y = ***m***x + ***b***. What do those mean again?
      * + ***y*** = y-value of a point on the line
        + ***x*** = x-value of a point on the line
        + ***m*** = slope We know slope is rise/run.
        + ***b*** = y-intercept y-value of the point where the line crosses the y-axis (vertical axis)
   4. See what they’ve done here? Tried to throw us off by replacing ***y*** with ***L*** and ***x*** with ***W***. Tricky.
      * + ***C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfy*** = ***mx*** + ***b*** Generic Equation
        + ***L*** = ***mW*** + ***b*** Equation for this problem
        + Find ***m*** and ***b*** and we’re good to go.
   5. Connect the dots with a straight edge of a calculator or spare pencil. See anything helpful?
      * + We can see the line crosses the y-axis at point (0,***0***) That means our b = ***0***.
   6. What about our ***m***? Pick two points on the line and figure out the slope. Make it easy; use (0,0) and the first point on the graph, (1, 10).
      * + ***m*** = Rise = y2 - y1

Run x2 - x1

* + - * y2 - y1  = (10 - ***0***) = 10 = ***10***

x2 - x1 (1 - 0) 1

* + - * ***m*** = ***10***
      * Plug those into our equation and we’re good.
      * ***L*** = ***mW*** + ***b***

***L*** = ***10W*** + ***0***

***L*** = ***10W*** is our ***equation***.

D is the correct answer.

1. Pure knowledge of math vocab question, with a friend for good measure.
   1. Let’s write our givens:
      * + ***Only*** Mode is 5 Mode is the number that appears the most in a series.
        + ***Median*** is 6 Median is the middle number in a series that goes from lowest to highest.
   2. What are we looking for? An answer that ***does not work*** for n.
   3. First, re-write the series in order. Leave n out because that’s what we’re trying to figure out.
      * + 5 , 5 , 5 , 6, 6, 6, 7 and n somewhere in there
   4. Now try the different answers and check the conditions.
      1. n = ***6***. 5, 5 , 5 , 5 , 6, 6, 6, ***6***, 7
         * Mode: There are four 5’s and four 6’s. That’s 2 modes. We’re told the ***only*** mode is 5. (A) ***does not work*** for n. We hit it on the first try. Check the others to be sure.
      2. n = ***7***. 5, 5 , 5 , 5 , 6, 6, 6, 7, ***7***
         * Mode: There are four 5’s. The mode is 5. So far so good. What about ***Median***?
         * ***Median***: The term in the middle is 6. That works.
      3. n = ***8***. 5, 5 , 5 , 5 , 6, 6, 6, 7, ***8***
         * Mode: Same thing as (B). It works.
         * ***Median***: Same thing as (B). It works.
         * Look at the numbers. Anything 7 or higher works, so you know (B) through (E) are not answers.

A is the correct answer.

1. Another Vahndahful Venn.
   1. What are we looking for? The ***elements in the intersection*** of ***Y*** and ***Z***.
   2. First, outline the intersection. The intersection is only where these two circles overlap, and the number elements are enclosed by ***both*** the ***Y*** and ***Z*** circles. Forget about circle X, that’s just there to throw us off. We’ve left in the ***5***, ***2***, and ***4*** from circle X to show that those don’t factor into this at all.

Y

Z

***5***

***2***

***3***

***5***

***4***

***3***

***7***

* 1. There are only two numbers, ***3*** and ***7***, surrounded by both ***Y*** and ***Z***.
     + - Add those and you have our number of ***elements***.
       - ***3*** + ***7*** = ***10***

C is the correct answer.

1. Friends and exponents. Know the exponent rules!
   1. Let’s write our givens:
      * + m = ***t***3
        + ***w*** = m2 + m
   2. What are we looking for? ***w*** in terms of ***t***. We’re looking for an equation.
   3. Plug and chug! Since m = ***t***3 , plug in ***t***3 in for m in the second equation.
      * + ***w*** = m2 + m
        + ***w*** = (***t***3)2 + ***t***3
        + When raising an exponent by an exponent we just add the exponents, right? That gives us ***t***5 for our first term. NO! We multiply here. (If the expression were (***t***3)(***t***2) it would be ***t***5, because we *add* exponents (2 + 5) when we are multiplying like bases (***t*** in this case). Rules!)
        + ***w*** = ***t***(3x2) + ***t***3
        + ***w*** = ***t***6 + ***t***3

E is the correct answer.

1. Friends and symbols. Excellent.
   1. Let’s write our givens:
      * + ***x*** is a positive integer.
        + ***x***∆ = (x - 1)(x + 1)
   2. What are we looking for? Which answer is ***equal*** to ***6***∆ - ***5***∆.
   3. Don’t let the ***x***∆ throw you. It’s the same thing as f(x), it just means it’s a function. Know that whenever the ∆ is to the right of a variable (or number) you just plug the variable (or number) preceding the ∆ into the right side of the equation for x. That’s it.
   4. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfThe key to this problem is to keep it simple. Look at the (x - 1)(x + 1). That should ring a bell. It’s the factored version of Difference of Perfect Squares. x2 - y2 = (x + y)(x - y). Do we care that the negative is in the first parenthesis in our equation? Not at all. FOIL the right side of our equation.
      * + (x - 1)(x + 1) = x2 -1x + 1x - 1
        + The -1 ***x*** and + 1***x*** cancel each other out so we’re left with ***x*** 2 -1.
        + ***x***∆ = ***x***2 - 1
   5. Back to what we’re looking for. Which answer is ***equal*** to ***6***∆ - ***5***∆? First figure out the value of ***6***∆ - ***5***∆.
      * + ***6***∆ - ***5***∆ Split them up.
        + ***6***∆ - ***5***∆
        + 62 - 1 - (52 - 1) Plug in the number. (Remember the rule above.)
        + 36 - 1 - (25 - 1)
        + 35 - 24 = 11
        + ***6***∆ - ***5***∆ = 11. We’re looking for what else is ***equal*** to 11. Get going with the answers.
   6. Try the different answers with the ***x***∆ = x2 - 1.
      1. ***2***∆ + **1**∆
         * ***2***∆ + ***1***∆
         * 42 - 1 + (12 - 1)
         * 16 - 1 + (1 - 1)
         * 15 + 0 = 15. That’s not 11. (A) is out.
      2. ***3***∆ + **2**∆
         * ***3***∆ + ***2***∆
         * 32 - 1 + (22 - 1)
         * 9 - 1 + (4 - 1)
         * 8 + 3 = ***11***. That’s it. 3∆ + 2∆ = ***6***∆ - **5**∆

B is the correct answer.

* 1. Run through (C), (D) and (E) if you want, but if you work carefully, you’ll have confidence in your answers.

1. Pure friends problem.
   1. Let’s write our givens:
      * + x2/y is an integer.
        + x/y is ***NOT*** an integer.
        + An integer is a whole number. We know that.
   2. What are we looking for? Possible values of ***x*** and ***y***. Go to the answers, and plug and chug.
      1. x = 1, y = 1
         * x2/y x/y
         * 12/1 1/1
         * x2/y = 1 x/y = 1 Not the answer. x/y must ***NOT be*** an integer.
      2. x = 3, y = 2
         * x2/y x/y
         * 32/2 3/2
         * x2/y = 9/2 x/y = 3/2 (B) is out. x2/y has to be an integer.
      3. x = 4, y = 2
         * x2/y x/y
         * 42/2 4/2
         * x2/y = 16/2 = 8 xy/y = 2 (C) is out. x/y must ***NOT be*** an integer.
      4. ***x = 6, y = 4***
         * x2/y x/y
         * 62/4 6/4
         * x2/y = 36/4 x/y = 3/2 Looks like a winner. We’ve come this far; try (E).
      5. x = 9, y = 3
         * x2/y x/y
         * 92/3 x/y
         * 92/3 9/3
         * 81/3 3 Answer (E) is out. x/y must ***NOT be*** an integer.
         * x2/y = 27 x/y = 3

D is the correct answer.

1. Function problem with absolute value. No problem.
   1. Let’s write our given:
      * + ***y*** = -2***x*** + ***6*** is the equation of the given line.
   2. What are we looking for? The graph of ***y*** = |-2***x*** + ***6***|.
   3. Know the absolute value rules. Whenever you see those bars ||, we know that our “answer” (in this case the y-value for a coordinate on our new line) has to be positive. Don’t (A), (C) and (E) all have points below the x-axis? That means y is negative at those points. (A), (C) and (E) are all out.
   4. We’re down to (B) and (D). Let’s start with our y-intercept (***6*** in our ***y*** = m***x*** + ***6*** equation).
      * + ***y*** = m***x*** + ***b***
        + ***y*** = -2***x*** + ***6***
        + 6 is the y-intercept (the y-value of the point where the line crosses the y-axis). In both (B) and (D) this point is (0,6), so the y–intercept doesn’t help us much. Next.
   5. Look at both (B) and (D) and let’s pick another x-value to see which graph works. We know the point where x = 0 is the same on both graphs, let’s try it where ***x*** = ***1***. Pick out the points on the two graphs.
      * + In graph (B) when ***x*** = ***1***, ***y*** = ***4***, so our point is (***1***, ***4***). See if that works in our equation.
        + ***y*** = |-2***x*** + ***6***|
        + ***4*** = |-2(***1***) + ***6***|
        + ***4*** = | -2 + ***6***|
        + ***4*** = |4|
        + ***4*** = ***4*** That works. Do the same with (D) just to make sure we’re right.
        + In graph (D) when ***x*** = ***1***, ***y*** = ***8***. Our point is (***1***, ***8***). See if that works in our equation.
        + ***y*** = |-2***x*** + 6|
        + ***8*** = |-2(***1***) + 6|
        + ***8*** = | -2 + 6|
        + ***8*** = |4|
        + ***8*** = 4 Nope. (D) is out.

B is the correct answer.

* 1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfNotice that we didn’t really use the graphs in the explanation except to pick points other than the y-intercept to see if they worked in the equation. The other way to do this problem, after you eliminate (A), (C) and (E), is to know that when you throw || around a function, the line just “bounces” off of the x-axis (horizontal one) to stay above the x-axis. The || keep all the y-values positive, and the slope just becomes the negative of whatever it was in the original (in this case, -2 instead of 2).

1. Geometry with friends. This is a friend-heavy section.
   1. Let’s write our givens:
      * + We’re dealing with a right circular cylinder. Know what that means. Base is a circle. Cylinder is straight up and down. Think right angles.
        + Diameter is ***d.***
        + Height is ***h.***
   2. What are we looking for?
      * + ***Volume*** of the smallest rectangular box that holds the cylinder. What?
        + ***Volume*** = ***Base*** x Height***\****
        + ***Volume*** = ***Base*** × ***h***
        + Find the base of the box and we’re good to go. ***h*** isn’t going to change, so put it aside for a second.
   3. Draw some pictures. Let’s just look at the base.
      * + Aren’t those red lines both ***d***? Indeed. So what’s the smallest rectangle that will enclose that circle? A square with sides of length ***d***. (We know the rectangle is a square because the base is a circle.) So the square with sides ***d*** is our new base.

***Base***

***d***

***h***

***d***

***d***

***d***

***d***

* + - * What’s the area of a square? It’s the length × width.
      * length × width
      * ***d*** × ***d***
      * ***d***2 That’s the area of our base. Multiply by ***h*** and we have our equation.
      * ***Volume*** = ***Base*** × ***h***
      * ***Volume*** = ***d***2 × ***h***
      * ***Volume*** = ***d***2***h***.

B is the correct answer.

***\*****We’ve simplified the formula for volume. In this problem, we don’t care what shape the base is; we multiply its area by the height to give us our volume. If the base is circular, the area of the base = r2, if it’s a rectangle, the base area = length x width. Find the area of your base, multiply it by the height, and you get your volume.*

1. “Math speak” with a system of equations. Towards the end of the section; supposed to be a hard one. Not for us.
   1. Let’s write our givens:
      * + Square of ***x*** is equal to 4 times square of ***y***.
        + ***x*** is 1 more than twice ***y***.
   2. What are we looking for? The value of ***x***.
   3. Convert the “math speak.”
      * + Square of ***x*** ***is equal*** to 4 ***times*** square of ***y***.

***x2*** ***=*** 4 ***×*** ***y***2

* + - * ***x2*** ***=*** 4***y***2 Equation 1
      * ***x*** ***is*** 1 ***more than*** ***twice*** ***y***.
      * ***x*** ***=*** 1 ***+***  ***2 ×*** ***y***
      * ***x*** ***=*** 1 ***+***  ***2y***
      * ***x*** ***=*** 1 ***+*** ***2y*** Equation 2
  1. Let’s solve our system. We’ve got ***x*** by itself in Equation 2 so just plug it into Equation 1.
     + - ***x2*** ***=*** 4***y***2
       - (1 ***+*** ***2y***)***2*** ***=*** 4***y***2 Unfortunately we have to FOIL the left side.
       - 1 ***+*** ***2y*** + ***2y*** + ***4y2*** ***=*** 4***y***2
       - 1 ***+*** ***4y*** + ***4y2*** ***=*** 4***y***2 Now simplify.

-***4y2***  ***-***4***y***2

* + - * 1 ***+*** ***4y*** + 0 ***=*** 0
      * 1 ***+*** ***4y*** = 0

-1 -1

* + - * ***4y*** = -1
      * ***4y*** = -1

***4 4***

* + - * ***y*** = ***-1/4*** Back to Equation 2 to find ***x***.
      * ***x*** ***=*** 1 ***+*** ***2y***
      * ***x*** ***=*** 1 ***+*** ***2***(***-1/4***)
      * ***x*** ***=*** 1 ***+*** -1/2
      * ***x*** ***=*** ***1/2***

E is the correct answer.

1. Coordinate Plane Problem. DRAW THE PICTURE.
   1. Let’s write our givens:
      * + Lines ***l*** and ***q*** are perpendicular. That means the angle between them is 90, and their slopes

are negative reciprocals.

* + - * ***l*** contains two points: (0,0) and (2,1)
      * ***q*** contains two points: (2,1) and (0,***t***)
      * Picture

(o,***t***)

(2,1)

(o,o)

* 1. What are we looking for? The value of ***t***.
  2. Looks like the y-value of (0,***t***) is ***5***, so it looks like ***t*** = ***5***.
  3. Since it’s hard to draw a to-scale picture for this diagram, we can check it algebraically.
     + - Let’s use the slope formula to find out the slope of ***l:***
         * ***l*** contains two points: (0,0) and (2,1)
         * Slope(***l)*** =***m*** = Rise = y2 - y1

Run x2 - x1

* + - * + ***m*** = Rise = 1 - 0 = 1

Run 2 - 0 2

* + - * + ***m*** = ½
        + Use the negative reciprocal to get the ***m*** for line ***q***. ***W***hat’s the negative reciprocal? Flip the top and bottom numbers and make it negative. That’s it.
        + Line ***l***: ***m*** = 1

2

* + - * + Line ***q***: ***m*** = -2

1

* + - * + Line ***q***: ***m*** = -2
      * Now use the slope formula again for line ***q*** to figure out ***t***.
        + ***q*** contains two points: (0,***t***) and (2,1)
        + Slope(***q***) = ***m*** = Rise = y2 - y1

Run x2 - x1

* + - * + ***-2*** = 1 - ***t*** 2 – 0 = 2. Multiply both sides by 2.

2 - 0

* + - * + ***-2*** × 2 = 1 - ***t*** × 2

2

* + - * + -4 = 1 - ***t***

-1 -1

* + - * + -5 = - ***t*** Get rid of the negatives on both sides and we’re done.
        + ***5*** = ***t***

E is the correct answer.

# Test 2 Section 5

1. Basic algebra problem. A confidence builder.
   1. Let’s write our given: 3***x*** = 0
   2. What are we looking for? The value of 1 + x +x2.
   3. Solve for ***x*** in the first equation.
      * + 3***x*** = 0

3 3

* + - * ***x*** = ***0***
  1. Plug that value into the next expression.
     + - 1 + x + x2
       - 1 + 0 + (0)2
       - 1

B is the correct answer.

1. Geometry word problem. Careful with your definitions.
   1. Let’s write our givens:
      * + Diameter of Circle ***A*** is 3 times diameter of Circle ***B.***
        + Diameter is 2 × Radius.
   2. What are we looking for?
      * + Ratio of radius of circle ***A*** to the radius of circle ***B***.
        + Radius ***A***

Radius ***B***

* 1. Convert the “math speak.”
     + - Diameter of Circle ***A*** is ***3 times*** diameter of Circle ***B.***
       - Diameter ***A*** = ***3×*** Diameter ***B***
       - Diameter ***A*** = ***3***(Diameter ***B***)
  2. Define a friend. We don’t have a number for the diameters of ***A*** 0r ***B***. Let the diameter of ***B*** = ***2***, because we can multiply that by 3 and get the diameter of ***A***. We didn’t go with our favorite, 1, because in the answer, we’re working with a radius, and we’d rather not deal with fractions.
  3. Find the Diameter of ***A***.
     + - Diameter ***B*** = ***2***
       - Diameter ***A*** = ***3***(Diameter ***B***)
       - Diameter ***A*** = ***3***(***2***)
       - Diameter ***A*** = ***6***
  4. Find the Radii of ***A*** & ***B***.
     + - Radius = Diameter/2
       - Radius ***A*** = ***6***/2 Radius ***B*** = ***2***/2
       - Radius ***A*** = ***3*** Radius ***B*** = ***1***
  5. Finish up. We’re looking for the ratio of Radius ***A*** to Radius ***B***.
     + - Radius ***A*** = ***3***

Radius ***B*** ***1***

* + - * ***3***/***1*** in Ratio Form is ***3***: ***1***.

E is the correct answer.

1. Series problem with friends.
   1. Let’s write our givens:
      * + Set ***N*** is a set of numbers with an average of 3.
        + Set ***M*** is derived by doubling every number in ***N***.
   2. What are we looking for?
      * + The ***average*** of ***M.***
   3. Not too much information is given here, so let’s keep it as simple as possible. First, we need a set of numbers that make up ***N*** so that ***N*** has an average of 3. Why don’t we pick the set {3,3}. It doesn’t tell us how many numbers, so keep it simple, and use only 2 terms. What’s the average of the numbers in {3,3}? Well, it’s 3. (3 + 3)/2 = 3. So ***N*** = (3,3}.
   4. Find ***M***.
      * + ***N*** = {3,3}
        + Now double those terms to get the terms in ***M***. 3 × 2 = 6.
        + ***M*** = {6,6}
   5. What’s the average of the terms of ***M***? Plug and chug.
      * + ***M*** = {6,6}
        + Average of ***M***: 6 + 6 = 6

2

* + - * The ***average*** of the terms of M is ***6***.

D is the correct answer.

1. Digits and exponents with friends. This problem is not as bad as it looks.
   1. Let’s write our givens: ***P***, ***R*** and ***T*** are digits in the ***positive*** 3-digit ***integer*** ***PRT***.
   2. What are we looking for? The decimal equivalent of ***PRT*** x 10-2.
   3. Let’s give numbers to ***P***, ***R*** and ***T***.
      * + ***P*** = ***1***
        + ***R*** = ***2***
        + ***T*** = ***3***
        + ***PRT*** = ***123***
   4. Now to the exponent. We have a base of 10. We know we’re moving the decimal, but which way? If the exponent is positive, we move the decimal to the right (to get a bigger number). So, if the exponent is negative, we move the decimal to the left (to get a smaller number). That’s it. How many places do we move the decimal? The number value of the exponent is 2. So we move the decimal 2 places. The exponent is ***negative***, so we move the decimal 2 places to the ***left***.
      * + ***123*** x 10-2 = ***1***.***23***
        + ***1***.***23*** = ***P***.***RT*** in the form of the answers.

C is the correct answer.

1. Friends problem.
   1. Let’s write our given: ***k*** + ***n*** < ***k***
   2. What are we looking for? The ***true*** statement.
   3. Try to eliminate answers by finding ***k***’s and ***n***’s that make the answer false.
      1. ***k*** > 0. Let’s say ***k*** = ***1***.
         * ***k*** + ***n*** < ***k***
         * ***1*** + ***n*** < ***1***
         * What if ***n*** = ***2***? ***1*** + ***2*** < ***1*** That doesn’t work. (A) is out.
      2. ***k*** = 0.
         * ***k*** + ***n*** < ***k***
         * 0 + ***n*** < 0
         * What if ***n*** = ***2***? ***0*** + ***2*** < ***0*** That doesn’t work. (B) is out.
      3. k < 0. Let’s say ***k*** = ***-1***.
         * ***k*** + ***n*** < ***k***
         * ***-1*** + ***n*** < -***1***
         * What if ***n*** is ***2***? ***-1*** + ***2*** < ***-1*** That doesn’t work. (C) is out.
      4. n > 0. Let’s say ***n*** = ***1***. Let’s say ***k*** = ***1***.
         * ***k*** + ***n*** < ***k***
         * ***1*** + ***1*** < ***1*** That doesn’t work. (D) is out. By elimination, (E) is the answer, but check it.
      5. n < 0. Let’s say ***n*** = ***-1***. Let’s say ***k*** = ***1***.
         * ***k*** + ***n*** < ***k***
         * ***1*** + ***-1*** < ***1*** That works.
         * What if ***k*** is -1? ***-1*** + ***-1*** < ***-1*** That works. So, (E) is ***true***.

E is the correct answer.

1. Geometry Problem. Together now…”MARK UP THE DRAWING!”

***3.5***

***x***

*Ramp*

* 1. Let’s write our givens:
     + - Slope of the ramp is 7/16.
       - ***y*** = ***3.5*** feet
  2. What are we looking for? The length of ***x***.
  3. The slope points towards a line function. What if we let the ramp equal our line, with 7/16 being our slope? All we would need for the slope formula are two points on the slope. We’ve almost got that.
     + - How do we get two points? Think outside the problem a little bit. Why don’t we let the point where the ramp hits the ground be our origin on a coordinate plane. That means that the point where the ramp hits the ground has the coordinates (***0***,***0***). And the ***x*** is the ***x***-value of the point (***x***, ***3.5***) on our ramp line where it hits the truck. We can use the slope formula to figure it out.

***3.5***

***x***

*Ramp*

(***0***,***0***)

(***x***,***3.5***)

* + - * Slope = (y2 - y1) Rise Let point 2 be (***x***, ***3.5***) and point 1 be (***0***,***0***).

(x2 - x1) Run

* + - * 7 = (***3.5*** - ***0***)

16 (***x*** - ***0***)

* + - * 7 = ***3.5*** Cross-multiply.

16 ***x***

* + - * 7 × ***x*** = ***3.5*** × 16
      * 7***x*** = 56 Simplify.
      * 7***x*** = 56

7 7

* + - * ***x*** = 8

A is the correct answer.

1. Parabola with friends. Know the generic formula for parabolas. Or plug in points to figure it out.
   1. Let’s write our givens:
      * + y = ***a***x2 + 2
        + ***a***  is a constant.
   2. What are we looking for? What the graph of y = ***a***x2 + 2 will look like compared to the original.

3

* 1. Easiest way to solve this is to know that if we have a smaller ***a*** in front of the x2 in a parabola, the graph is going to be ***wider.***

B is the correct answer.

* 1. If we forgot that rule, then we can try plugging in different numbers for ***a*** and find points on the original and new graph. Let’s use ***a*** = ***1*** because we’re dividing by 3 for the new parabola. Also, for our points, try to pick x’s that will give x2’s that are divisible by 3 so we don’t have nasty fractions.
     + - Original New
       - y = ***1***x2 + 2 y =***1***x2 + 2

3

***y = x2 + 2*** ***y =x2 + 2***

(***-3***, ***11***)

(***3***, ***11***)

(***-3***, ***5***)

(***3***, ***5***)

***3***

|  |  |  |  |
| --- | --- | --- | --- |
| Original | | New | |
| ***y = x2 + 2*** | | ***y =x2 + 2***  ***3*** | |
| x | y | x | y |
| 3 | 11 | 3 | 5 |
| -3 | 11 | -3 | 5 |
| 6 | 38 | 6 | 14 |
| 9 | 83 | 9 | 29 |

* + - * For x = 3, the y’s in the two equations are below.
      * y = x2 + 2 y =x2 + 2

3

* + - * y = (3)2 + 2 y =(3)2 + 2

3

* + - * y = 9 + 2 y = 9 + 2

3

* + - * y = ***11*** y = ***5***
  1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\717RERNY\MC900229885[1].wmfThis problem has to do with the number in front of x2 and its effect on y. In our case, the ***a*** is smaller in the new equation, so we know that y will not be as high as in the original equation. Notice how we don’t even care about the “ + 2” in the equations. That just shows us where the parabola crosses the y-axis, (where x=0) and it’s not going to change when we change the number that is multiplied by the x2 term.

1. Logic Problem
   1. Let’s write our givens:
      * + 3 hats: ***Red***, ***Blue*** and White
        + 3 sweaters: ***Red***, ***Blue*** and White
        + 3 jeans: ***Red***, ***Blue*** and White
        + Wants to wear a ***Red***, ***Blue*** and White outfit
   2. What are we looking for? The number of combinations of red/blue/white outfits that she can wear.
   3. Draw the bones of a chart and fill it in. We know every outfit has all three colors.

|  |  |  |
| --- | --- | --- |
| Hat | Sweater | Jeans |
| ***Red*** | ***Blue*** | White |
| ***Red*** | White | ***Blue*** |
| ***Blue*** | ***Red*** | White |
| ***Blue*** | White | ***Red*** |
| White | ***Red*** | ***Blue*** |
| White | ***Blue*** | ***Red*** |

* + - * That’s ***6*** different combinations.

B is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. “Math speak” Problem
   1. Let’s write our given: When twice a ***number*** is increased by 5, the result is 14.
   2. What are we looking for? The ***number***. Let’s call it ***x***.
   3. Translate the “math speak.”
      * + ***Twice*** a ***number*** ***is*** ***increased by*** 5, the result is 14.
        + ***2× x*** + 5 = 14
        + ***2x*** + 5 = 14.

-5 -5

* + - * ***2x*** = 9.

2 2

* + - * ***x*** + = ***9/2*** or ***4½***

***9/2*** is the correct answer.

1. Geometry with parallel lines.
   1. Let’s write our givens:
      * + l || m
        + ***y*** = 3***x***  Equation 1
   2. What are we looking for? The value of ***y***.

k

l

m

***y***°

***x***°

* 1. Mark up the picture.
     + - Remember the rules about parallel lines. Aren’t angles x and y “Same Side Interior”? Yes, indeed. What do those add up to? 360°? Nope. 180°.
       - ***x*** + ***y*** = 180 Equation 2
  2. Let’s write our equations and plug and chug. Substitute Equation 1 into Equation 2.
     + - ***y*** = 3***x*** Equation 1
       - ***x*** +  ***y*** = 180 Equation 2
       - ***x*** + (3***x***) = 180
       - 4***x*** = 180

4 4

* + - * ***x*** = 45 Plug x back in to find ***y***.
      * ***y*** = 3***x***
      * ***y*** = 3(45)
      * ***y*** = ***135***

***135*** is the correct answer.

1. Geometry Problem
   1. Let’s write our givens:
      * + Rectangular box is 4 x 4 x 8.
        + CD cases are 4 x 4 x ¼.
   2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfWhat are we looking for? Max ***number*** of CD’s that will fit in the box.
   3. We can draw the picture, but there’s already one there, and it would just show a bunch of lines up and down for the CD cases. Unless we want to draw a perfectly to-scale picture, let’s go another way. (This is one of the few times we won’t scream DRAW THE PICTURE; cherish it.)
   4. Look at the dimensions.
      * + Box 4 x 4 x 8
        + CD 4 x 4 x ¼
        + With which dimensions do we have any room for manipulation? In the length of the box (8) and the width of a CD case (1/4). Divide them, and we will have the max number of CDs that will fit in the box.
        + 8 ÷ ¼ is the same as 8 x 4
        + ***32*** = Max ***number*** of CDs in the box.

***32*** is the correct answer.

1. Friends and fractions.
   1. Let’s write our given:
      * + 3***x*** + ***y*** = 6

***y*** 5

* 1. What are we looking for? The ***value*** of ***x***/***y***.
  2. Cross multiply the equation.
     + - 3***x*** + ***y*** = 6

***y*** 5

* + - * 6***y*** = 5(3***x*** + ***y***)
      * 6***y*** = 15***x*** + 5***y***

***-***5 ***y*** - 5 ***y***

* + - * ***y*** = 15***x*** Now divide by ***x*** to get the ***x*** and ***y*** on the same side.
      * ***y*** = 15***x***

***x x***

* + - * ***y*** = 15 = 15 What now? We’ve got y/x, how do we get x/y? Flip both sides.

***x*** 1Remember, any number is that number over 1. So 15 = 15/1.

* + - * ***x*** =  ***1***

***y*** ***15***

***1/15*** is the correct answer.

1. Data Problem
   1. Let’s write our givens: 3 stores; 2 years of profit; profit data in the table.
   2. What are we looking for? The ***average increase in profit*** for the 3 stores from year 1 to year 2.
   3. We have two years of profits. We need the increase in profits. Add another column to the table.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Year 1 | Year 2 | Increase |
| Store A | 5,000 | 6,200 | 1,200 |
| Store B | 6,000 | 7,350 | 1,350 |
| Store C | 10,000 | 12,700 | 2,700 |
| ***Total*** | ***21,000*** | ***26,250*** | ***5,250*** |

* + - * For store A, the increase is: 6,200 - 5,000 = 1,200
      * We can do that for all of the stores or just find the *total* increase and save some time.
  1. Find the average. What’s an average? Sum of the terms (increase in profit at each store), divided by the number of terms (in this case stores).
     + - 1,200 + 1,350 + 2,700 =

3

* + - * C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmf ***5,250*** See how this is the same as the ***total increase*** above? (We didn’t need 3 to find the increase at each the store.)
      * ***1,750*** = ***average increase in profit***

***1750*** is the correct answer.

1. Absolute value friend problem.
   1. Let’s write our given: f(***x***) = |3***x*** - 17|
   2. What are we looking for? A value of ***a*** that makes f(***a***) < ***a***.
   3. Don’t let the friends scare you! f(***a***) < ***a*** is the same as f(***x***) < ***x***.
   4. Pick a number and see if it works. Where do we like to start? That’s right, with 0 and 1.
      * + Try ***a*** = 0. So ***a*** = ***x*** = ***0***
        + f(***x***) = |3***x*** - 17|
        + f(***0***) = |3(***0***) - 17|
        + f(***0***) = | -17|
        + f(***0***) = 17
        + When ***a*** (=***x***) = ***0***, f(***a***) = f(***x***) = 17. Does that meet the condition?
        + f(***x***) < ***x***
        + f(***0***) < ***0***
        + ***17*** < ***0*** That doesn’t work. Let’s try -1.
        + f(***x***) = |3***x*** - 17|
        + f(***-1***) = |3(***-1***) - 17|
        + f(***-1***) = | -20|
        + f(***-1***) = 20
        + When ***a*** (=***x***) = ***-1***, f(***a***) = f(***x***) = 20. Does that meet the condition?
        + f(***x***) < ***x***
        + f(***-1***) < ***-1***
        + ***20*** < ***-1*** That didn’t work either.
   5. What if we could get the value inside the || closer to zero, maybe that would help. We want ***x*** to be bigger than f(***x***), so that’s a start.
      * + The expression inside is 3***x*** -17. How could we get that close to zero? The 3***x*** term needs to be 17 or maybe a little bigger. What if ***x*** = ***7***? Let’s try it.
        + ***x*** = ***7***
        + f(***x***) = |3***x*** - 17|
        + f(***x***) = |3(***7***) - 17|
        + f(***x***) = |21 - 17|
        + f(***x***) = |4|
        + f(***x***) = 4 Now try the condition.
        + f(***x***) < ***x***
        + f(***7***) < ***7***
        + ***4*** < ***7*** So when ***x*** = ***7***, our condition is satisfied. ***7*** is an answer.

***7*** is a correct answer. Any answer within this range works: 17/4 < ***x*** < 17/2

1. Word Problem
   1. Let’s write our givens:
      * + Jar with 50 pieces of candy: 25 red pieces and 25 green pieces
        + 3 reds and 4 greens taken from jar.
        + 13 more pieces taken from jar.
   2. What are we looking for? Lowest number of the 13 extra pieces taken that must be red for the ***Total Red Pieces*** > ***Total Green Pieces***. ***x*** is the number of reds taken on the second pass.
   3. Let’s set up our skeleton.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Red Pieces | Green Pieces | Total |
| First pass | 3 | 4 | 7 |
| Second pass | ***x*** |  | 13 |
| **Total** |  |  |  |

* 1. We need ***Total Red Pieces*** > ***Total Green Pieces***. Let’s pick an ***x***. Let’s try ***x*** = ***1***.
     + - If ***x*** = ***1***, then green pieces taken on the second pass must be 12. Does that satisfy the condition?

|  |  |  |  |
| --- | --- | --- | --- |
|  | Red Pieces | Green Pieces | Total |
| First pass | 3 | 4 | 7 |
| Second pass | ***1*** | 12 | 13 |
| **Total** | ***4*** | ***16*** | **20** |

* Nope. There are more green pieces. What about ***x*** = ***12***?

|  |  |  |  |
| --- | --- | --- | --- |
|  | Red Pieces | Green Pieces | Total |
| First pass | 3 | 4 | 7 |
| Second pass | ***12*** | 1 | 13 |
| **Total** | ***15*** | ***5*** | **20** |

* Red pieces are more, but aren’t we looking for the lowest number for ***x*** that makes total reds greater than total greens? We need to come down a bit.Let’s try ***x*** = ***7***.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Red Pieces | Green Pieces | Total |
| First pass | 3 | 4 | 7 |
| Second pass | ***7*** | 6 | 13 |
| **Total** | ***10*** | ***10*** | **20** |

* Close, but not quite. One more red, and we should be good. Try ***x*** = ***8***.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Red Pieces | Green Pieces | Total |
| First pass | 3 | 4 | 7 |
| Second pass | ***8*** | 5 | 13 |
| **Total** | ***11*** | ***9*** | **20** |

* There we go. ***8*** is the lowest number of reds that needs to be taken on the second pass to give a total of more reds than greens.

***8*** is the correct answer.

1. Numbers Rules
   1. Let’s write our givens:
      * + Positive integer less than 1,000.
        + The integer is tri-factorable. Tri-factorable means the number is the product of 3 consecutive integers (e.g., 1,2,3 or 20,21,22, multiplied together).
   2. What are we looking for? The number of ***positive*** integers less than 1,000 that are tri-factorable.
   3. Pick a set of 3 consecutive integers and try it.
      * + Let’s start with ***1,2,3***.
        + ***1*** × ***2*** × ***3*** = 6 That works. But we need to go up to 1,000.
        + What 3 numbers multiply to 1,000?
        + 10 × 10 × 10 = 1000. That isn’t an answer, but it gives us a clue as to how high we can go.
        + What about ***9,10,11***?
        + ***9*** × ***10*** × ***11*** = 990 That works, too. So we have our lowest integer group and our highest.
   4. Now figure out how many consecutive integer groups are between (and include) ***1,2,3*** and ***9,10,11***.
   5. Looks like ***9***, right? Our first set starts with 1, and our last set starts with 9. What about sets that start at 0 and 10? Try them.
      * + ***0*** × ***1*** × ***2*** = 0 That doesn’t work; the integer must be ***positive***. 0 is not positive.
        + ***10*** × ***11*** × ***12*** = 1,320 Nope, the result must be less than 1,000.

***9*** is the correct answer.

1. Word problem with friends. And a sneaky system of equations.
   1. Let’s write our givens:
      * + ***Carrier A cost*** is $1.00 for first minute and $0.07 every additional minute after that.
        + ***Carrier B*** ***cost*** is $0.06 for every minute of a call.
        + For a call of ***t*** minutes, ***Carrier A cost*** = ***Carrier B*** ***cost***.
   2. What are we looking for? The value of ***t*** minutes.
   3. We need some equations. Set one up for ***Carrier A*** using ***t***. Convert the “math speak.”
      * + ***Carrier A cost*** ***is*** $1.00 for first 20 minutes  ***+*** $0.07 every additional minute after that.
        + ***Carrier A cost*** ***=*** $1.00 ***+*** $0.07 x (***Something***)
        + What’s the ***Something***? Let’s figure it out.
        + If a call is 21 minutes, there is 1 minute that is charged at $0.07 (in addition to the $1 for the first 20 minutes). What’s an expression using ***t*** to reflect those extra minutes that are greater than 20?
        + 21 - 20 = 1
        + ***t*** - 20 = 1. Seems like a good one.
        + Does that equation give us extra minutes for other call lengths? Try ***t*** = 30.
        + ***t*** - 20 = 1.
        + ***30*** - 20 = 10. Makes sense. In a 30-minute call, there are 10 minutes over 20 minutes.
        + ***Something*** = ***t*** - 20. Plug it into our equation.
        + ***Carrier A cost*** ***=*** $1.00 ***+*** $0.07 × (***t*** - 20)
        + ***Carrier A cost*** = $1.00 ***+***$0.07(***t*** - 20) **Equation 1**

Find an equation for ***Carrier B***. This is one is easy after doing ***A***. It’s $0.06 per minute.

* + - * ***Carrier B*** ***cost*** ***is*** $0.06 ***for*** ***every minute*** of a call
      * ***Carrier B*** ***cost*** ***=*** $0.06 ***×***  ***t***
      * ***Carrier B*** ***cost*** = $0.06***t* Equation 2**
  1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfNow we can set up our equation using the last given to find ***t*** where ***Carrier A cost*** = ***Carrier B*** ***cost***.
     + - ***Carrier A cost*** = ***Carrier B*** ***cost***
       - $1.00 ***+*** $0.07(***t*** - 20) = $0.06***t*** Simplify and finish up.
       - $1.00 ***+*** $0.07***t*** - $1.4 = $0.06***t***
       - $0.07***t*** - $0.4 = $0.06***t***

-$0.06***t*** - $0.4 = -$0.06***t***

* + - * $0.01***t*** - $0.4 = $0

+$0.4 = +$0.4

* + - * $0.01***t*** = $0.40Divide both sides by $0.01.

$0.01 $0.01

* + - * ***t*** = ***40***

***40*** is the correct answer.

1. Geometry with friends. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + 10 squares, each side is ***k***.
        + Perimeter of entire figure is ***p***.
        + Area of the figure is ***a***.
        + ***p*** = ***a***
   2. What are we looking for? The value of k.

***k***

***k***

***k***

***k***

***k***

***k***

***k***

***k***

***k***

***k***

***k***

***k***

***k***

* 1. Label the picture.
     + - Does that tell us anything? Yes. Add up all the ***k***’s and that gives us the perimeter, ***p***.
       - There are 16 ***k***’s.
       - 16***k*** = ***p*** **Equation 1**
  2. What about ***a***? Do we have an equation for that using ***k***? Not exactly, but we have an equation for the area of ***one*** square. A square’s area is side × side. We have 10 squares, so if we find the area of one, we can multiply it by 10 to get the area of the whole figure.
     + - Area of ***one*** square = side × side
       - Area of ***one*** square = ***k*** × ***k***
       - Area of ***one*** square = ***k***2 Multiply by 10 to get ***a***, the area of the whole figure.
       - ***a*** = 10***k***2 **Equation 2**
  3. What now? Use the last given, ***p*** = ***a***, and then solve for ***k***.
     + - ***p*** = ***a***
       - 16***k*** = 10***k***2 Divide by ***k***.
       - 16***k*** = 10***k***2 Simplify.

***k k***

* + - * 16 = 10***k***

10 10

* + - * ***1.6*** =  ***k***

***1.6*** is the correct answer.

# Test 2 Section 8

1. Fraction question. Not sure? It says “fraction” in the problem. What’s a fraction? ***Part***/**Whole**.
   1. Let’s write our given: Film takes ***90*** minutes.
   2. What are we looking for? The fraction of the film that is done after ***15*** minutes.
   3. Plug and chug.
      * + ***Part*** = ***15***

***Whole*** ***90***

***15***  = ***1***

***90*** ***6***

B is the correct answer.

1. Geometry Problem. Use the basic rules of triangles to figure this one out.
   1. What are we looking for? The ***longest*** side.
   2. What are the givens? The picture, and also ∠JHK is a right angle (90°). Mark that in the picture.

L

H

K

J

* 1. Start eliminating answers. Is there any way that any of the sides of the smaller triangle HJL can be the longest? Nope. Cross out answers (A) HJ, (C) HL and (E) JL. We’re down to two.
  2. Of the remaining choices, (B) HK and (D) JK, which one is longer? If we don’t guess by just looking at the picture, then we can use the Pythagorean Theorem (a2 + b2 = ***c***2). ∆JHK is a right

triangle because ∠JHK is a right angle (90°). What is ***c*** in the Pythagorean Theorem? It’s the hypotenuse, or the longest side. That’s it. ***JK*** is our ***c***, so ***JK*** is the longest side.

D is the correct answer.

1. A function problem hidden in a table.
   1. What are we looking for? The value of ***p***, which is f(n) when n is 4.
   2. The best thing to do here is look for patterns in f(n). See any? Looks like f(n) goes up ***6*** every time n goes up by 1. Try it.
      * + 7 + ***6*** = 13
        + 13 + ***6*** = 19;
        + 31 + ***6*** = 37. So how do we get to ***p***? Let’s add ***6*** to 19.
        + 19 + ***6*** = 25. Does that work to get to 31?
        + 25 + ***6*** = 31. Looks like a winner. ***p*** = ***25***

C is the correct answer.

* 1. If we really wanted to, we could find the slope of the line using two points, find the y-intercept, find the formula of the line f(n) and plug in 4. We’d get to the same place, but what we did above took about 10 seconds.

1. “Math Speak” Word Problem
   1. Let’s write our givens:
      * + ***Charlie*** has built houses for 5 years less than twice as long as ***Maly***.
        + ***Maly*** has built houses for ***n*** years.
   2. What are we looking for? The expression that gives us the number of years that Charlie has built houses.
   3. Convert the “math speak” into an equation.
      * + ***Maly*** has built houses for ***n*** years

***Maly*** = ***n***

* + - * ***Charlie*** has built houses for ***5*** ***years*** ***less*** than twice as long as ***Maly***
      * ***Charlie*** = ***-5*** 2× ***n***
      * That still looks weird, but with a little manipulation we can get what we need. 5 less than a number means that 5 is *subtracted from* that number. So:
      * ***Charlie*** = (2 × ***n***) ***-5***
      * ***Charlie*** = 2***n*** ***-*** ***5***

C is the correct answer.

1. Geometry problem with lines. MARK UP THE DRAWING.
   1. Let’s write our givens:
      * + P is on line AD
        + PC ***bisects*** ∠BPD What does bisect an angle mean? Cut in half. Mark it.

B

C

D

P

A

***?***

80°

* 1. What are we looking for? ***∠CPD***.
  2. Now our rules on lines and angles. How many degrees in a straight line? 180°. So:
     + - 80° + ∠BPC + ***∠CPD*** = 180°
       - ∠BPC = ***∠CPD*** because PC ***bisects*** (cuts in half) ∠BPD. Let’s make a friend, and let those angles be ***x***.
       - ∠BPC = ***∠CPD*** = ***x***
       - 80° + ∠BPC + ***∠CPD*** = 180°
       - 80° + ***x*** + ***x*** = 180°
       - 80° + 2***x*** = 180°

-80° -80°

* 2***x*** = 100°
* 2***x*** = 100°

2 2

* ***x*** = 50°
* ***∠CPD*** = 50°

B is the correct answer.

1. Integer problem. Know your rules.
   1. Let’s write our given: ***x*** is an odd integer.
   2. What are we looking for? The expression that gives us next odd integer after ***x*.**
   3. What are odd integers? Odd numbers like -1, 5, -131, -2,345, etc. Let’s pick one as our ***x***. We love 0’s and 1’s, so start there. Is 0 odd? No. ***1*** it is.
   4. Using ***1***, see which answer gives us the next ***odd*** integer.
      1. ***x*** - 1

***1*** - 1 = 2 Even. (A) is out.

* + 1. ***x***  + 1

***1*** + 1 = 2 Even. Nope.

* + 1. ***x*** + 2

***1*** + 2 = 3 Odd and greater than 1. (C) could work. Check the others.

* + 1. ***x*** + 3

***1*** + 3 = 4 Even. (D) is out.

* + 1. 2***x*** - 1

2(***1***) - 1 = 1 No dice. That equals ***x***.

The only answer that works is (C), ***x*** + 2.

C is the correct answer.

* 1. If you were wondering about answer (E), it gives a correct answer for ***x*** = 3, but it breaks down once you get to x =5 and above. Keep it simple. Use 1.

1. Coordinate plane problem with friends. No problem.
   1. Let’s write our given:
      * + TO = PO. What does that mean? Since O is the origin, it means that there are similarities between the x and y coordinates of points T and P. Just how similar?

T

O

P (***a***,***b***)

* 1. What are we looking for? The coordinates of point T.
  2. Let’s give some values to our friends. Keep it simple; remember, we love 0’s and 1’s. 0’s don’t work here, because all of our points would be at the origin.
     + - ***a*** = ***1*** We know ***a*** has to be positive because it’s to the right of the y-axis.
       - ***b*** = -***1*** We know ***b*** has to be negative because it’s below the x-axis.
  3. Plug in the values for point P to get the values of T. T has the negative ***x-value*** from point P, and the same ***y-value*** as point P.
     + - P = (***1***, -***1***), so:
       - T = (-***1***, -***1***)
       - P = (***1***,-***1***) T = (***-1***,-***1***)

P = (***a***, ***b***) T = (***-a***, ***b***)

A is the correct answer.

1. Word problem with a lot of info. Write down all the info.
   1. Let’s write our givens:
      * + Box has wood beads, red glass beads and blue glass beads.
        + Glass beads are 4 times wood beads.
        + Probability of red bead is 3 times a blue bead.
        + 12 red beads in the box.
   2. What are we looking for? ***Total beads*** in the box.
   3. Draw the skeleton and fill in givens.

***Wood*** ***Red*** ***Blue Total***

***?***

***?***

***12***

=

+

+

***?***

* 1. Convert the “math speak.”
     + - Glass beads are 4 times ***Wood*** beads

***Red*** + ***Blue*** = 4 × ***Wood*** Remember, glass beads includes ***Red*** and ***Blue***.

***Red*** + ***Blue*** = 4***Wood***

* + - * Probability of a ***Red*** is 3 times a ***Blue***

***Red*** = 3 × ***Blue***

***Red*** = 3***Blue***

* 1. Use our formulas to plug and chug!
     + - ***Red*** = 3***Blue***

***4***

***?***

***12***

=

+

+

***?***

***Wood*** ***Red*** ***Blue Total***

* + - * ***12*** = 3***Blue***
      * ***12*** = 3***Blue***

3 3

* + - * ***4*** = ***Blue***
      * ***Red*** + ***Blue*** = 4***Wood***
      * ***12*** + ***4*** = 4***Wood***
      * ***16*** = 4***Wood***
      * ***16*** = 4***Wood***

***4***

***4***

***12***

=

+

+

***?***

***Wood*** ***Red*** ***Blue Total***

4 4

* + - * ***4*** = ***Wood***
      * ***Wood*** + ***Red*** + ***Blue*** = ***Total***
      * ***4*** + ***12*** + ***4*** = ***20***

A is the correct answer.

1. Coordinate Plane Problem
   1. What are we looking for? The reflection of the given graph about the x-axis.
   2. What does reflection mean? Think mirror. It means the figure is flipped over an axis, in this case the ***x-axis***. The ***x-axis*** is the mirror.
   3. So, we are flipping this thing over the x-axis. Does it say anything about cutting away part of the line or adding more humps? No. When you look in the mirror, do you see 3 of you or half of you? No. Look at (C), (D) and (E). (C) has an extra hump, and (D) and (E) both have portions of the line cut out. All three of those are out. We’re left with (A) and (B).

(***A***) (***B***)

* 1. Look at the above lines (***A***) and (***B***) on the same graph as the original. Which axis is our mirror? The ***x-axis*** (the horizontal one, so it’s like flipping a page in a wall calendar). (***B***) is a reflection of the original, but only if our mirror is the y-axis (like turning the pages of a book). (***A***) is the only answer left, and it also matches a reflection of the original over the **x-axis**.

A is the correct answer.

1. Algebra problem with exponents. Joyous.
   1. Let’s write our givens:
      * + (x + y)2 = 100
        + (x - y)2 = 16
   2. What are we looking for? The ***value*** of ***xy***.
   3. Where do we start? These are ugly equations. Notice that the two expressions equal perfect squares 100 and 16. That’s a start.
      * + Square root both sides of both equations.
   4. Now we’ve got two equations and two friends. A nice system of equations.
      * + ***x*** + ***y*** = 10 Equation 1
        + ***x*** - ***y*** = 4 Equation 2
   5. Plug and chug. Let’s start with Equation 1 and get the ***y*** by itself. (You can do ***x*** if you want to.)
      * + ***x*** + ***y*** = 10
        + -***x*** -***x***

***y*** = 10 - ***x*** Now plug that value of y into Equation 2.

* + - * ***x*** - ***y*** = 4
      * ***x*** - (10 - ***x***) = 4
      * ***x*** - 10 + ***x*** = 4
      * 2***x*** - 10 = 4

+10 +10

* + - * 2***x*** = 14
      * 2***x*** = 14

2 2

* + - * ***x*** = ***7***  Take that value of x and plug it back into Equation 1.
      * ***x*** + ***y*** = 10
      * 7 + ***y*** = 10

-7 -7

* + - * + ***y*** = ***3***  Now we’ve got ***x*** and ***y***. What are we looking for again? ***xy***.
      * ***xy*** = ***?***

***7***×***3*** = ***21***

C is the correct answer.

1. A friend with an inequality and some number lines.
   1. First off, forget about the answers. Focus on the algebraic expression and get ***x*** by itself. ***x*** is what we’re looking for, so it can’t hurt.
      * + -1 ≤ 4***x*** - 5

+5 +5

* + - * 4 ≤ 4***x***
      * 4 ≤ 4***x***

4 4

* + - * 1 ≤ ***x*** which is the same as ***x***  ≥ 1 ***x*** is greater than or *equal to* 1*.*
      * Now look at our answers. We need a number line where ***x*** is (at least) equal to one. That would be represented by a closed dot. So (B), (C) and (D) are all out.
      * We’re left with (A) and (E). (E) equals (closed dot) and starts at the right point (1), but then line goes to the left, meaning that ***x*** ≤ 1. Not right. (A) is the only answer that we’re left with, and it’s the only line where ***x*** is both greater than and equal to 1.

A is the correct answer.

1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfGeometry problem. This is really an intuitive question. We can try to use the given info, and find each side of each rectangle, but that’s a trap. Focus on what *inscribe* means.
   1. Inscribe means that each point of the rectangle is on the circle. Look at the picture and think of the center of one of those old record players (ask your parents what that is). If you turn the record slightly, doesn’t that shift the rectangles? One shift is 2 more rectangles, another shift is 2 more, so we’re already at 4 more rectangles.
   2. Look at the above picture. The black rectangles are the original, and the green, red and blue ones are slight shifts. There are an infinite number of shifts, or turns of the record, that would allow us to make more than 4 other rectangles in the circle with the same perimeter as the original.

E is the correct answer.

1. Holy exponents, Batman! We have to be familiar with exponents to work this problem.
   1. Let’s write our givens:
      * + 2***n*** + 2***n***+1 = ***k***
        + n is a positive integer
   2. What are we looking for? ***2n+2*** in terms of ***k***. Then answer has to have a ***k*** in it. Now what? Let’s give our friend ***n*** a value.
   3. What do we like to start with? 0 and 1. But look at our givens. ***n*** is a positive integer, so 0 is gone. Let’s use ***n*** = ***1*** and see if we get a ***k***.
      * + 2***n*** + 2***n***+1 = ***k***
        + 2***1*** + 2***1***+1 = ***k***
        + 2 + 22 = ***k***
        + 2 + 4 = ***k***
        + ***6*** = ***k*** So when ***n*** = 1, ***k*** = ***6***.
   4. What about ***2n+2*** when ***n*** = 1?
      * + ***2n+2***
        + ***21+2***
        + ***23***  = ***8***
        + ***2n+2*** = ***8***
   5. So when ***n*** = 1, ***k*** = ***6*** and ***2n+2*** = ***8***. Create an equation, and put ***2n+2*** on one side and ***k*** on the other.
      * + ***2n+2*** = ***?k*** The ? means that we have to do something to ***k*** to get it to ***2n+2***.
        + ***8*** = ***?6*** How can we get from ***6*** to ***8***? We have to multiply it by something, and that’s our ***?.***
        + ***8*** = ?***6*** Get that ***?*** by itself.

6 6

* + - * 4/3 = ***?*** Now check to see if it works.
      * When ***n*** = 1, ***k*** = ***6***. Also ***?k*** = ***2n+2*** = ***8***. Does it check out? Plug and chug.
      * ***2n+2*** = ***?k***

***8*** = ***4/3×***(***6***)

***8*** = ***24/3***

***8*** = ***8*** It checks out. So, ***2n+2*** = 4/3***k***

B is the correct answer.

1. Triangle Problem
   1. Let’s write our givens:
      * + AB › AC
        + The triangle is isosceles. KNOW WHAT THAT MEANS: 2 sides and 2 angles (the ones opposite the equal lines) are equal.
   2. Draw the picture to match the givens and to be more to scale. It could be like either of the pictures below. Isosceles doesn’t mean that the 2 equal sides have to be longer than the 3rd, it just means that two sides in the triangle are equal. In both cases, AB › AC.

C

A

B

***y****°*

*z°*

***x****°*

B

***z****°*

***x****°*

*y°*

C

A

AB = BC BC = AC

x0 = z0 x0 = y0

* 1. What are we looking for? An answer that MUST BE false. Let’s check them.
     1. AB = BC. This is out because our first picture shows this being true.
     2. BC = AC. Our second picture proves this can be true. It’s out.
     3. ***x*** = ***y***. True in second picture. Out.
     4. ***x*** = ***z***. True in first picture. Gone.
     5. ***y*** = ***z***. Not possible. Look at the pictures. It’s also the only one not eliminated.

E is the correct answer.

1. Percentages with a wrinkle.
   1. Let’s write our givens:
      * + ***Tom’s Total Expenses*** were ***$240***.
        + ***Tom’s Hotel Expenses*** were ***20% of the trip expenses***.
        + Tom shared the room cost ***equally*** with 3 other people (4 people total in the room).
   2. What are we looking for? The ***Total Cost of the Hotel Room***.
   3. Focus only on the hotel room; do we really care about the burgers and plane tickets? First, figure out his money spent on the hotel room.
      * + ***Tom’s Hotel Expenses*** were 20% of ***Tom’s Total Expenses***
        + ***Tom’s Hotel Expenses*** = 20% × ***Tom’s Total Expenses***
        + ***Tom’s Hotel Expenses*** = 20% × ***$240***
        + ***Tom’s Hotel Expenses*** = ***$48***
   4. Remember the other given. Tom shared the room cost ***equally*** with 3 other people (4 people total in the room). So all ***4*** people in the room paid ***$48***.
      * + ***4*** × ***$48*** = ***Total Cost of the Hotel Room***
        + ***$192*** = ***Total Cost of the Hotel Room***

D is the correct answer.

1. Friends with no numbers. We need a picture.
   1. Let’s write our givens:
      * + Square gameboard with ***n*** rows that are ***n*** squares each.
        + ***k*** is number of outside squares (boundary).
   2. What are we looking for? A possible number for ***k***.
   3. Draw a picture. Let’s start simply here, with ***n*** = ***4***. So, we have 4 rows with 4 blocks each.

***4*** rows with ***4*** blocks each

* 1. What’s ***k*** in this situation? Count up the ***blue boxes***.
* ***k*** = ***12*** when ***n*** = ***4***. Not one of our answer choices. Let’s try one more ***n*** to see if we can spot a pattern.

***3*** rows with ***3*** blocks each

* 1. Let ***n*** = ***3***.
     + - ***k*** = ***8*** when ***n*** = ***3***.
  2. Notice anything about those k’s? They’re both divisible by 4. So, we have a pattern. Look at our answers; there’s only one divisible by 4, and it’s ***52***.

E is the correct answer.

* 1. Working this entire problem for every box up to 14 (the box that has ***52*** as ***k***) would take forever. Again, there’s some intuition here. If you get stumped, look at the answers and see if you can find a pattern. The best thing is to start small, and try different sizes of gameboards.

# Test 3 Section 2

1. System of equations to get us warmed up.
   1. Let’s write our givens:
      * + ***y*** = ***x*** - 5 **Equation 1**
        + 20***y*** - 5***y*** = 15 **Equation 2**
   2. What are we looking for? The value of ***x***.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfLook at the equations. We could plug in the value of ***y*** which is (***x*** - 5) into the second equation, but there’s an easier way. Let’s start with Equation 2.
      * + 20***y*** - 5***y*** = 15
        + 15***y*** = 15
        + ***y*** = ***1*** Now we’ve got ***y***. Plug it into the Equation 1.
        + ***1***  = ***x*** - 5

+5 +5

***6*** = ***x***

A is the correct answer.

1. Word Problem
   1. Let’s write our givens:
      * + 9 buttons total
        + ***4*** ***blues;*** ***3 reds;*** rest are ***yellow***
   2. What are we looking for?
      * + ***Probability*** of pulling a ***yellow*** button (when only pulling out 1 button).
   3. First figure out how many yellow buttons we have.
      * + Total = ***Blue*** + ***Red*** + ***yellow***
        + 9 =  ***4***  + ***3***  + ***yellow***
        + 9 = 7 + ***yellow***

-7 -7

* + - * ***2*** = ***yellow***
  1. Now, how do we figure the probability of pulling out one ***yellow***? That’s ***yellow*** /Total.
     + - ***yellow*** = ***2***

Total 9

C is the correct answer.

1. Geometry Problem
   1. Study the given picture and visualize.
   2. What are we looking for? The figure that will give us a complete circle when we put it together with the given figure. No overlap and no gaps.
   3. If you can’t see it, start drawing in the answers (***red outline*** below).
      1. With our picture below, we can see it doesn’t work. (A) is out.
      2. Our circle is a little bulgy because we are not artists, but you get the point. This one looks like a winner. If you like, try the others to confirm.

B is the correct answer.

1. Data and percentage problem with a circle.
   1. Study the picture. How many degrees are in a circle? 360°. Remember that.
   2. What are we looking for? The number of ***flavors*** that are less than 25% of total sales.
   3. What does 25% of a pie chart look like? What’s 25% of 360°?
      * + 25% of 360° or 360° = ***90°*** That’s a right angle.

4

* + - * Count up the ***flavors*** that look like they have central angles less than ***90°*** and we’re done.

***Strawberry***

***Chocolate***

***Vanilla***

***Mint***

***Peach***

***Pecan***

* + - * Looks like ***Pecan***, ***Peach***, ***Mint*** and ***Strawberry*** all have central angles smaller than ***90°***. That’s ***4*** ***flavors***.

D is the correct answer.

1. Triangles, lines and friends. It’s a party.
   1. What are we looking for? The ***value*** of ***x*** + ***y*** + ***z***.
   2. Mark up the drawing. First, let’s find the value of that third angle (***?***) in the triangle. The angles of a ∆ add up to 180°.
      * + ? + 37° + 58° = 180°

***y***°

***z***°

37***°***

58***°***

***?***

***x***°

37***°***

58***°***

***85°***

***x***°

***y***°

***z***°

* + - * ***?*** = ***85°***
  1. With this, we can solve for ***x****,* ***y & z***. How many degrees are there in a straight line? 180°. Write some equations to find our values.
     + - ***x***° + ***85***° = 180° Subtract 85° from both sides.
       - ***x***° = ***95***°
       - ***y***° + 37° = 180° Subtract 37° from both sides.
       - ***y*** ° = ***143***°
       - ***z*** ° + 58° = 180° Subtract 58° from both sides.
       - ***z*** ° = ***132***°
  2. Add ‘em up and we’re done.
     + - ***x*** + ***y*** + ***z*** = ***value***
       - ***95*** + ***143*** + ***132*** = ***360***

E is the correct answer.

1. Plain ole algebra.
   1. Let’s write our given: 6***x*** + 4 = 7
   2. What are we looking for? The ***value*** of 6***x*** - 4.
   3. Solve for ***x*** in the given.
      * + 6***x*** + 4 = 7

-4 = -4

* + - * 6***x*** = 3
      * 6***x*** = 3

6 6

* + - * ***x*** = 1/2
  1. Plug that ***x*** into the ***value*** expression.
     + - 6***x*** - 4 = ***value***
       - 6(***1/2***) - 4 = ***value***
       - 3 - 4 = ***value***
       - -1 = ***value***

B is the correct answer.

1. Geometry joy.
   1. Let’s write our given: Pentagon ABCDE is equilateral.
   2. What are we looking for? ***Ratio*** of ***arc ABC*** to ***arc AEC***.
   3. If the pentagon is equilateral, all the sides, AB, BC, CD, DE and EA are equal. Is it safe to assume all those arcs are equal as well? It sure is.

***A***

***B***

***C***

***D***

***E***

* 1. How many arc segments make up ***arc ABC*** and ***arc AEC***?
     + - ***arc ABC*** is made up of AB and BC. That’s ***2*** segments.
       - ***arc AEC*** is made up of CD, DE, and EA. That’s ***3*** segments.
  2. Plug and chug.
     + - ***arc ABC*** = ***2*** = ***Ratio***

***arc AEC*** ***3***

* + - * ***2/3*** in ratio form is ***2*** to ***3***

B is the correct answer.

1. Number line problem.
   1. Let’s write our given: The tick marks are ***equally*** spaced.
   2. What are we looking for? The ***letter*** that corresponds to (-1/2)2.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfSimplify the fraction. REMEMBER PEMDAS!! If you don’t, this problem will get you.
      * + -1 2 = (-1)2 = ***1***

2 (2)2 ***4***

* 1. Back to the number line.
     + - How big are those spaces? From ***0*** to  ***1*** there are 4 equal spaces. So one space is 1/4. Mark ‘em.

***0***

***1***

***-1***

***A***

***B***

***E***

***C***

***D***

***3***/***4***

***1***/***2***

***1***/***4***

* + - * Look familiar? That’s the value of our simplified fraction. So start at 0, go one space to the right (***1/4***), and that’s going to be our ***letter***. (***D***) it is.

D is the correct answer.

1. Friends.
   1. Let’s write our givens:
      * + ***t*** > ***w***
        + There’s another friend ***s***.
   2. What are we looking for?
      * + How much greater the sum of ***s*** and ***t*** is than the sum of ***s*** and ***w***.
   3. Name another (another??!) friend. ***x***. Let that be how much greater ***s*** + ***t*** is than ***s*** + ***w***.
   4. Write an equation that shows ***x*** by translating the “math speak.”
      * + ***x*** + (***s*** + ***w***) = (***s*** + ***t***).
        + The ***x*** goes on the (***s*** + ***w***) side because that is smaller than (***s*** + ***t***).
   5. Simplify the equation to get ***x*** by itself.
      * + ***x*** + ***s*** + ***w*** = ***s*** + ***t***

***-s*** = ***-s***

* + - * ***x*** + ***w*** = ***t***
      * ***x*** + ***w*** = ***t***

-***w*** -***w***

* + - * ***x*** = ***t*** - ***w***

C is the correct answer.

* 1. If working with variables is too much, we can also define them (give them number values) and solve. We’ve done this below. We get to the same place. Don’t be afraid of friends. They are just like numbers.
     + - Let ***t*** = ***2***.
       - Let ***w*** = ***1***. (***t*** > ***w***… ***2*** > ***1*** …that works)
       - Pick anything for ***s***. Let’s say ***s*** = ***5***.
  2. Plug our values into the equation and solve it.
     + - ***x*** + (***s*** + ***w***) = (***s*** + ***t***)
       - ***x*** + (***5*** + ***1***) = (***5*** + ***2***)
       - ***x*** + 6 = 7 Subtract 6 from both sides.
       - ***x*** + = ***1***
       - But 1 isn’t a possible answer because all the choices have ***s***’s, ***t***’s, and ***w***’s. Forget about the ***s*** ones. We could have picked anything for ***s***, and it doesn’t really matter. Focus on ***t*** and ***w***. We need to get our ***1*** in terms of ***t*** and ***w***.
       - ***t*** = ***2*** and ***w*** = ***1***. How can we get to our ***1***?
       - ***2*** - ***1*** = ***1***
       - ***t*** - ***w*** = ***1***

1. Friendly exponents.
   1. Let’s write our given:
      1. Equation: P(t) = 3,000 x 2t/4
   2. What are we looking for? How big the ***increase*** is from t = ***4*** to t = ***16***.
   3. Calculate the equation for our two t’s.
      * + t = ***4*** t = ***16***
        + 3,000 x 2(***4***/4) 3,000 x 2(***16***/4)
        + 3,000 x 21 3,000 x 2(4)
        + 3,000 x 23,000 x 16
        + ***6,000*** ***48,000***
   4. How do we calculate an ***increase***? Subtract the two values we just got.
      * + ***Increase*** = ***48,000*** - ***6,000***
        + ***Increase*** = ***42,000***

D is the correct answer.

1. Average friends. Not your best friends, just average.
   1. Let’s write our given: Average of 3, ***s*** and ***t*** is 5.
   2. We know what averages are. The sum of the terms divided by the number of terms. Write it out.

* 3 + ***s*** + ***t*** = 5

3

* 1. What are we looking for? The ***value*** of ***s*** + ***t***. Let’s isolate that in our equation and simplify.
     + - 3 + (***s*** + ***t***) = 5

3

* + - * 3 + (***s*** + ***t***) ***×*** ***3*** = 5 ***× 3***

3

* + - * 3 + (***s*** + ***t***) = 15

-3 -3

* + - * (***s*** + ***t***) = ***12***

E is the correct answer.

1. Vahndahful Venns. This is almost the exact same question as in Test 2, Section 2. These things do repeat.
   1. What are we looking for? The number of ***elements that are common*** in ***A*** and ***B***. The “common” and the “and” point us to an intersection. Outline the intersection. The intersection is the area where these two circles overlap, and where the number elements are enclosed by ***both*** the ***A*** and ***B*** circles. Forget about circle C, that’s just there to throw us off. We’ve left in the ***4***, ***8***, and ***6*** from circle C to show that those don’t factor into this at all.

***A***

***6***

***8***

***7***

***3***

***4***

***5***

***2***

***B***

* 1. There are only two numbers, ***5*** and ***2***, surrounded by both ***A*** and ***B***.
     + - Add those and we have our number of ***elements***.
       - ***5*** + ***2*** = ***7***

D is the correct answer.

1. Word problem. System of Equations.
   1. Let’s write our givens:
      * + 1,000 students in next year’s class
        + 800 students ***accepted*** so far
        + Of 800: 60% are *female* and 40% are *male*
   2. What are we looking for? How many ***new males*** must be accepted in the next bunch of acceptances so

Total females = Total males.

* 1. First, how many students are left to be accepted?
     + - ***Accepted***  + To Be Accepted = 1,000
       - 800 + To Be Accepted = 1,000
       - To Be Accepted = 200
  2. How many females and males are already ***accepted***?
     + - Females Males
       - 60% of Accepted 40% of Accepted
       - 60%(800) 40%(800)
       - ***480*** ***320***
  3. Write some equations.
     + - Total class has to be evenly split between girls and guys:
       - 480 + ***New*** ***Females*** = 320 + ***New Males*** Equation 1
       - Remaining students to be accepted must sum to 200:
       - ***New*** ***Females*** + ***New Males*** = 200 Equation 2
  4. Solve the system. Let’s make it easy on ourselves and isolate ***New*** ***Females*** so we can get rid of them when we get to the second part of the solving.
     + - ***New*** ***Females*** + ***New Males*** = 200

- ***New Males*** - ***New Males***

* + - * ***New*** ***Females*** = 200 - ***New Males*** Now plug that into Equation 1.
      * 480 + ***New*** ***Females*** = 320 + ***New Males***
      * 480 + (200 - ***New Males***) = 320 + ***New Males***
      * 680 - ***New Males*** = 320 + ***New Males*** Simplify.

+ ***New Males*** = + ***New Males***

* + - * 680 = 320 + 2(***New Males***)

-320 -320

* + - * 360 = 2(***New Males***)
      * 360 = 2(***New Males***)

2 2

* + - * 180 = ***New Males***

D is the correct answer.

1. Unequal friends.
   1. Let’s write our givens:
      * + ***t*** 2 - ***k***2 < 6 **Inequality 1**
        + ***t*** + ***k*** > 4 **Inequality 2**
        + ***t*** and ***k*** are positive integers. No negatives; no fractions.
   2. What are we looking for? The value of ***t***.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfWhat now? We can try replacing the < and > with = and solving the system, but it doesn’t work (we found that out 30 minutes after starting the problem).
   4. Let’s start with the second inequality, ***t*** + ***k*** > 4. We need two positive integers that satisfy that. Should we try 50 and 75? Not a good idea, because while it works for this second inequality, remember that we also need ***t*** 2 - ***k***2 to be less than 6. And it won’t work.
      * + So, let’s find the lowest numbers for ***t*** and ***k*** that will sum to just more than 4. How about 2 and 3? Which one should be which? Look at the first inequality, ***t*** 2 - ***k***2 < 6. If ***t*** = 2, this will not work. So we need the ***t = 3***.
        + ***t*** = ***3 k*** = ***2*** Does that satisfy **Inequality 2**?
        + ***t*** + ***k*** > 4
        + ***3*** + ***2*** > 4 That works. Now plug those values into **Inequality 1**.
        + ***t*** 2 - ***k***2 < 6
        + ***3***2 - ***2***2 < 6
        + 9 - 4 < 6
        + 5 < 6 Also true. We’ve confirmed our t and k. ***t*** = ***3***

C is the correct answer.

* 1. This is what we would call a “feel” problem. We can (and did when writing this book) get bogged down in working with the difference of squares and trying to make this a system of equations. But by keeping it simple, knowing that **Equation 1** forces the numbers to be small, we found 2 numbers that added up to just more than 4. We plugged them into both equations, and it worked. When it gets troublesome, find the easiest part of the problem, pick some easy numbers, and just start working!

1. “Math Speak”
   1. What are we looking for? The answer that’s the ***equivalent*** of ½ of 23 percent of 618.
   2. Translate it. If we try all the answers in our calculators, it’s going to bog us down.
      * + ½ ***of*** 23 ***percent*** ***of*** 618
        + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmf½ ***×*** 23 % ***×*** 618 Notice that (E) is out. There’s no % sign.

They want us to pick that one because it’s different.

* + - * ½ x 618 = 309. Just because the percentage is in the middle, doesn’t mean we

We can’t multiply the two numbers on the ends together.

* + - * 2***3*** % ***×*** 309 Reverse translate!
      * ***2***3 % ***of*** 309.

A is the correct answer.

1. Functions of functions. Nice.
   1. Let’s write our given: g(x) = f(3x + 1)
   2. What are we looking for? The ***value*** of ***g(2)***.
   3. Functions like this are just inputs and outputs. Look at the left side of the given.
      * + g(x): Functions are figures on graphs, so they need a general form. g(x) is that general form.
        + (x): The (x) is just the input. Put in a number for (x), and it spits out a specific output for g(x).
   4. Look at the right side of the equation. This is the “inner workings” of the function, the processor. We take whatever input (x) is on the left side, plug that number into wherever there is an x on the right side, and we get the output. That’s it.
   5. Don’t get confused by the f and g. When there’s a value for (x) on the left side, put that value in for (x) on the right side to generate the output.
   6. Solve it.
      * + g(x) = f(3x + 1)
        + ***g(2)*** = f(3(2) + 1)
        + ***g(2)*** = f(6 + 1)
        + ***g(2)*** = ***f(7)***
        + But there’s no 7 in the f(x) column! Easy tiger, what did we say about the (x)? It’s an input. Look at the table.

|  |  |
| --- | --- |
| x | f(x) |
| ? | ***f(7)*** |

* + - * That f(x) looks like the left side of the equation. What does that mean? The x is the same in both expressions, so x = 7. Look at our table.
      * When x = 7, ***f(7)*** = -5.
      * We said above ***g(2)*** = ***f(7)***, so ***g(2)*** = -5. That’s it.

A is the correct answer.

* 1. Run through (C), (D) and (E) if you want, but if you work carefully, you’ll have confidence in your answers.

1. Geometry. Remember your formulas.
   1. Let’s write our givens:
      * + AB, AC, BD and CD are ***semi-circular*** arcs. That means half-circles.
        + The diameter of the large circle, AD, is 6.
        + The dots divide AD into 6 equal parts.
   2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfWhat are we looking for? The ***area of the shaded regions***.
   3. Look at the picture. Let’s redraw the figure to create more circles.

***A***

***D***

***B***

***C***

***A***

***D***

***B***

***C***

Don’t these two figures

have the same shaded

area?

* 1. The picture on the right seems to be a little easier to deal with. We’ve got a big circle with a medium one taken out, and then a little one put back in. Can we make that a math expression? Of course.
     + - Big Circle with a medium one ***taken out*** and a little one ***added*** back in.
       - ***Big*** Circle Area ***-*** ***Medium*** Circle Area ***+*** ***Little*** Circle Area
  2. Now some numbers. What’s given? AD = 6, and it’s 6 equal segments long. How long is each segment?
     + - 6/6 = 1. Each segment is 1.
  3. What’s area of a circle? . r is the radius.
     + - ***Big*** Circle. What’s the radius? 6? NO! That’s diameter. The radius is ½ the diameter. Count the segments. ***Big*** radius is 3.
       - Area of ***Big*** Circle = π(3)2 = ***9***π
       - ***Medium*** Circle. What’s the radius? 2 segments. ***Medium*** radius is 2.
       - Area of ***Medium*** Circle = π(***2***)2 = ***4***π
       - ***Little*** Circle. What’s the radius? 1 segment. ***Little*** radius is 1.
       - Area of ***Little*** Circle = π(1)2 = ***1***π
       - Back to our expression.
       - ***Big*** Circle Area ***-*** ***Medium*** Circle Area ***+*** ***Little*** Circle Area = ***shaded area***
       - ***9***π  ***-*** ***4***π ***+*** ***1***π = ***shaded area***
       - ***6π*** = ***shaded area***

C is the correct answer.

1. Lines and Planes. We need to know our definitions.
   1. Let’s write our givens:
      * + 2 points make a line.
        + 6 points are in a ***plane***.
        + No 3 points are on same line.
   2. What are we looking for? The number of ***lines*** that can be drawn by pairs of the 6 points.
   3. What’s a plane? A flat surface that goes on forever. Like a desert. A piece of paper can be a plane, so let’s use this one, put 6 points on it and draw some ***lines***.

***1***

***2***

***3***

***4***

***5***

***6***

* 1. We’ve got ***Black***, ***Blue***, ***Red***, ***Purple*** and ***Orange*** lines. Count and add ‘em up.
     + - ***Black***  = ***6***
       - ***Blue*** = ***3***
       - ***Red*** = ***2***
       - ***Purple*** = ***3***
       - ***Orange*** = ***1***
       - ***Lines*** = ***15***

A is the correct answer.

1. Function party with lots of friends. This could get ugly.
   1. Let’s write our givens:
      * + We have a function, ***f***. What it is we don’t know.
        + ***f***(x + y) = ***f***(x) + ***f***(y)
        + ***a*** = ***b***
   2. What are we looking for? The ***statements*** that are ***true*** when ***a*** = ***b***. Great, that helps.
   3. Where do we start? First, we need to remember that all those friends just represent numbers. x could be 1, 5, 831, -5697, or maybe even ***a***. Since the answers all have *a* and *b*, let’s make it *a*. So let x = ***a*** and y = ***b***. That’s a good place to start. Now rewrite the givens.
      * + f(x + y) = f(x) + f(y)
        + f(***a*** + ***b***) = f(***a***) + f(***b***) That’s better. We’ve gotten rid of x and y.
   4. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfRemember ***a*** = ***b***. Let’s plug some numbers in for ***a*** and ***b***. Make it easy on us. How about 1 and 1?
      * + ***a*** = ***1***
        + ***b*** = ***1***
   5. What about the function, f? What if f(***a***) = ***a***? Let’s use that as our f. We could use a complex function, but why make it more difficult on ourselves? Keep it simple. This way, it doesn’t matter what we put in the ( ), it could be -123, 520\*@, or 1MM. f(-123) = -123. f(520\*@) = 520\*@. f(1MM) = 1MM. Whatever our input is, that’s our output. Ours are much simpler. ***1*** and ***1***.
      * + ***a*** = ***1*** ***b*** = ***1***
        + f(***a***) = ***a*** f(***b***) = ***b***
        + f(***1***) = ***1*** f(***1***) = ***1***
        + f(***a***) = 1 f(***b***) = ***1***
   6. We’ve got the right side of our equation. Now the left.
      * + f(***a*** + ***b***) We defined a and b above. Use the same values.
        + f(***1*** + ***1***)
        + f(2) What did we say before? Input is the same as output.
        + f(2) = 2
        + f(***a*** + ***b***) = 2
   7. Now we have the pieces; check to see if the equation works.
      * + f(***a*** + ***b***) = f(***a***) + f(***b***)
        + 2 = 1 + 1
        + That works. So, a = 1 and b = 1 are good numbers. Now try the different choices in the statements.

f(***a*** + ***b***) =2×f(***a***)

2 = 2×1 That works. I is a good answer.

f(***a*** + ***b***) =[f(***a***)]2

2 = [1] 2

2 = 1 Nope. II is out.

f(***b***) + f(***b***) = 2×f(***a***)

1 + 1 = 2×1

2 = 2 That works too. III is a good answer.

* + 1. ***I*** and ***III*** are the only choices that work.

C is the correct answer.

1. Ending with geometry and friends. What a way to end… MARK UP THE DRAWING!
   1. Let’s write our givens:
      * + Rectangular swimming area has an ***area*** of 4,000 m2.
        + Dimensions of pool are ***x*** and ***y***.
        + Rope divides the swimming area into three equal rectangles.
   2. What are we looking for? The ***total length of rope*** needed in terms of ***y***.
   3. First, let’s get warmed up and find an expression for the area of the swimming area with ***x*** and ***y***.
      * + ***Area*** = length × width
        + ***4,000*** = ***x*** × ***y***
        + ***4,000*** = ***xy*** We want to answer in terms of ***y***, so get ***x*** by itself.
        + ***4,000*** = ***xy***

***y y***

* + - * ***4,000*** = ***x***

***y***

* 1. Now the picture. Look at those 2 interior rope lines. Aren’t they the same length as ***x***? Indeed. Mark ‘em.

***x***

***x***

***x***

***x***

***y***

* 1. Aren’t all of our ropes now labeled? Now all we have to do is get an expression with all y’s to represent them and we’re done. Glad we had our little warm-up above.
     + - ***Length of rope*** = ***y*** + ***x*** + ***x*** + ***x*** + ***x*** Simplify that. Combine like terms.
       - ***Length of rope*** = ***y*** + 4(***x***) Now plug in our value of ***x*** from above.
       - ***Length of rope*** = ***y*** + 4(***4,000***)

***y***

* + - * ***Length of rope*** = ***y*** + 16,000

***y***

B is the correct answer.

# Test 3 Section 5

1. Averages to start with.
   1. Let’s write our givens:
      * + 3 guys: ***Fred***, ***Norman***, ***Dave***
        + Total comic books = 128
        + ***Dave*** owns ***44***.
   2. What are we looking for? The ***average*** number of books owned by ***Fred*** and ***Norman***.
   3. We know what averages are. The sum of the terms divided by the number of terms. Write it out.
      * + ***Fred*** + ***Norman*** = ***average***

2

* 1. We have to go through another equation first, because we don’t know ***Fred*** + ***Norman***. Use total books.
     + - 128 = Total comic books
       - 128 = ***Fred*** + ***Norman*** + ***Dave***
       - 128 = ***Fred*** + ***Norman*** +  ***44***

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* + - * ***84*** = ***Fred*** + ***Norman*** Look familiar? That’s the top of our average equation.
      * ***Fred*** + ***Norman*** = ***average***

2

* + - * ***84*** = ***average***

2

* + - * ***42*** = ***average***

A is the correct answer.

1. Circles and the Coordinate Plane. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Circle is tangent to x-axis and y-axis.
        + Circle is tangent to x-axis at point (6,0).
   2. What are we looking for? ***Coordinates*** of point ***P***.
   3. To the drawing. What does “tangent” mean? That the line touches the circle at one point. It also means that a radius from the center to that point forms a 90° (or right) angle with the line. What? Look at the second picture. The ***blue*** line is a ***radius*** and it is tangent to the y-axis. The ***red*** line is a ***radius*** and is tangent to the x-axis. The radii of a circle are equal, right? Of course. So we’ve got right angles and equal sides. Sounds like a square.

***(6,0)***

***P***

***(6,0)***

***P***

* 1. How can we find the ***coordinates*** of point ***P***? It’s on the y-axis, so the the point doesn’t move left or right. That means the x-value is 0. So we’ve got (o,y). What is that y?
  2. Back to the box. If the blue and red lines are equal, and we’ve already figured out that our box is a square, aren’t all the sides equal? Yes. What’s the distance from the origin (0,0) to (6,0)? 6. The y-values are the same, and the x-values are 0 and 6. The distance all comes from the x-values, so one side of our square is 6. That means ***all*** the sides of the squares are 6.
  3. Back to point ***P***. Finish up.
     + - Distance ***P*** = 6, because it’s a side of the square.
       - ***P*** extends from the origin (0,0) to point (0,y).
       - This means our y has to be 6. Because both x’s are 0, all the distance comes from y. So our ***coordinates*** of point ***P*** are ***(0,6)***.

B is the correct answer.

1. Word Problem
   1. Let’s write our givens:
      * + Service fee of $1.00 per day
        + $0.10 per copy made
        + ***n*** = copies made
   2. What are we looking for? A ***total charge*** expression using ***n*** for ***one*** ***day*** of copies.
   3. Two things going on here, the per day charge and the per copy charge. We’ve got one variable, ***n***. We need another for the ***days***. Let ***days*** = ***d***. Let’s write an equation.
      * + ***total charge*** = $1.00 × ***d*** + $0.10 × ***n*** We know ***d***; it was given. ***d*** = ***1***. Plug it in.
        + ***total charge*** = $1.00 × ***1*** + $0.10 × ***n*** That ***1*** drops out.
        + ***total charge*** = $1.00 + $0.10 × ***n***
        + ***total charge*** = 1.00 + 0.10***n***

D is the correct answer.

1. 3 friends in different pairings.
   1. Let’s write our givens:
      * + Three letters: ***a*** , ***b*** and ***c***
        + ***a*** paired with an ***a*** gives the pair a value of 2.
        + ***a***  paired with any other letter gives the pair a value of 1.
        + All other pairs have a value of 0.
   2. What are we looking for? The ***sum*** ***of the*** ***values of the pairs***.
   3. Using the rules, value the pairs.
      * + ***aa*** = 2 ***bc*** = 0

***ab*** = 1  ***aa*** = 2

***ac*** = 1  ***ba*** = 1

* + - * ***sum*** ***of the*** ***values of the pairs*** = 2 + 1 + 1 + 0 + 2 + 1
      * ***sum*** ***of the*** ***values of the pairs*** = ***7***

B is the correct answer.

1. Another square. MARK. UP. THE. DRAWING.
   1. Let’s write our givens:
      * + ***ABCD*** is a square.
        + ***AC*** = 4
   2. What are we looking for? The ***area*** of ***ABCD***.
   3. What is the ***area*** of a square? ***side × side***, or ***s2***. If we find the lengths of ***AB*** and ***BC*** we’ll be good to go.
   4. Special triangle bells should be going off in our heads. A diagonal of a square forms 45°/45°/90° triangles.

***B***

***C***

***4***

***A***

***D***

***B***

***C***

***A***

***s***

***s***

***s***

* 1. To the figure! Put in the right angles, too.
  2. ∆***ABC*** is a 45°/45°/90° triangle . Draw it and label it. In this case, ***s*** for the side of the square is the same as the ***s*** for the legs of the triangle.
     + - We know ***AC*** = 4. So:
       - s = 4 Get s by itself.
       - (s) 2 = 42
       - ***s2***× 2 = 16
       - ***s2***× 2 = 16 Divide by 2.
       - ***s2*** = 8

A is the correct answer.

1. Proportionality. Without numbers. No problem.
   1. Let’s write our givens:
      * + ***x*** ≠ 0
        + ***x*** is inversely proportional to ***y***.
   2. What are we looking for? The expression that is ***directly proportional*** to 1/ ***x*** 2.
   3. What do directly and inversely proportional mean? Let’s start with directly proportional.
      * + Let’s create an equation with our ***x*** & ***y***, and add ***k***.
        + We’ll let ***k*** be a constant so that ***y*** changes directly in proportion to ***x***. Let’s say ***k*** is ***2***.
          - ***xk*** = ***y***

***x2*** = ***y*** ***k*** is a constant, so ***y*** changes *directly* in proportion to ***x***.

* + - * + Let ***x*** = ***1***. (***1***)***2*** = ***y***
        + Try ***x*** = ***2***. (***2***)***2*** = ***y*** So when ***x*** goes up by 1***, y*** goes up by a factor of 2.
      * How about inverse proportions? This time let ***k*** be a constant so that ***y*** changes *inversely* in proportion to ***x***. Let ***k*** = ***2***.
        + ***y*** = ***k***

***x***

* + - * + Let ***x*** = ***1***. ***y*** = ***2***

***1***

* + - * + ***y*** = ***2***
        + Let ***x*** = ***2***. ***y*** = ***2***

***2***

* + - * + ***y*** = ***1***
        + Let ***x*** = ***4***. ***y*** = ***2***

***4***

***y*** = ***1*** So when ***x*** goes up by 1***, y*** goes down by a factor of 2.

***2***

* 1. Look at those proportions. With directly proportional, the ***x*** and ***y*** are both on the same side (the “top”) of a fraction. (Any number can be written as a fraction. It is the same as that number over 1 [e.g., 32 is the same as 32/1].) In inversely proportional, ***x*** and ***y*** are on opposite sides of the fraction. ***x*** is on the bottom, ***y*** is on the top.
  2. In our expression that’s given 1/ ***x*** 2, the ***x*** is on the bottom. ***x*** is inversely proportional to ***y***, so that means ***y*** has to be on top. But it’s not just ***x***, it’s ***x*** 2. So on the top, instead of ***y***, we’ve got ***y***2.

E is the correct answer.

1. Geometry Problem. DRAW THE PICTURE.
   1. Let’s write our givens:
      * + 8-sided polygon. It’s an octagon.
        + 8 equal sides; 8 equal angles
   2. What are we looking for? The number of triangles that can be drawn from point ***A***, a point on the octagon.
   3. ***Draw the picture.*** Draw all the ***diagonals*** from A. Name the other points, too, so we don’t get confused.

***A***

***H***

***G***

***E***

***D***

***F***

***C***

***B***

* 1. Start naming triangles.
     + - ∆ABC
       - ∆ACD
       - ∆ADE
       - ∆AEF
       - ∆AFG
       - ∆AGH That’s ***6 triangles***.

C is the correct answer.

1. A friends party. By far the most difficult question we’ve come across so far. Let’s give it a go.
   1. Let’s write our givens:
      * + ***k*** and ***m*** are constants.
        + The equation is true for ***all values*** of ***x***.
   2. What are we looking for? The value of ***m***.
   3. This thing is ugly. Not sure where to start? Looks like we have an ***x*** 2 on the right side. Can we get one of those on the left side? Sure. Let’s FOIL the left side.
      * + (***x*** -8)(***x*** - ***k***)
        + ***x***2 - ***kx*** - 8***x*** +8***k***
        + That doesn’t really seem to help, but write the whole equation.
        + ***x***2 - ***kx*** - 8***x*** +8***k*** = ***x***2 - 5***kx*** + ***m*** The ***x***2 drop away. That’s a small victory.
        + -***kx*** - 8***x*** +8***k*** = -5***kx*** + ***m*** We’ve simplified the equation. We’ll use it later.
   4. Now we need to think about the second given. This equation works for ***all values*** of ***x***. Go to our favorite number, 0. If the equation is true for all values of ***x***, o must work. Let ***x*** = ***0***.
      * + -***k***(***0***) - 8(***0***) + 8***k***  = -5***k***(***0***) + ***m***
        + 0 - 0 + 8***k*** = 0 + ***m***
        + 8***k*** = ***m***
        + Now we’ve got a direct relationship between ***k*** and ***m***. Multiply ***k*** by 8 and that’s our ***m***.
   5. Let’s assign some values for our ***k*** and ***m***. What about ***k*** = o? That doesn’t work, because when ***k*** = 0, ***m*** = 0, and that’s not one of our answer choices. Let’s try our second favorite number, 1. Let ***k*** = 1. When ***k*** = 1, ***m*** = 8. At this point, we need to set up a tableto keep us organized.

|  |  |
| --- | --- |
| ***k*** | ***m*** |
| 1 | 8 |
| 2 | 16 |
| 3 | 24 |

* 1. Remember, whatever ***k*** and ***m*** are, the equation must work for ***all values*** of ***x***. We’ve got some checking to do. Use our equation from above so it’s easier on us. Check the first pair of ***k*** and ***m*** in our table, ***k*** = ***1***, ***m*** = ***8***.
     + - -***kx*** - 8***x*** + 8***k*** = -5***kx*** + ***m***
       - -(***1***)***x*** - 8***x*** + 8(***1***) = -5(***1***)***x*** + ***8***
       - -9***x*** + 8 = -5***x*** + ***8***
  2. Now pick an ***x***. We know it works for ***x*** = ***0***, since that’s how we found 8***k*** = ***m***. So, let’s try ***x*** = ***1***.
     + - -9***x*** + 8 = -5***x*** + ***8***
       - -9(***1***) + 8 = -5(***1***) + ***8***.
       - -9 + 8 = -5 + 8.
       - -1 = 3 That doesn’t work. So ***k*** ≠ 1 and ***m*** ≠ 8. On to the next pair

in the table.

* 1. ***k*** = ***2*** , ***m*** = ***16***. Plug those in.
     + - ***-kx*** - ***8x*** + ***8k*** = -***5kx***  +  ***m***
       - -(***2***)***x*** - 8***x*** + 8(***2***) = -5(***2***)***x*** + ***16***
       - -10***x*** + 16 = -10***x*** + ***16*** That shows our pair works, but let’s try ***x*** = ***1***.
  2. Again, we know ***x*** = ***0*** works, so try ***x*** = ***1***.
     + - -10***x*** + 16 = -10***x*** + ***16***
       - -10(***1***) + 16 = -10(***1***) + ***16***
       - -10 + 16 = -10 + 16
       - 6 = 6 So ***m*** = ***16*** gives us an equation that works.

B is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write in” answer.*

1. Rate Problem
   1. Let’s write our given: Bird flew a constant speed and traveled 62 miles in 4 hours.
   2. What are we looking for? The number of ***miles*** the bird flew in 3 hours.
   3. Define a friend. Let ***x*** = ***miles*** the bird traveled in 3 hours.
   4. Write a proportion and solve.
      * + 62 miles = ***x*** miles

4 hours 3 hours

* + - * 62 = ***x*** Cross-multiply and finish up.

4 3

* + - * 62 × 3 = ***x*** × 4
      * 186 = 4***x***
      * 186 = 4***x*** Divide by 4.

4 4

* + - * ***x*** = ***93/2*** or ***46.5***

***93/2*** is the correct answer.

1. Geometry Problem. DRAW THE PICTURE.
   1. Let’s write our givens:
      * + Q, R, S, T lie on a circle with center P.
        + Radius of circle is 1.
   2. What are we looking for? ***Value*** of PQ + PR + PS + PT.

***P***

***R***

***T***

***S***

***Q***

* 1. Picturetime.
     + - Remember the definition of a radius? The line from the center of the circle to any point on the line. So, it doesn’t matter where we place our points; all the lines, PQ, PR, PS and PT, form radii. The radius of the circle is given; it’s 1.
  2. Write an equation for the value and finish up.
     + - ***Value*** = PQ + PR + PS + PT
       - ***Value*** = 1 + 1 + 1 + 1
       - ***Value*** = ***4***

***4*** is the correct answer.

1. Exponential friends.
   1. Let’s write our givens:
      * + 10***ab*** = 10,000
        + ***a*** and ***b*** are ***positive integers***.
   2. What are we looking for? One possible ***value*** of ***a***.
   3. Forget the ***a*** and ***b*** for a second. 10 raised to what exponent gives us 10,000? Don’t worry about logs and all that; just try some numbers on your calculator.
      * + Try 10 to the exponent of 3. 103 = 1,000. Close, but not yet.
        + What about 4? 104 = 10,000. That’s it.
   4. Now to ***a*** and ***b***. We know our exponent has to equal 4. In our given though, our exponent is ***ab***. We just figured out that our exponent has to be 4, so ***ab*** = ***4***. What are some combinations of positive integers that multiply to give us 4? How about 1,4 and 2,2? Those both work, so ***a*** could be ***1***, ***2*** or ***4***.

***1***, ***2*** or ***4*** is the correct answer.

1. Coordinate Plane Problem
   1. Let’s write our givens:
      * + 2***x*** - 3***y*** = ***c*** is a line.
        + The line passes through point (***5***,***-1***).
   2. What are we looking for? The value of ***c***.
   3. We don’t even need to know the equation of lines or draw the coordinate plane for this problem. We have x and y and an equation.
      * + (***x***,***y***)
        + (***5***,***-1***) Plug that into the equation of the line to find c.
        + 2***x*** - 3***y*** = ***c***
        + 2(***5***) - 3(***-1***) = ***c***
        + 10 + 3 = ***c***
        + ***13*** = ***c***
2. is the correct answer.
3. Analyzing a scatterplot. Study the graph carefully.
   1. Let’s write our given: Scatterplot shows male and female students in a given year.
   2. What are we looking for? ***Year*** with the greatest ***absolute value*** of difference between females and males.
   3. What does that mean? We don’t care if there are more females or males in a given year; we care only about the value of the difference.
   4. Two ways to do this:
      1. Method 1. Pick a year. Look at the graph and find the number of males and females.
         * + 1990: 600 females 600 males Absolute value of difference: 0
           + 1991: 700 females 650 males Absolute value of difference: 50
           + ***1992***: 800 females 650 males Absolute value of difference: ***150***
           + 1993: 800 females 850 males Absolute value of difference: 50
           + 1994: 650 females 750 males Absolute value of difference: 100
           + 1995: 800 females 750 males Absolute value of difference: 50
           + 1996: 900 females 850 males Absolute value of difference: 50
           + 1997: 1,100 females 1,150 males Absolute value of difference: 50
           + 1998: 1,050 females 1,100 males Absolute value of difference: 50
           + 1999: 1,000 females 1,000 males Absolute value of difference: 0
      2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfMethod 2. Look at 1990 and 1999. The numbers of females and males are equal those years. If we draw a line from those points, we will have a line that illustrates equal numbers of females and males. We see that some of the years are close to that line, and some are far from that line. When a year is close to the line, the difference is smaller than when it is farther way. Find the year that is farthest from the line, and we have our greatest difference. ***1992*** it is again.

***1992*** is the correct answer.

1. “ Math Speak” Word Problem
   1. Let’s write our given: Five times a ***number*** is the same as the number added to five.
   2. What are we looking for? The value of the ***number***.
   3. First make a friend. Let the ***number*** = ***x***.
   4. Translate the “math speak.”
      * + ***Five*** ***times*** a ***number*** ***is*** ***the same as*** the ***number*** ***added*** ***to*** ***five***.
        + ***5*** ***×*** ***x*** ***=*** ***x*** ***+***  ***5***
        + ***5x*** ***=*** ***x*** ***+*** ***5*** Simplify.
        + ***5x*** ***=*** ***x*** ***+*** ***5*** Subtract ***x*** from both sides.
        + 4***x*** = 5
        + 4***x*** = 5

4 4

* + - * ***x*** = ***5/4*** or ***1.25***

***5/4*** is the correct answer.

1. Angles. MARK UP THE DRAWING!
   1. Let’s write our givens:
      * + OD bisects ∠AOF
        + OC bisects ∠AOE
        + OB bisects ∠AOD
        + x = 40
        + y = 30
   2. What are we looking for? The ***measure*** of ∠***BOE***.
   3. To the drawing. Mark all those bisecting lines and put in the values of x and y. What does bisect mean? It means it cuts the angle in half.

***E***

***O***

***A***

***B***

***C***

***D***

*40°*

***F***

*30°*

***?***

* 1. This is ugly. But doable. Look at the ***green (?)*** angle, ∠***BOE.*** That’s what we’re looking for. Can we figure out any pieces of that right away? Indeed. First, look at the ***purple*** angles with ***2 marks***. We know that’s OB bisecting ∠AOD. We also know one of the angles (∠A***OB***). It’s ***40°***. So the other one (∠***BOD***)has to be ***40°***. Now we’ve got two small segments of ∠***BOE***.
  2. Next, find ∠***DOE***, and we’ll have all our pieces. This takes a few steps. First, write an equation.
     + - ∠***BOE*** = ∠***BOD*** + ∠***DOE***
       - ∠***BOE*** = ***40°*** + ∠***DOE*** Now we need to find∠***AOD***.
       - ∠***AOD*** =∠***AOB*** + ∠***BOD*** How can we find ∠***BOD***? Look at the big ***red*** angles.
       - ∠***AOD*** = ***40°*** + ***40° (***because OB bisects ∠AOD)
       - ∠***AOD*** = ***80°***
       - ∠***AOD*** = ∠***DOF*** (because OD bisects ∠AOF)
       - ***80°*** = ∠***DOF***
       - ∠***DOF*** = ∠***DOE*** + ∠***EOF***
       - ***80°*** = ∠***DOE*** + ***30°***
       - ***-30° -30°***
       - ***50°*** = ∠***DOE*** Now we’ve got all our pieces of ∠***BOE***.
       - ∠***BOE*** = ∠***BOD*** + ∠***DOE*** Back to our equation.
       - ∠***BOE*** = ***40°*** + ***50°***
       - ∠***BOE*** = ***90°***

***90*** is the correct answer. Don’t put in the ° in the grid sheet.

1. Integers. Friends.
   1. Let’s write our givens:
      * + All positive integers are in the sequence.
        + Each integer k appears k times.
        + Each integer is either the same as or 1 greater than the number before it.
        + The integer 12 first appears as the ***n***th term.
   2. What are we looking for? The value of ***n***.
   3. Look at the sequence: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4
      1. Where do we see the first 4? We can count all the integers before it. There are 6 integers before the first four, so the 4 first appears at place number 7.
      2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfAnother way to look at it is to add up the value of each integer before the 4 appears. That would be:
         * 1 + 2 + 3 = 6
         * But that just gets us through the 3’s. We need to add one to get to place number 4.
         * 6 + 1 = 7. That gets us to the same place.
      3. Now for our 12. We can write down the entire sequence up to the first 12 and count the integers, or we can use the method above to easily figure out the ***place value*** of the 12, which is our ***n***.
         * The numbers before 12 are 1 through 11. Add them up.
         * 1 + 2 + 3 + 4 + 5 + 6 + 7 +8 + 9 + 10 + 11 = 66
         * Remember though, that just gets us through the 11’s. We need to add one to get the ***place number*** 12.
         * 66 + 1 = ***67***. That’s our ***n***, the ***place value*** for where 12 first appears.

***67*** is the correct answer.

1. Geometry word problem. MARK UP THE DRAWING!
   1. Let’s write our givens:
      * + Paper strip is 80 inches long.
        + Notch formed by removing an ***equilateral triangle***.
   2. What are we looking for? The ***total length*** of notched edge.
   3. Look at the picture. Those triangles are equilateral, so we know that those small edges are 1. Mark ‘em.

***3***

***1***

***1***

***1***

* 1. So, it looks like we have a length of ***5*** for every segment that includes one straight edge (***3***) and 2 sides of the notched triangle (***1*** each). We need to find out how many of those repeating segments we have. If we know that, we can multiply the segments by our segment length (5) and have total notched length.
     + - Notched Segment Length × # of Segments = ***Total Length*** of notched edge.
       - Look at just the long side as if there were no notch. The length of that is ***4*** (***3*** + **1**). Divide the length of the paper strip (80) by that segment length (4) to find out how many segments we have.
       - # of Segments = Length of Paper Strip

***4***

* + - * # of Segments = ***80/4***
      * # of Segments = ***20***
      * ***Total Length*** of notched edge = Notched Segment Length × # of Segments
      * ***Total Length*** of notched edge  ***= 5***× ***20***
      * ***Total Length*** of notched edge =  ***100***

***100*** is the correct answer.

1. Parabolas and the coordinate plane. MARK UP THE GRAPH!
   1. Let’s write our givens:
      * + PQRS is a square with area 64.
        + Q, R and O lie on the graph of ***y*** = ***ax***2.
        + ***a*** is a constant.
   2. What are we looking for? The value of ***a***.
   3. What do we need to find ***a*** in our function? Values for ***x*** and ***y***. How about a point? That works; (***x***, ***y***) would give us two numbers to plug in. Now, how do we get there?
   4. Let’s start with the parabola. What do we know about parabolas that have a vertex at the origin? We know that the distance of two points at the same “height”, or y-value, is the same distance from the y-axis (have the same x-values). So Q and R are the same distance from the y-axis (let’s call that point ***z;*** it’s on the line QR at the y-axis).
   5. On to the square. If the area of the square is 64, what’s the length of a side?

***P***

***Q***

***S***

***R***

***O***

z

***8***

***8***

***8***

***4***

* + - * Area = side2
      * 64 = side2 Remember perfect squares? 82 = 64.
      * 64 = 82

***8***

* + - * 8 = side
      * ***PQ***, ***QR***, ***RS*** and ***SP*** = ***8***. Mark that.
      * So, ***QR*** = ***8*** and point z is right in the middle of line ***QR***.
      * That means ***Q***z=z***R*** = ***4***.
      * Looking at our marked up graph, can we find the coordinates of a point?

Let’s try ***R***?

* + - * Point ***R*** is ***4*** to the right of the y-axis, and it’s ***8*** above the x-axis. Get that in (***x*** , ***y***) form.
      * (***x*** , ***y***)
      * (***4*** , ***8***) are the coordinates of point ***R***.
  1. We needed an ***x*** and ***y*** value for our function. We have ‘em. Plug and chug.
     + - ***y*** = ***ax***2
       - ***8*** = ***a*** ×(***4***)2 Remember the exponent is just on the ***x***, not the ***a***.
       - ***8*** = ***a*** × 16
       - ***8*** = 16***a*** Divide both sides by 16.
       - ***1/2*** = ***a*** or ***a*** = ***0.5***

***1/2*** is the correct answer.

# Test 3 Section 8

1. Fractions.
   1. Let’s write our given: ¾ of a ***number*** is 18.
   2. What are we looking for? ¼ of the ***number***.
   3. First we need to define a friend. ***Let x*** = ***number***. Now convert the “math speak”t***o find*** ***x***.
      * + ¾ ***of*** a ***number*** ***is*** 18
        + ¾ ***×***   ***x =***  18
        + 3 ***x =*** 18

4

* + - * 3 ***x ×*** 4 ***=*** 18 ***×*** 4

4

* + - * 3 ***x*** ***=*** 72 Divide by 3.
      * ***x*** ***=*** ***24*** We’ve got ***x***. Find ¼ of ***x***.
      * ¼ ***of*** the ***number*** ***is*** ***our answer***.
      * ¼ ***×***   ***x =***  ***our answer***
      * ¼ ***×***   ***24 =***  ***our answer***
      * ***6 =***  ***our answer***

C is the correct answer.

1. Function with a weird symbol.
   1. Let’s write our givens:
      * + ***k*** represents an integer.
        + ***k***\* = ***k***(***k*** - 1)
   2. What are we looking for? ***Value*** of 5\*.
   3. This is just like a f(x) function, but instead of an x, they use a ***k***, and instead of f( ), they use \*. It means the same thing. Plug in the value of ***k*** into the right side of the function wherever there is a ***k***, and it spits out the answer. In our case, ***k*** = ***5***.
      * + ***k***\* = ***k***(***k*** - 1)
        + ***5***\* = ***5***(***5*** - 1)
        + ***5***\* = ***5***(4)
        + ***5***\* = ***20***

B is the correct answer.

1. Data Analysis. Study the graph carefully.
   1. Let’s write our givens:
      * + Diameter of pupils is on y-axis. Age is on x-axis.
        + Day is a solid line. Night is the dashed line.
   2. What are we looking for? Estimate of ***age*** when pupils during the day are the same as at night.
   3. Day ***equals*** night sounds like an intersection, where both pupils are the same.
   4. Where do the Day and Night lines cross? Right around age ***45***. That’s where diameter of the pupil is the same, day or night.

D is the correct answer.

1. Word problem with fractions.
   1. Let’s write our givens:
      * + Toni drives to work Monday through Friday.
        + 1 hour commuting to work; 1 hour commuting home
   2. What are we looking for? The ***fraction*** of ***total hours*** Toni spent ***commuting*** in the 5 day period.
   3. Write an equation for the ***fraction*** and solve it.
      * + ***Fraction*** = ***total commuting hours***

***total hours***

* + - * Figure out ***commuting hours***.
      * How many days from Monday to Friday? ***5***
      * ***Five*** days ***of*** ***one*** hour to work ***and one*** hour home
      * ***5*** ***×*** ( ***1*** ***+ 1*** )
      * ***5***×(***1*** + ***1***)
      * ***5***× (***2***)
      * ***10*** = ***total commuting hours***
      * Figure out ***total hours***.
      * How many hours in a day? 24. How many days? ***5***
      * ***Total hours*** = ***5*** × 24
      * ***Total hours*** = ***120***
      * ***Fraction*** = ***total commuting hours***

***total hours***

* + - * ***Fraction*** = ***10***

***120***

* + - * ***Fraction*** = ***1/12***

A is the correct answer.

1. Friends, factors and exponents. No problem.
   1. Let’s write our given: √3 = x + 1
   2. What are we looking for? The ***value*** of ***(x + 1)2***.
   3. We can find the value of ***x*** first, and then plug that in. But take a step back and look at it. What if we square both sides of our given equation?
      * + √3 = x + 1
        + (√3)2 = (x + 1)2
        + ***3*** = ***(x + 1)2*** Isn’t that what we’re looking for? Indeed. 3 is our answer.

C is the correct answer.

1. Triangle problem. DRAW THE PICTURE.
   1. Draw the triangle.

r°

x°

t°

10

8

9

* 1. What are we looking for? The ***true*** statement.
  2. What do we know about triangles? The long side is always opposite the largest angle, and the short side is always opposite the smallest angle.
     + - 10 > 9 > 8 So:
       - x > r > t That’s the same as:
       - ***t < r < x***

A is the correct answer.

1. Absolute Value Problem
   1. Let’s write our given: |6 – 5***y***| > 20
   2. What are we looking for? A possible value of ***y***.
   3. We can do this two ways. Get ***y*** by itself in the given, or just plug in possible answers. Let’s try plugging in.
      * + ***y*** = ***-3*** (Answer A)
        + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmf|6 – 5***y***| > 20
        + |6 – 5(***-3***)| > 20
        + |6 + 15 | > 20
        + |21| > 20 That’s a true statement. We’re done. ***y*** can be ***-3***.
        + We can try all the other answers, but -3 is the answer.

A is the correct answer.

* 1. To get ***y*** by itself, we can simplify the given inequality.
     + - |6 – 5***y***| > 20 Split it into two inequalities
       - 6 – 5***y*** > 20 6 – 5***y*** < -20
       - -6 -6 -6 -6
       - -5***y*** > 14 -5***y*** < -26 Dividing by a negative flips the ***<*** and ***>*** signs.

-5 -5 -5 -5

* + - * ***y*** ***<*** -14/5 ***y*** ***>*** 26/5 Ugly. We get mixed fractions.

***y*** ***<*** -2 4/5 ***y*** ***>*** 5 1/5

* + - * No answers are greater than 5 1/5, and only one answer is less than -2 4/5, answer A. Same place. But plugging in was a lot easier, and then we didn’t have to face getting tripped up on flipping the signs.

1. Right triangles. Know the rules. Know some Pythagorean triplets\*, and this problem takes 3 seconds.
   1. Let’s write our givens:
      * + ∆ABC is a right triangle.
        + The legs of ∆ABC are 3 and 4.
   2. What are we looking for? The ***perimeter*** of a corresponding triangle to ∆ABC with side lengths twice those of ∆ABC. What does that mean?
      * + Corresponding triangles have the same shape and angles.

***A***

***C***

***B***

***4***

***3***

***5***

***A***

***C***

***B***

***4***

***3***

* 1. Draw the picture of ∆ABC. The legs are 3 and 4.
  2. The hypotenuse is 5. Why? Because 3,4,5 is a Pythagorean triplet. Or we can use the Pythagorean Theorem, which is ***a***2 + ***b***2 = ***c***2.
     + - ***a***, ***b*** and ***c*** are sides of the triangle in this equation. Don’t get thrown by the points of this triangle being named A, B, C.  ***a*** and ***b*** represent legs, and ***c*** is the hypotenuse.
       - ***a***2 + ***b***2 = ***c***2
       - ***3***2 + ***4***2 = ***c***2
       - ***9*** + ***16*** = ***c***2
       - 25 = ***c***2
       - √25 = √***c***2
       - 5 = ***c***
  3. Now we have the sides of ∆ABC. Doesn’t it make sense that if our corresponding triangle has all sides ***twice*** the length of the sides of ∆ABC, that the perimeter would also be ***twice*** as big? Sure does.
     + - 3 + 4 + 5 = Perimeter of ∆ABC
       - 12 = Perimeter of ∆ABC
       - ***Perimeter*** of Corresponding Triangle = ***Twice*** the Perimeter of ∆ABC.
       - ***Perimeter*** of Corresponding Triangle = ***2×*** (12)
       - ***Perimeter*** of Corresponding Triangle = ***24***

D is the correct answer.

***\**** *Pythagorean triplets are good things to know. Some are (3,4,5), (5,12,13), (7,24,25). Being able to spot* *these* *can really speed up work on problems.*

1. Data problem. Mark up the table.
   1. What are we looking for? The ***company*** that has the ***highest ratio*** of foreign locations to US locations.
   2. First convert the “math speak.”
      * + Highest ratio of ***foreign locations*** to ***US locations***
        + ratio = ***foreign locations***

***US locations***

* 1. Now to the table. We need to add another column so we can divide ***foreign*** by ***US***.

|  |  |  |  |
| --- | --- | --- | --- |
| Company | US Locations | Foreign Locations | Foreign/US Ratio |
| ***A*** | ***1,400*** | ***4,000*** | 2.86 |
| ***B*** | ***1,300*** | ***3,300*** | 2.54 |
| ***C*** | ***1,012*** | ***3,618*** | 3.58 |
| ***D*** | ***1,242*** | ***2,268*** | 1.83 |
| ***E*** | ***762*** | ***2,100*** | 2.76 |

* + - * For Company A: ***Foreign***  = ***4,000***  = 2.86

***US 1,400***

* 1. Which one of those ***Foreign***/***US*** ratios looks the greatest? ***3.58;*** our winner is ***Company*** ***C***.

C is the correct answer.

1. Coordinate Plane Problem. Study the picture well.
   1. Let’s write our given: Line ***n*** is perpendicular to line ***l***.
   2. What are we looking for? The ***slope*** of line ***n.***
   3. Two ways to do this. Go straight to line ***n*** and find 2 points. We have one point, (0,0). We can eyeball another point on line ***n*** at (1,3). What’s the slope formula?
      * + ***Slope*** = y2 - y1 Rise

x2 - x1 Run

* + - * ***C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfSlope*** = 3 - 0

1 - 0

* + - * ***Slope*** = 3

1

* + - * ***Slope*** = ***3***

E is the correct answer.

* 1. The other way to do this is using the slope rule for perpendicular lines. Perpendicular lines have slopes that are ***negative reciprocals*** of each other. Take the slope of one line, flip the numbers in the top and bottom of the fraction, and change the sign. This gives us the slope of the perpendicular line.
  2. Look at line ***l***. We have two points, (0,1) and (3,0). The slope of of ***l*** is:
     + - ***Slope*** of ***l*** = 0 - 1

3 - 0

* + - * ***Slope*** of ***l*** = -1 Flip the numbers and change the sign to get slope of ***n***.

3

* + - * ***Slope*** of ***n*** = +3

1

* + - * ***Slope*** of ***n*** = ***3***
      * We get to the same place, but if you’re sketchy about finding the coordinates of the points on ***n***, use the points of ***l*** that are given and know for sure.

1. A couple friends here. This is similar to the tricky question in Section 5, but we only have one constant here.
   1. Let’s write our givens:
      * + ***k*** is a constant.
        + The equation is true for ***all values*** of ***x***.
   2. What are we looking for? The value of ***k***.
   3. On to the equation. Plug in a value for ***x***. We love o, but ***x*** = 0 will zero out our ***k*** term. Try ***x*** = ***1***.
      * + 2***x*** + 5 = 3***kx*** + 5
        + 2(***1***) + 5 = 3***k***(***1***) + 5
        + 2 + 5 = 3***k*** + 5
        + 7 = 3***k*** + 5 Subtract 5 from both sides.
        + 2 = 3***k*** Divide both sides by 3.
        + 2/3 = ***k***

E is the correct answer.

1. Number Problem. A little tricky.
   1. Let’s write our given: Eleven ***different*** integers add up to 0.
   2. What are we looking for? The ***least*** ***number*** of these integers that must be positive.
   3. At first glance, it looks like our answer is five, but that’s not right. Sketch out a number line. We know these numbers need to be around zero so the positives and negatives can cancel each other out.

***0***

***-1***

***-2***

***-3***

***-4***

***-5***

***1***

***2***

***3***

***4***

***5***

* 1. See how the -5 and 5 cancel each other out, as do all the pairs. We’re left with 0 when we add them up. But we’re trying to find the ***least*** ***number*** of positive numbers. Is there anything in the problem that says we must have the same number of negative and positive numbers? Nope. The integers just have to be ***different*** values.
     + - Could this work for lesser numbers of positive values? What about:
       - -6 + -5 + -4 + -3 + -2 + -1 + 0 + 2 + 3 + 4 + 12
       - Yes that works. Our negatives = -21, and our positives = 21. Total sum = 0. We just need all of our negatives to add up to our positives. One positive could cover all the negatives, but the positive number is going to be big. Look at the expression below.
       - -10 + -9 + -8 + -7 + -6 + -5 + -4 + -3 + -2 + -1 + 0 + 55
       - That sums to zero. We have one positive number. ***One*** is our ***least number*** of positive integers.

B is the correct answer.

1. Combination Problem
   1. Let’s write our givens:
      * + We have some tokens.
        + Each token has a value of ***1***, ***5*** or ***10***.
   2. What are we looking for? Total ***number*** of combinations that add up to 17.
   3. We could draw factor trees, but let’s just start with the biggest number (10) and try some combos.

***10*** + ***5*** + ***1*** + ***1*** = 17 That’s 1. Any more with a ***10***? Replace the ***5*** with 5 ***1***’s.

***10*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** = 17 Now try it with no ***10*** and 3 ***5***’s.

***5*** + ***5*** + ***5*** + ***1*** + ***1*** = 17 Now replace one ***5*** with ***1***’s.

***5*** + ***5*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** = 17 Replace another ***5*** with ***1***’s.

***5*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** = 17 Now all ***1***’s.

***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** + ***1*** = 17

* 1. We’ve exhausted the possibilities. There are ***6*** combos.

E is the correct answer.

1. A funky graph problem.
   1. Let’s write our given: ***y*** = f(***x***) drawn in the book.
   2. What are we looking for? The graph of ***y*** = 2f(***x***).
   3. We are not drawing all the graphs here, and instead we’ll talk a little about the theory of these functions. This thing is weird; it’s got some line segments, what looks like a parabola, and then another line segment. We could break it all out, but that would take more time than 3 math sections. Here’s the explanation.
      * + Basically, when we have a number (like 2, or 50) multiplied by the whole function, it just magnifies the function. It’s like grabbing the function at the top and bottom and stretching it vertically.
        + In ***y***  = 2f(***x***), we have the same function f(***x***), but the whole thing is multiplied by 2 to get the y-value.
        + To work this out, first let’s take a simple function like this: f(***x***) = 1***x*** + 1. So for f(***x***): ***y*** = 1***x*** +1 because ***y*** = f(***x***) (given)
        + When ***x*** = 1, ***y*** = 2. Point (1,2)
        + Now we’re going to use 50f(***x***) instead of 2f(***x***) to show this point. So, when we’re lookingfor 50f(***x***) we haveto multiply whatever the f(***x***) is by 50 to get our ***y***.
        + ***y*** = 50f(***x***)
        + ***y*** = 50(1***x*** + 1) Remember, f(***x***) = 1***x*** + 1.
        + ***y*** = 50(1(1) +1) Let ***x*** = 1.
        + ***y*** = 50(2)
        + ***y*** = 100
        + So, when ***x*** = 1, ***y*** = 100. (1,100) is our point here.
        + See how the effect on our new y is magnified because we multiplied the original f(***x***) by 50?
   4. Now check out our answers.
      1. This doesn’t look like a stretching out of the original graph, it’s just the original graph shifted up. That would be like saying the new function is f(***x***) + 2. The “y-intercept” has gone up two, so the graph shifts up. There is no change to the slope of the line or the shape of the parabola. (A) is out.
      2. The opposite of (A). The graph is just the same function, it’s just lower than the original. (B) is out.
      3. A new twist. This one is compressed. The line is steeper, the parabola is skinnier, but this isn’t what we need. This does not look stretched out at all. (C) is out.
      4. Now we have something. The line is steeper, the parabola is bigger, and it looks like the graph is grabbed from the top and bottom and stretched. This looks pretty good.
      5. This is the exact same graph as the original. How can we multiply something by 2 and have it give us the exact same thing? Not possible. (E) is out.

D is the correct answer.

1. Another number sequencing problem.
   1. Let’s write our givens:
      * + First term is 2.
        + Every term after that is ***-2×*** the proceeding term.
   2. What are we looking for? ***How many*** of the ***first 50 terms*** are less than 100?
   3. At first this looks daunting because of the 50 terms. It’s not bad.
   4. Start with our given numbers and plug in.
      * + 2, -4, 8 What’s the next term?
        + ***-2×*** (8) = -16 That’s less than 100. Next term.
        + ***-2×*** (-16) = 32 Still good. Keep going.
        + ***-2×*** (32) = -64 Still good. But that number is going up pretty quickly.
        + ***-2×*** (-64) = 128 That does not work.
   5. We can’t stop there though, because when we multiply 128 by -2, we get -256, which is less than 100. At this point we have: 2, -4, 8, -16, 32, -64, 128, -256... That’s 8 terms. But every other term is negative, how do we capture all those without doing this for 50 terms? Look at the numbers again, but forget about the negatives.
      * + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmf2, 4, 8, 16, 32, 64, 128, 256, ...
        + See a possible exponent and a common base in there? What about 2 as a base?
        + ***2***1 = 2, ***2***2 = 4, ***2***3 = 8, ***2***4 = 16, ***2***5 = 32, ***2***6 = 64, ***2***7 = 128, ***2***8 = 256. ***2*** is our base. Our positive sequence is:
        + ***2***1, ***2***2, ***2***3, ***2***4, ***2***5, ***2***6, ***2***7, ***2***8, ... , ***2***50
        + Now get our negatives back in there. Every other term starting with the 2nd term (even terms) is negative.
        + ***2***1, -***2***2, ***2***3, -***2***4, ***2***5 , -***2***6, ***2***7 , -***2***8 , … , -***2***50
        + 2, -4 , 8, -16, 32, -64, 128, -256, … , (very negative number)
        + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfSo we’ve got 50 terms, and every other one is negative. That means half of them are negative. 50/2 = 25. So ***25*** terms are negative, which means they are less than 100. Are there any positive terms less than 100? Indeed.
        + Remember ***2***1, ***2***3, ***2***5 (2, 8, 32). They are also less than 100. That’s ***3*** more terms that work.
   6. Add up our total terms that are less than 100.
      * + ***25*** + ***3*** = ***28*** terms are less than 100.

C is the correct answer.

1. Super 3-D Geometry Problem. Going to be tough to draw a picture, but try to.
   1. Let’s write our givens:
      * + Cube with volume of 8 cm3
        + Cube is inscribed in a sphere (3-D circle).
   2. What are we looking for? The ***diameter*** of the sphere.
   3. Start easy.
      * + What’s the length of a side of the cube? Well, the volume of a cube is side3, so:
        + side3 = 8
        + side = 2 That might help us, but who knows.
        + The ***diameter*** is the length of a line through the center of the sphere that touches both edges of the sphere.
   4. Picture time. Just use the cube first. We know that the cube touches the circle at all of the cube’s vertexes (corners), so draw a couple in there. Give the points on the corners some names to see if that will help us.

***A***

***B***

***C***

***D***

***E***

* 1. Now it gets abstract. This whole cube is surrounded by a sphere, and we’re looking for a line to go through the center of the sphere. Since all the vertexes touch the sphere, doesn’t this cube have to be centered in the sphere? Indeed it does. So if we can find the center of the cube, we’ll be closer.
  2. Look at line ***AD***. It goes from two opposite corners of the cube. That line goes through the center of the cube. If it goes through the center of the cube, it must go through the center of the sphere. It touches two points on the sphere, so isn’t that our definition of a ***diameter*** of the sphere? Yes. Look at the pictures below.
     + - The ***red circle*** is one ring on the sphere. It includes points A, E, D and B. Looking at it in 2 dimensions (right picture) shows that the center of the circle (which is the center of the sphere) is the center of the rectangle AEDC. That center point is the ***green dot*** in the middle of ***AD***.

3-D Sphere 2-D ring

***C***

***D***

***E***

***A***

***A***

***B***

***C***

***D***

***E***

* + - * We’ve confirmed that line AD goes through the center of the sphere, and we were given that it touches two points on the sphere. ***AD*** is our ***diameter***. Find that, and we’re done.
  1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfForget about the sphere now. We know ***AD*** is our ***diameter***. Back to the cube. We know the sides are 2, so label a couple of them. Right triangle bells should be going off in our heads.

***A***

***B***

***C***

***D***

***E***

***2***

***2***

***2***

* 1. We can’t find AD in one step, so first look at the triangles formed in our cube.
     + - A cube means right angles, so ∠ACD and ∠ABC are both right angles (90°).
       - Look at ∆ADC. We have one side, DC, which is 2. If we had AC, we could find ***AD***. What about ∆ABC? We have two sides, AB and BC, which are both 2. What do we know about a right triangle with legs that are the same length? It’s a 45°/45°/90° triangle. Pull that triangle out and draw it.
       - The second triangle shows the formulas for a 45°/45°/90° triangle.

***C***

***2***

***2***

***A***

***B***

***s***

***s***

***s√2***

***45°***

***45°***

***45°***

***45°***

* + - * s = 2. So AC = s√2.
      * AC = s√2
      * AC = 2√2
  1. Now we have two sides of ∆ADC. Pull that triangle out and label it. Remember ∠ACD is a right angle.

***D***

***2***

***2√2***

***A***

***C***

***?***

* + - * This is just a plain ole right triangle, so we need to use the Pythagorean Theorem.
      * a2 + b2 = c2 a and b are the legs, c is the hypotenuse. Plug and chug.
      * AC2 + CD2 = ***AD***2
      * (2√2)2 + (2)2 = ***AD***2 Square the expressions on the left side.
      * (22×[√2]2) + (2)2 = ***AD***2
      * (4 × 2 ) + (4) = ***AD***2
      * (8) + (4) = ***AD***2
      * 12 = ***AD***2 Take the square root of both sides.
      * √12 = √(***AD***2)
      * ***√12*** = ***AD*** Calculator time.
      * ***3.46*** = ***AD***

D is the correct answer.

* 1. We don’t need the calculator, we could have simplified under the radical.
     + - ***√12*** = √(3×4) The √4 is 2, so we can pull 4 out from under radical and put a 2 in front.
       - ***√12*** = ***2√3***

# Test 4 Section 3

1. Unequal friends. And a NOT.
   1. Let’s write our given: 3***b*** + 1 < 10
   2. What are we looking for? Which ***answer*** CANNOT be ***b***?
   3. Isolate the ***b*** in the inequality.
      * + 3***b*** + 1 < 10 Subtract 1 from both sides.
        + 3***b*** < 9 Divide both sides by 3.
        + ***b*** < 3 So which ***answer*** doesn’t work? ***3***

E is the correct answer.

1. Exponents Problem
   1. Let’s write our given: 24***x*** = 16
   2. What are we looking for? ***x***.
   3. Get the 16 to a base of 2 so we can work with this. 22 = 4, 23 = 8, 24 = 16.
      * + 24***x*** = 16
        + 24***x*** = 24 If our bases are the same (2), then the exponents must be equal. Lose the 2’s.
        + 4***x*** = 4 Divide both sides by 4.
        + ***x*** = 1

A is the correct answer.

1. Friends with a wrinkle. Old friends.
   1. Let’s write our givens:
      * + r - 2
        + r + 5
   2. What are we looking for? How much ***greater*** than r - 2 is r + 5? Sounds like subtraction.
   3. Lets’s give our variable a value. Go to our favorite, 0. Let r = 0.
      * + r - 2 r + 5
        + 0 - 2 0 + 5
        + ***-2*** ***5*** Now just subtract.
        + ***5*** - (***-2***) = 7.
        + ***5*** + ***2*** = ***7*** That’s the difference.

E is the correct answer.

1. Geometry Problem
   1. Let’s write our givens:
      * + Rectangle with no lid and sides as given
        + All the vertical edges are cut and the box is flattened.
   2. What are we looking for? The ***picture*** of the flattened box.
   3. Mark up the box. Add in another few sides to match them with the answers.

***4***

***2***

***2***

***3***

***4***

* 1. Now check out the answers.
     1. Is there any way we can have diagonals when this thing is flattened? No way. Those make it seem like the sides of the box are still connected. We were given that the vertical edges were cut, so they are not connected anymore. (A) is out.
     2. Ditto for (B). So, (C), (D) and (E) are left.
     3. Look at the bottom of the box. When the box is flattened, this is the rectangle in the middle. What are the dimensions of that rectangle? 3 by 4. In (C), the middle rectangle is 2 by 3. So (C) is out.
     4. Same reasoning as (C). The middle rectangle is 2 by 4. (D) is out.
     5. We’re left with (E). The middle rectangle is the only one out of (C), (D) and (E) that has the dimensions of 3 by 4.

E is the correct answer.

1. Geometry Problem. MARK UP THE PICTURE!!
   1. Let’s write our given: Movement is up or to the right.
   2. What are we looking for? How many ***paths*** there are from A to D that don’t include B or C.
   3. We can only move up or to the right. Draw the routes.

***C***

***B***

***A***

***D***

***C***

***B***

***A***

***D***

***C***

***B***

***A***

***D***

***C***

***B***

***A***

***D***

* + - * No more possible routes from A to B that don’t go through B or C. There are ***4 paths***.

B is the correct answer.

1. Friends and Fractions.
   1. Let’s write our given: 3/7 of ***n*** = 42
   2. What are we looking for? The ***value*** of 5/7 of ***n***.
   3. Translate the “math speak” to find ***n***.
      * + 3/7 ***of*** ***n*** = 42
        + 3/7 ***×***  ***n*** = 42 Isolate ***n***.
        + 3 ***×***  ***n*** ***× 7*** = 42 ***×*** 7 Multiply both sides by 7/3.
        + 7 3 3

***n*** = 42 ***× 7***

3

* + - * ***n*** = 294

3

* + - * ***n*** = ***98*** Now find 5/7 of that.
      * 5/7 ***×***  ***n*** = ***value***
      * 5/7 ***×***  ***98*** = ***value***
      * ***70*** = ***value***

A is the correct answer.

1. Geometry with proportions. MARK UP THE DRAWING.
   1. Let’s write our givens:
      * + Open square box divided into 6 sections (rectangles): ***A***, ***B***, ***C***, ***D***, ***E***, ***F***.
        + ***D***, ***E*** and ***F*** have ***twice*** the area of ***A***, ***B*** and ***C***.
   2. What are we looking for? ***Probability*** of a single marble being dropped in and landing in ***F***.
   3. To the drawing.

***C***

***B***

***A***

***D***

***E***

***F***

* 1. Give those rectangles some values. Keep it simple. Let ***A***, ***B*** and ***C*** = ***1***. That means ***D***, ***E*** and ***F*** = ***2***.

***1***

***1***

***1***

***2***

***2***

***2***

* 1. Now the probability. What is the total value of these boxes?
     + - ***1*** + ***1*** + ***1*** + ***2*** + ***2*** + ***2*** = ***9***
       - Our probability is going to be something over ***9***. We’re looking for the chance that one marble falls into F. Our probability is:
       - ***F*** We know ***F*** = ***2***.

***9***

* + - * ***2*** = ***Probability***

***9***

E is the correct answer.

1. Odd friends.
   1. Write the given: ***a*** and ***b*** are odd integers.
   2. What are we looking for? The ***expressions*** that are ***odd*** integers.
   3. Give ***a*** and ***b*** values. Keep it simple. 1 is odd, right? And it doesn’t say ***a*** and ***b*** have to be different. So, let them both be equal to 1. ***a*** = ***1*** , ***b*** = ***1***.
   4. Plug and chug to see which of our expressions work.

(***a*** + 1)***b***

(***1*** + 1)***1***

(2)***1***

2 That’s even. Choice (I) is out.

(***a*** + 1) + ***b***

(***1*** + 1) + ***1***

(2) + ***1***

***3*** That works. (II) is in.

(***a*** + 1) - ***b***

(***1*** + 1) - ***1***

(2) - ***1***

***1*** That works. (III) is in.

E is the correct answer.

1. Decimals.
   1. Let’s write our givens:
      * + We’re given a decimal: 5.***1***0***1***00***1***000***1***0000***1***…
        + The first ***1*** is followed by 1 zero, the second ***1*** by 2 zeros, etc.
   2. What are we looking for? The total number of zeros between the 98th and 101st ***1***.
   3. What does this sequence look like from the 98th to the 101st ***1***? Remember the ***1*** comes ***before*** its corresponding zeros.
      * + ***1***, (98 0’s), ***1***, (99 0’s), ***1*** , (100 0’s), ***1***… Now just add up the zeros
        + 98 + 99 + 100 = ***297***

D is the correct answer.

1. Regular algebra problem.
   1. Let’s write our given: f(***x***) = 3 - 2***x***2

***x***

* 1. What are we looking for? The ***value*** of f(2).
  2. Simple plug and chug. Put a ***2*** wherever there is an ***x*** and solve.
     + - f(***x***) = 3 - 2***x***2

***x***

* + - * f(***2***) = 3 - 2(***2***)2 Remember PEMDAS.

***2***

* + - * f(***2***) = 3 - 2(4)

***2***

* + - * f(***2***) = 3 - 8

***2***

* + - * f(***2***) = ***-5***

***2***

D is the correct answer.

1. Geometry Problem. MARK UP THE PICTURE!!
   1. Let’s write our givens:
      * + Picture ***not*** to scale
        + *l* ⊥ *n*
        + x > 90
   2. What are we looking for? The ***true*** statement.
   3. Draw the picture. In the book, it looks like *l* and *m* are parallel, but can that be true if x > 90? No, because if x > 90, line *m* will slope down from left to right.

***n***

***l***

***m***

***y°***

***x°***

So, (E) is out.

* 1. Now to the other answers.
     1. y < 90. How many degrees are there in a straight line? 180°. So x + y = 180. If x > 90, then y must be less than 90. This one seems pretty good. Let’s check the other ones.
     2. y > 90. If x > 90 (given), then x + y > 180. That would put a kink in our straight line n. (B) is out.
     3. y = 90. Same reasoning as (B). x + y > 180. Not a straight line. (C) is out.
     4. *n*  ⊥ m. A little trickier, but that means x and y both equal 90. We’re given x > 90. (D) is out.

A is the correct answer.

1. Coordinate Plane Problem
   1. Let’s write our givens:
      * + ***y*** = 5***x*** - 10 is the equation of the line.
        + The line crosses the ***x***-axis at point (***a***,***b***). ***a*** is the ***x***-value. ***b*** is the ***y***-value.
   2. What are we looking for? The value of ***a***.
   3. We don’t need a picture here. What is the ***x***-axis? It’s the horizontal line of the coordinate plane. A point on the x-axis does not have any “up” or “down” to it. That means for a point (***x***, ***y***) on the x-axis, ***y*** = ***0***.

So, the point we are looking for is then (***x***, ***0***). Let’s use our given equation, ***y*** = 5***x*** - 10, and plug in ***y*** = ***0***.

* + - * ***y*** = 5***x*** - 10
      * ***0*** = 5***x*** - 10 Add 10 to both sides.
      * 10 = 5***x*** Divide both sides by 5.
      * ***2*** = ***x***
      * Our point that crosses the ***x***-axis is (***2***, ***0***). We’re looking for ***a,*** which is the x-value.
      * ***a*** = ***2***

D is the correct answer.

1. Data and a median.
   1. Let’s write our givens:
      * + Table shows the noon temperatures of cities A – G.
        + ***Median*** temperature is ***40°***.
   2. What are we looking for? The noon temperature that ***City D*** (represented by ***t°***) ***cannot*** have, given that the ***median*** temperature is ***40°***.
   3. What’s a ***median***? The middle value. So first, line up the temperatures from lowest to highest. Then, after we put in our ***t°***, we will be able to identify the middle value, or ***median.*** 
      * + 27° 33° ***40°*** 44° 50° 68°
        + We’re looking for an answer *where 40° is not in the middle*. Let’s look at our answer choices, and plug them into our line of temperatures.
          - ***29°***: 27° ***29°*** 33° ***40°*** 44° 50° 68° (A) ***40°*** is in the middle. Nope.
          - ***35°***: 27° 33° ***35° 40°*** 44° 50° 68° (B) out.
          - ***39°***: 27° 33° ***39° 40°*** 44° 50° 68° (C) out.
          - ***40°***: 27° 33° ***40° 40°*** 44° 50° 68° (D) out.
          - ***42°***: 27° 33° 40° ***42°*** 44° 50° 68° (E) ***work***s, since the middle term is ***42°,*** not ***40°***.

E is the correct answer.

1. Geometry Problem. MARK UP THE PICTURE!!
   1. What are we looking for? The ***perimeter*** of the figure.
   2. Draw the picture. Notice those right angles at the top? That makes the outer shape a square, and three of the sides are 6. Fill in the missing side of the square.

***30°***

***30°***

***6***

***6***

***6***

***6***

* 1. What about that triangle at the bottom? Can we figure out those angles? Yes, because the angles of a square are 90°. That means that the 2 bottom angles (of the square) are 90°. What about the angles for the triangle? They must be 90° – 30° = ***60°***. Write them in.

***30°***

***6***

***60°***

***6***

***6***

***60°***

***6***

***30°***

* 1. We know two angles of the triangle. What’s the 3rd? ***60°***. So we have a ***60°***/***60°***/***60°*** triangle. Equiangular triangles are also equilateral, so all the sides are the same. One of the sides is 6, so they all are 6. Mark ‘em.

***30°***

***6***

***60°***

***6***

***6***

***60°***

***6***

***6***

***6***

***30°***

* 1. Now we have all the sides of our original 5-sided figure. Add them up to find the ***perimeter***.
     + - ***6*** + ***6*** + ***6*** + ***6*** + ***6*** = ***30***

D is the correct answer.

1. Prime factors. First time prime has appeared.
   1. Let’s write our givens:
      * + ***m*** is greatest prime factor of 38.
        + ***n*** is greatest prime factor of 100.
   2. What are we looking for? The ***value*** of ***m*** + ***n***.
   3. What does prime factor mean? A factor is just a number that can be divided equally into another number (in this case ***m*** or ***n***). A prime number can only be divided by itself and 1.
   4. Start a list of prime numbers:
      * + 1,2, 3, 5, 7, 11, 13, 17…
   5. Next go to 38 to find ***m***. Factor trees are great here.

38

19 2

* + - * That was easy. 19 can’t be divided by anything other than 1 and 19.
      * ***m*** = ***19***
  1. Factor tree 100 to find ***n***.

100

50 2

25 25

5 5 5 5

* + - * Same logic. 5 can’t be divided by anything other than 1 and 5.
      * ***n*** =  ***5***.
  1. Find ***m*** + ***n***.
     + - ***m*** + ***n***
       - ***19*** + ***5*** = ***24***

C is the correct answer.

1. More coordinate planes. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + Line ***l*** has a positive slope.
        + Line ***l*** passes through (0,0).
        + Line ***k*** is perpendicular to line ***l.***
   2. What are we looking for? Which statement ***must be*** ***exclusively true*** for line ***k***.
   3. Draw the base case picture. Remember, perpendicular means that the lines form a right angle.

***l***

* 1. Now draw the answer choices, and see if we can find a situation where the lines are perpendicular but the answer choices are not solely true, so we can eliminate them.
     1. Line ***k*** passes through (0,0)

***l***

***k***

***k***

***k***

(A) is out.

We have drawn ***k*** perpendicular to line ***l*** passing through point (0,0). But we have also drawn 2 other ***k*** perpendicular to ***l*** that do not pass through (0,0). Since ***k*** can pass through other points, (A) is out.

* + 1. Line ***k*** has a positive slope.

***l***

***k***

***k***

No ***k*** can be upward sloping and perpendicular to ***l***. (B) is out.

* + 1. Line ***k*** has a negative slope.

***l***

***k***

***k***

***k***

These 3 ***k***’s are all perpendicular to ***l*** and have negative slopes. Is there any way ***k*** can have a positive slope? No. (C) looks pretty good.

* + 1. Line ***k*** has a positive x-intercept.

***l***

***k***

Our ***k*** here is perpendicular to ***l*** and shows that ***k*** can have a positive x-intercept. But look back at the graph in (A). ***k*** can also have a negative x-intercept and a (0,0) intercept. So, (D) is out.

* + 1. Line ***k*** has a negative y-intercept.

***l***

***k***

Our ***k*** is perpendicular to ***l.*** We already know ***k*** can have a negative intercept (see (A)). We show here that ***k*** can also have a positive y-intercept. (E) is out.

C is the correct answer.

1. A strange function with friends.
   1. Let’s write our givens:
      * + ⇧ defined as ***a*** ⇧ ***b*** = ***a*** + ***b a*** ≠ ***b***

***a*** - ***b***

* + - * ***1*** ⇧ ***2*** = ***2*** ⇧ ***x***
  1. What are we looking for? The value of ***x***.
  2. Start with the ***1*** ⇧ ***2, let a = 1*** & ***b = 2,*** and plug it into our first given.
     + - ***a*** ⇧ ***b*** = ***a*** + ***b***

***a*** - ***b***

* + - * ***1*** ⇧ ***2*** = ***1*** + ***2*** = **3 = -3**

***1*** - ***2***  **-1**

* + - * ***1*** ⇧ ***2*** = -**3**
      * So, since ***1*** ⇧ ***2*** = ***2*** ⇧ ***x*** , then ***2*** ⇧ ***x*** = -***3***
  1. So, back to our first given, and this time plug in using ***2*** ⇧ ***x*** = -***3*** 
     + - ***a*** ⇧ ***b*** = ***a*** + ***b***

***a*** - ***b***

* + - * ***2*** ⇧ ***x*** = ***2*** + ***x***  = -***3*** Cross multiply and simplify.

***2*** - ***x***

* + - * ***2*** + ***x***  = -***3***

***2*** - ***x***

* + - * ***-3***×(***2*** - ***x***) = ***2*** + ***x***
      * -6 +3***x*** = ***2*** + ***x*** Subtract ***x*** from both sides.
      * -6 +2***x*** = ***2*** Add 6 to both sides.
      * 2***x*** *= 8* Divide by 2.
      * ***x*** = 4

A is the correct answer.

1. Word problem with friends.
   1. Let’s write our givens:
      * + One shirt can be bought for ***x*** dollars.
        + Each additional shirt costs ***x*** – ***z***.
   2. What are we looking for? An ***expression*** for the total cost of buying ***n*** shirts.
   3. Break it down. We need an expression for the cost of the first shirt, and then for the additional shirts.
      * + The first shirt costs ***x***. That never changes. The first part of our expression has to be ***x***.
        + ***Expression*** = ***x*** + (Total cost of additional shirts)
        + ***Expression*** = ***x*** + (Number of additional shirts)×(cost of each additional shirts)
        + ***Expression*** = ***x*** + (Number of additional shirts)×( ***x*** – ***z***)
        + Now we need an expression in terms of ***n*** for the number of additional shirts.
   4. Let’s start with a simple situation; the customer buys 3 shirts.
      * + The first shirt costs ***x*** dollars.
        + Each additional shirt costs ***x*** – ***z*** dollars.
        + Shirt 1 Shirt 2 Shirt 3

***x x*** – ***z x*** – ***z***

* + - * We know we have 3 shirts, so ***n*** = ***3***. How many cost ***x*** – ***z***? The 2 additional ones. What is 2 in terms of ***n***? It’s ***n*** - 1. So the number of additional shirts is ***n*** - 1. Try 4 shirts.
      * Shirt 1 Shirt 2 Shirt 3 Shirt 4

***x x*** – ***z x*** – ***z x*** – ***z***

* + - * 4 shirts, so ***n*** = ***4***. How many cost ***x*** – ***z***? The 3 additional ones. So (***n*** - 1) for the number of additional shirts makes sense. Plug that back into the ***expression***.
      * ***Expression*** = ***x*** + (Number of additional shirts)×( ***x*** – ***z***)
      * ***Expression*** = ***x*** + (***n*** - 1) ×( ***x*** – ***z***)
      * ***Expression*** = ***x*** + (***n*** - 1)( ***x*** – ***z***)

A is the correct answer.

1. Geometry Problem. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + QR is an arc of the circle with center P.
        + Length of QR is ***6π***.
   2. What are we looking for? The ***area*** of PQR.
   3. Draw the picture, including the complete circle.

***30°***

***Q***

***P***

***R***

***6π***

* 1. If we find the area of the circle, we can find the ***area*** of PQR.
     + - Start with the percentage of the circle that’s represented by sector ∠QPR.
         * 360° in a circle; ∠QPR is 30°. That means:
         * Sector/circle = 30°/360°- = 1/12. That means we can divide the area of the circle by 12 to get the area of sector ∠QPR.
       - What do we need for the area of a circle? The ***radius***. We can get that by finding the circumference of the circle. Write a proportion for the circumference of the circle and the sector.
         * 30° = ***length of QR*** .

360° circumference of circle

* + - * + 1 = ***6π*** . Cross multiply.

12 circumference of circle

* + - * + 12 × ***6π*** = 1 × circumference of circle
        + ***72π*** = circumference of circle
        + Now use the equation for circumference of a circle, 2π***r***, to find the ***radius***.

Circumference = 2π***r***

***72π*** = 2π***r*** Simplify to get ***r*** by itself.

2π 2π

***36*** = ***r*** We have our ***radius***.

* 1. Now we can find the area of the circle using π***r***2.
     + - Area of circle = π***r***2
       - Area of circle = π(***36***)2
       - ***Area of circle*** = ***1,296π***
  2. Divide the area of the circle by 12 to get the area of ∠QPR.
     + - ***Area*** of PQR = ***Area of circle*** /12
       - ***Area*** of PQR = ***1,296π*** /12
       - ***Area*** of PQR= ***108π*** Leave the π in there because it’s in the answers.

A is the correct answer.

1. Finishing with friends.
   1. Let’s write our givens:
      * + 75 more women than men
        + ***n*** men enrolled
   2. What are we looking for? An ***expression*** for the percentage of men enrolled, in terms of ***n***.
   3. First let’s find an expression for the men and women.
      * + men = ***n***
        + women = 75 ***more than*** men Convert the “math speak.”
        + women = 75 ***+ n***
   4. Now for the percentage.
      * + Part = Men .

Whole Men + Women

* + - * Men = ***n*** .

Men + Women ***n*** + (***n*** + 75)

* + - * Men = ***n*** .

Men + Women 2***n*** +75

* + - * We’re not done yet. Remember, to get to a percentage from a fraction, we have to multiply by 100 and put in the % sign.
      * Men% = 100 x ***n*** %.

Men + Women 2***n*** +75

* + - * Men % = 100***n*** %.

Men + Women 2***n*** +75

E is the correct answer.

* 1. Try it with numbers. If ***n*** = 2, the men/(men + women) fraction is 2/(2x2 +75) = 2/79. When we divide 2/79 we get 0.0253. That’s not a percentage. Multiply .0253 by 100 and put in the percent sign, we get 2.53%. That’s a percentage.

# Test 4 Section 6

1. Fractions and a symbol.
   1. Let’s write our given:
      * + 3 + ◊ = 7 1

2 2

* 1. What are we looking for? The ***value*** for ◊ that makes the statement true.
  2. The denominators are both two, so we need to get the top of these fractions to a point where we set them equal and solve for ◊. So, let’s start by converting 7 ½ into an improper fraction.
     + - 7 ½ = 15/2 Now we have:
       - 3 + ◊ = 15 Set the numerators equal and solve for ◊.
       - 2 2
       - 3 + ◊ = 15

-3 -3

* + - * ◊ = ***12***

D is the correct answer.

1. Lines and a tranversal. MARK UP THE PICTURE!
   1. Let’s write our given: ***l*** ∥ ***m***
   2. What are we looking for? ***Sum*** of which ***two angles*** equals ***∠2*** + ***∠4***.
   3. To the drawing.

***l***

***m***

***1***

***2***

***3***

***4***

***5***

***6***

***7***

***8***

* 1. Just by looking at the picture, it looks like ***∠8*** = ***∠4*** and ***∠6*** = ***∠2***. Do the rules confirm that? ***∠8*** and ***∠4*** are same side corresponding angles. Those have to be equal. ***∠6*** and ***∠2*** are also same side corresponding angles. The “same side” part means that the angles are on the same side of the transversal (the line that cuts the two parallel lines). If ***∠4*** = ***∠8***, and ***∠2*** = ***∠6***, then ***∠4*** + ***∠2*** = ***∠8*** + ***∠6***.

D is the correct answer.

1. Data problem.
   1. What are we looking for? ***unemployed women***.
   2. Fill in the table. Let’s start by looking at the women row. If we knew how many women were employed, we could subtract that from the total women to find the ***unemployed women***.
   3. We can find employed women by subtracting men employed (27,000) from the total employed (48,000).
      * + Total employed – Men employed = Women employed
        + 48,000 – 27,000 = ***21,000***
   4. Find ***unemployed women*** by subtracting ***employed women*** (***21,000***) from total women (21,500).
      * + Total women – ***employed women*** = ***unemployed women***
        + 21,500 – ***21,000*** = ***500***
        + ***unemployed women*** = ***500***

|  |  |  |  |
| --- | --- | --- | --- |
|  | Employed | Unemployed | Total |
| Men | 27,000 |  |  |
| Women | ***21,000*** | ***500*** | 21,500 |
| Total | 48,000 |  | 50,500 |

A is the correct answer.

1. Function with friends.
   1. Let’s write our givens:
      * + ***A*** is the net amount in dollars.
        + ***k*** is the number of cars washed.
        + ***A***(***k***) = 4***k*** -30 is the function.
   2. What are we looking for? The ***net amount*** when ***15*** cars are washed.
      * + Don’t let the ***A***(***k***) fool us. That’s just a f(x) function, where ***A*** is the f and ***k*** is the x input. Plug ***k*** = ***15*** into the function, and it will spit out our answer.
        + ***A***(***k***) = 4***k*** - 30
        + ***A***(***15***) = 4(***15***) - 30
        + ***A***(***15***) = 60 - 30
        + ***A***(***15***) = ***30***

E is the correct answer.

1. A friend party. Remember, friends are just like numbers.
   1. Let’s write our givens:
      * + ***xr*** = ***v***
        + ***v*** = ***kr***
        + ***rv ≠*** 0
   2. What are we looking for? The ***expression*** for ***k***.
   3. We need to isolate ***k***. Do that first.
      * + ***v*** = ***kr*** Divide by ***r***.

***r r***

* + - * ***v*** = ***k***

***r***

* 1. None of our answers have ***v*** or ***r***. Substitute the value of ***v*** from the first given and see what happens.
     + - ***xr*** = ***v*** or ***v*** = ***xr*** First given equation.
       - ***v*** = ***k***

***r***

* + - * ***xr*** = ***k*** Those ***r***’s cancel each other out.

***r***

* + - * ***x*** = ***k***

D is the correct answer.

1. Proportions. And Eggs.
   1. Let’s write our givens:
      * + Eggs are either white or brown.
        + Ratio of white to brown eggs is 2/3.
   2. What are we looking for? The ***value*** that ***cannot*** be the number of total eggs.
   3. Write the ratio.
      * + White = 2

Brown 3

* 1. If this were a small basket, and the numbers in the ratio were the only eggs, how many would we have?
     + - White + Brown = Total
       - 2 + 3 = 5
       - What if the basket were twice as big? 2 × 5 = 10 total eggs.
       - 3 times as big? 3 × 5 = 15 total eggs.
  2. See the pattern here? The total number of eggs has to be a multiple of 5. Check the answers.
     + - 10, 15, 30, and 60 are all multiples of 5. (A), (C), (D) and (E) are out.
       - ***12*** is not a multiple of 5.

B is the correct answer.

1. Radical friends.
   1. Let’s write our givens:
      * + 18 =
        + ***r*** and ***t*** are positive integers.
        + ***r*** > ***t***
   2. What are we looking for? A ***possible value*** of ***rt***.
   3. We need to be comfortable with radicals. Look at the 18. What are some factors of 18? 2 and 9 should pop out at us, because 2 x 9 = 18 and 9 is a perfect square. Rewrite 18.
      * + 18
   4. What now? Look at ***r***. We know the ***r*** > ***t*** , so the number on the outside of the radical has to be bigger than the number underneath the radical. = 3, so we can take that 9 out from under the radical, and put a 3 on the outside.
      * + Simplify.
   5. Does that satisfy ***r*** > ***t*** ?
      * + ***r***  = 54 , ***t*** = 2
        + ***r*** > ***t***
        + ***54*** > ***2*** Yes that’s true.
   6. So we have two potentials for ***r*** and ***t***. Plug those into ***rt***.
      * + ***rt***
        + ***54*** × ***2*** = ***108***

C is the correct answer.

1. Triangles and friends. MARK UP THE PICTURE!
   1. What are we looking for? The value of ***c*** in terms of ***a*** and ***b***.
   2. To the picture. Label the 3rd angle in the top triangle. Let’s call it ***x***.

***c***°

***b***°

***b***°

***b***°

***a***°

***a***°

***x***°

* 1. We can’t find ***c*** immediately, but if we find the value of ***x*** in the top triangle, we can find ***c***.
  2. ***x*** is one of three angles (the other two are “no-names”) that make up a straight line.
  3. The 2 no-name angles are in two other triangles that have 2 angles defined as ***a*** and ***b***.
     + - There are 180° degrees in a triangle, so our two no-name angles are:
       - No-name angle = 180° - ***a***° - ***b***°. Label them.

***c***°

***b***°

***b***°

***b***°

***a***°

***a***°

***x***°

180° - ***a***° - ***b***°

* 1. Back to our straight line. A straight line has 180°. So ***x***° and the two (180° - ***a***° - ***b***°) must add up to 180°.
     + - ***x***° + 2(180° - ***a***° - ***b***°) = 180° Get ***x***° by itself.

- 2(180° - ***a***° - ***b***°) - 2(180° - ***a***° - ***b***°)

* + - * ***x***° = 180° - 2(180° - ***a***° - ***b***°) Ugly, but now we have ***x***. Simplify.
      * ***x***° = 180° - 360° + 2***a***° + 2***b***°
      * ***x***° = -180° + 2***a***° + 2***b***°
  1. Back to the top triangle. ***x***° + ***b***° + ***c***° = 180° Substitute our expression for ***x***°.
     + - ***x***° + ***b***° + ***c***° = 180°
       - -180° + 2***a***° + 2***b***° + ***b***° + ***c***° = 180° Combine like terms. And simplify.
       - -180° + 2***a***° + 3***b***° + ***c***° = 180° Add 180° to both sides.

2***a***° + 3***b***° + ***c***° = 360° Get ***c*** by itself.

* + - * -2***a***° -3***b***° = -2***a***° - 3***b***°
      * ***c***° = 360° - 2***a***° - 3***b***°

E is the correct answer. We kept the ° in there, but it doesn’t really matter. Same answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. Exponent with a friend.
   1. Let’s write our given: ***t3*** = ***351***
   2. What are we looking for? The ***value*** of ***4t3***.
   3. We can find the value of ***t*** by finding the cube root of ***351***, but step back for a second. We’re looking for ***4t3***. That just means whatever ***t***3 is, we multiply it by ***4***. ***t3*** = 351. So ***4*** x ***351*** is our answer.
      * + ***t3*** = ***351***
        + ***4t3***
        + ***4*** × ***351*** = ***1,404***

***1,404*** is the correct answer.

1. Number Line Problem
   1. Let’s write our given: 2 points on a number line are 53 and 62.
   2. What are we looking for? The ***point*** that is exactly in the middle of 53 and 62. Average anyone?
   3. We don’t even need a picture. The number in the middle of two others is just the average of the two.
   4. An average is the sum of the terms divided by the number of terms.
      * + 53 + 62 = 115 = ***57.5***

2 2

***57.5*** is the correct answer.

1. Triangle Problem. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + 2 angles of the same measure.
        + Lengths of 2 sides are ***50*** and ***30***.
   2. What are we looking for? The ***least*** possible value for the ***perimeter***.
   3. Draw the picture. Remember that when 2 angles are the same in a triangle, it is an isosceles triangle. An isosceles triangle has 2 sides that are equal. 2 sides are ***30*** and ***50***, so the 3rd side has to be either ***30*** or ***50***.

***30***

***50***

***50***

***30***

***30***

***50***

* 1. What’s ***perimeter***? The sum of all the sides. Add them up for both triangles and see which one is smaller.
     + - ***30*** + ***30*** + ***50*** = ***110*** Perimeter 1
       - ***30*** + ***50*** + ***50*** = ***130*** Perimeter 2
       - ***110*** < 130, so ***110*** is the least value for the ***perimeter***.

***110*** is the correct answer.

1. Friends and factors.
   1. Let’s write our givens:
      * + ***x***2 - ***y***2 = 77
        + ***x*** + ***y*** = 11
   2. What are we looking for? The value of ***x***.
   3. ***x*** 2 - ***y***2 is the difference of perfect squares. Remember the formula: (***x*** 2 - ***y***2) = (***x*** + ***y***)( ***x*** - ***y***). First, plug in the numbers from the givens.
      * + (***x*** 2 - ***y***2) = (***x*** + ***y***)( ***x*** - ***y***)
        + 77 = 11 × ( ***x*** - ***y***) Simplify.
        + 77 = 11 × ( ***x*** - ***y***)

11 11

* + - * 7 =  ***x*** - ***y*** Subtract ***x*** to get ***y*** by itself .

-***x*** -***x***

* + - * 7 - ***x*** = -***y*** Multiply by -1.
      * -1(7 – ***x***) = -1(-***y***)
      * ***x*** - 7 = ***y***
  1. We now have two equations, ***x*** + ***y*** = 11 (given) and ***x*** - 7 = ***y***. Let’s solve for x.
     + - ***x*** + ***y*** = 11 Plug in the value of ***y*** into the given equation.
       - ***x*** + (***x*** - 7) = 11
       - 2***x*** - 7 = 11

+7 +7

* + - * 2***x*** = 18
      * 2***x*** = 18

2 2

* + - * ***x*** = ***9***

***9*** is the correct answer.

1. Geometry Problem. Pizza pie. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Pizza is cut into slices.
        + The angle of each slice is greater than 20° and less than 30°.
   2. What are we looking for? A ***possible number*** of pizza slices.
   3. Pick a value for the ***angle*** of the slice. What about right in the middle of 20° and 30°? Try ***25°***.
   4. T0 the picture.

***25°***

* + - * How many degrees are in a circle? 360°. That’s the whole pizza. One slice is ***25°***, so divide 360° by ***25°*** and we’ll have one ***possible number*** of pizza slices.
      * Whole Pizza = 360° = 14.4 slices.

One Slice ***25°***

* + - * That doesn’t quite work. We like even slices. What if we chop off the .4 from the 14.4 slices? If we divide 360° by that number (14), and the answer is between 20° and 30°, we’ve got an answer. Try it.
      * 360° = ***angle*** = 25.71°. That’s between 20° and 30°. ***14*** slices works.

14

***14*** is a correct answer. (***13***, ***15***, ***16*** and ***17*** also work)

1. Sequencing problem.
   1. Let’s write our givens:
      * + First term is ***a***.
        + Each term after the first is 3 × the preceding term.
        + First 5 terms add up to 605.
   2. What are we looking for? The ***value*** of ***a***.
   3. Find the first 5 terms. We’ve already got 2: ***a***, 3***a***, …
      * + 1st term = ***a***
        + 2nd term = 3***a***
        + 3rd term = 3(3***a***) = 9***a***
        + 4th term = 3(9***a***) = 27***a***
        + 5th term = 3(27***a***) = 81***a***
   4. Write an expression for the sum of the terms.
      * + ***a*** + 3***a*** + 9***a*** + 27***a*** + 81***a*** = 605 Combine like terms.
        + 121***a*** = 605 Divide by 121.
        + ***a*** = ***5***

***5*** is the correct answer.

1. Geometry Problem. MARK UP THE DRAWING!
   1. Let’s write our givens:
      * + ***QS***/***QV*** = ***1***/***3***
        + ***PT***/***PR*** = ***3***/***4***
   2. What are we looking for? The ***value*** of area of ∆PST

area of ∆PQR

* 1. Area of a right triangle is ½(base × height).
  2. To the drawing. The fractions in the givens give us values of QS, QV, PT and PR. Mark them.

***1***

***V***

***P***

***Q***

***R***

***S***

***T***

***4***

***3***

***3***

***1***

***V***

***P***

***Q***

***R***

***S***

***T***

***4***

***3***

***3***

***2***

* + - * ***QS*** = ***1***
      * ***QV*** = ***3***
      * ***PT*** = ***3***
      * ***PR*** = ***4***
  1. We have the base (***PR*** = ***4***) and height (***QV*** = ***3***) of ∆PQR.
  2. We have the base (***PT*** = ***3***) of ∆PST. We don’t have the height (***SV***) of ∆PST. We can find it, though. We have ***QV*** and ***QS***. Subtract ***QS*** from ***QV*** to get ***SV***.
     + - ***QV*** - ***QS*** = ***SV***
       - ***3*** - ***1*** = ***2***
       - The height of ∆PST = ***2***. Mark that.
  3. Use ½(base × height) to find the areas of ∆PST and ∆PQR.
     + - Area of ∆PST = 1/2 (base × height)
       - Area of ∆PST = 1/2 ( ***PT*** × ***SV*** )
       - Area of ∆PST = 1/2 ( ***3*** × ***2*** )
       - Area of ∆PST = 1/2 × (6)
       - Area of ∆PST = 3
       - Now ∆PQR.
       - Area of ∆PQR = 1/2 (base × height)
       - Area of ∆PQR = 1/2 ( ***PR*** × ***QV*** )
       - Area of ∆PQR = 1/2 ( ***4*** × ***3*** )
       - Area of ∆PQR = 1/2 × (12)
       - Area of ∆PQR = 6
  4. Substitute the areas into our fraction to find the ***value*** of the fraction.
     + - The ***value*** = area of ∆PST

area of ∆PQR

* + - * The ***value*** = 3/6 = ***½*** or ***.5***

***1/2*** is the correct answer.

1. Friend in a function.
   1. Let’s write our givens:
      * + h(***x***) = 14 + ***x***2

4

* + - * h(2***m***) = 9***m***
  1. What are we looking for? A possible value of ***m***.
  2. A regular function looks like f(***x***). In our case the f is an h, and our ***x*** is an ***m***. The second given just means that when we use 2***m*** as an input in the function, it spits out 9***m***. Plug 2***m*** into the function for ***x***.
     + - h(***x***) = 14 + ***x***2

4

* + - * h(2***m***) = 14 + (2***m***)2 Simplify.

4

* + - * h(2***m***) = 14 + 4***m***2 The 4’s go away.

4

* + - * h(2***m***) = 14 + ***m***2
      * Look at the second given, h(2***m***) = 9***m***. That means when we put in 2***m***, the output is 9***m***.
      * 9***m*** = 14 + ***m***2 Looks like a factoring situation. Get the terms on the same side.
      * -9***m*** = -9***m***
      * 0 = 14 + ***m***2 -9***m*** Rearrange the terms and factor to get this into the format of a

quadratic equation. Know the rules!

* + - * 0 = ***m***2 - 9***m*** + 14 14 has two factor pairs: 14, 1 and 2,7. Can’t get a 9 from 1 and 14.
      * 0 = (***m*** - 7)(***m*** - 2)
      * ***m*** = 7 or ***m*** = 2.

***7*** or ***2*** is a correct answer.

1. Data Analysis with Clocks.
   1. Let’s write our givens:
      * + 10 ***A*** clocks that chime n times on the nth hour and once on the half hour.
        + 5 ***B*** clocks that chime n times on the nth hour.
        + 3 ***C*** clocks that chime one time on the hour and one on the half hour.
   2. What are we looking for? The ***total chimes*** in the 90-minute period from 7:15 to 8:45.
   3. Understandings: ***n*** chimes on the ***n***th hour means at 2:00 the clock chimes 2 times, at 3:00 the clock chimes 3 times, etc. Between 7:15 and 8:45, there are only 3 times we care about, 7:30, 8:00 and 8:30.
   4. Make a different table showing the number of chimes at different times for the different types of clocks.

|  |  |  |  |
| --- | --- | --- | --- |
| Time | Clock ***A*** | Clock ***B*** | Clock ***C*** |
| 7:30 | 1 |  | 1 |
| 8:00 | 8 | 8 | 1 |
| 8:30 | 1 |  | 1 |

* 1. Count the total chimes per each type of clock.
     + - 1 + 8 + 1 = 10 chimes per ***A*** clock
       - 8 chimes per ***B*** clock
       - 1 + 1 + 1 = 3 chimes per ***C*** clock
  2. Multiply the chimes of each type of clock by the number of each type of clock.
     + - 10 chimes × (10 ***A*** clocks) = ***100*** total ***A*** chimes
       - 8 chimes × (5 ***B*** clocks) = ***40*** total ***B*** chimes
       - 3 chimes × (3 ***C*** clocks) = ***9*** total ***C*** chimes
  3. Add up the A, B and C chimes to get total chimes.
     + - ***100*** + ***40*** + ***9*** = ***149 total chimes***

***149*** is the correct answer.

1. Patterns and placement.

\*

\*

\*

\*

\*

\*

\*

\*

* 1. Let’s write our givens:
     + - 5 cards are:
       - The card can’t be on either end.
  2. What are we looking for? The ***total number*** of different arrangements of cards that are possible.
  3. Start making combos. Do it in an orderly fashion, so we catch them all.
     + - in the middle and first.
       - These are all the combos with first and in the 3rd spot. There are ***6***. If we keep in the

middle, there are 3 other cards, , and that can be 1st. Just like when is 1st, these

other 3 cards in the 1st position will also have ***6*** combinations.

* + - * So, we have ***4*** cards that can be first when the is in the 3rd spot, and they each have 6 combinations.

That’s ***4*** x ***6*** = ***24*** combos for ***one***location of the card. But we can also move .

It can be in ***3*** spots (2nd, 3rd, or 4th), because it can’t be on either end (1st or 5th).

* + - * (combos for one placement of ) × (number of places for ) = ***total arrangements***
      * ***24*** × ***3*** = ***72***

***72*** is the correct answer.

# Test 4 Section 9

1. “Math Speak” to start us off.
   1. Let’s write our givens:
      * + Same number of ***boys*** and ***girls*** on the bus when it leaves
        + At first stop 4 ***boys*** get off.
        + After first stop, there are twice as many ***girls*** as ***boys***.
   2. What are we looking for? The number of ***girls*** on the bus.
   3. This is a *system of equations* problem. We’ve got two friends, ***boys*** and ***girls***, so we need two equations.
   4. Convert the “math speak” at the beginning:
      * + Same number of ***boys*** and ***girls***
        + ***boys*** = ***girls***
   5. Now ***after*** the first stop:
      * + ***4*** ***boys*** get off
        + ***boys*** - ***4***
        + ***Twice*** as many ***girls*** as ***boys***
        + ***2*** ×(***boys*** - ***4***) = ***girls***
        + ***2boys*** - ***8*** = ***girls***
   6. Remember, ***boys*** = ***girls***, so now substitute in ***girls*** for ***boys*** in our equation.
      * + ***2boys*** - ***8*** = ***girls***
        + ***2girls*** - ***8*** = ***girls*** Simplify.

-***2girls*** -***2girls***

* + - * - ***8*** = -***girls*** Multiply by -1.
      * ***8*** = ***girls***

C is the correct answer.

1. Function rules.
   1. What are we looking for? The  ***graph*** that has a ***negative*** slope and ***positive*** y-intercept.
   2. This is a pure definition problem.
      * + What is a y-intercept? It’s the point where the graph crosses the y-axis (the vertical axis). If that must be positive, then the line has to cross the y-axis ***above*** the origin (0,0). (B), (C) and (E) are out. (A) and (D) both cross the y-axis in positive territory.
        + What’s a negative slope? That means the line slopes *down from left to right*. (A) slopes up from left to right. (A) is out. We’re left with (D). (***D***) slopes down from left to right, and crosses the y-axis in positive territory.

D is the correct answer.

1. Data analysis with donuts.
   1. What are we looking for? The ***closest approximation*** of the cost per donut when a box of 6 is purchased.
   2. This is an average problem. Average = (***total cost***)/(***number of donuts***). Plug in and solve.
      * + (***total cost***)/(***number of donuts***) = Average cost per donut
        + (***$1.89***) / (***6***) = ***$0.315***
        + ***$0.315*** is closest to ***$0.30***.

B is the correct answer.

1. More donut analysis.
   1. What are we looking for? The ***least*** ***amount*** of money needed to buy 21 donuts.
   2. Look at the average cost from problem 3). It’s $0.315. That’s less than buying 1 donut by itself at $0.40. When 12 donuts are purchased, the average cost is $0.299 ($3.59/12). So, when we buy more donuts, it costs less. If we could buy only 12 donut boxes, we’d find the cheapest combo. But 21 isn’t a multiple of 12, so we need to buy a mix of purchases, as follows:
      * + 1 box of 12 + 1 box of 6 = 18 Now we need to buy 3 individual donuts.
        + 1 box of 12 + 1 box of 6 + 3 individuals = 21 Plug in our cost info and solve.
        + 1×($3.59) + 1×($1.89) + 3×($0.40) = ***least*** ***amount***
        + $3.59 + $1.89 + $1.20 = ***least*** ***amount***
        + ***$6.68*** = ***least*** ***amount***

B is the correct answer.

1. Functions.
   1. Let’s write our given: Our function is ***h***.
   2. What are we looking for? ***h***(5).
   3. Don’t get confused by ***h***. It’s just like an f in f(x). So our 5 is our x. That’s the input. Our output in a function is the y-value that comes out when we put in x. Look at the point on the graph where x = 5. The y-value (the output) for this point is closest to ***3***.

C is the correct answer.

1. Angles. MARK UP THE PICTURE!
   1. Let’s write our given: 3 angles: 2***x***°, 3***x***° and 4***x***°
   2. What are we looking for? The value of ***x***.
   3. To the picture. How many degrees are there all the way around this figure? It’s a circle, so ***360°***.

***2x°***

***3x°***

***4x°***

***360°***

* 1. Write the equation and solve.
     + - 2***x***° + 3***x***° + 4***x***° = ***360°***
       - 9***x***° = ***360°*** Divide by 9.
       - ***x***° = ***40°***

C is the correct answer.

1. Friends and negative exponents (that are fractions too!)
   1. Let’s write our givens:
      * + ***x***, ***y*** and ***z*** are positive integers.
        + ***x***-1/2 = 1/3
        + ***yz*** = 16
        + ***z*** > ***y***
   2. What are we looking for? The ***value*** of ***x*** + ***z*.**
   3. We need a value of x and z. Start with x.
      * + ***x***-1/2 = 1/3 With a negative exponent we can flip the base’s position from the

numerator to the denominator and change the sign of the exponent.

* + - * 1 = 1 If the fractions are equal, then our denominators must be equal.

***x***½ 3

* + - * ***x***½ = 3 ½ in the exponent means it’s equal to the square root of the base (in this case ***x***).
      * = 3 Square both sides.
      * 2 = 32
      * ***x***= ***9*** One part of our answer down.
  1. Figure out ***z***.
     + - ***yz*** = 16
       - 16 is a perfect square. ***yz*** = ***42***. But that doesn’t work because ***z*** > ***y***. What about ***yz*** = ***24***?
       - ***24*** = 16, and ***4*** > ***2***, so ***y*** = ***2*** and ***z*** = ***4***. Now we have both pieces of our answer.
  2. Solve for the ***value*** of ***x*** + ***z***.
     + - ***value*** = ***x*** + ***z***
       - ***value*** = ***9*** + ***4***
       - ***value*** = ***13***

D is the correct answer.

1. Circle on the coordinate plane. MARK UP THE DRAWING!
   1. Let’s write our given:
      * + Center of semi-circle is (4,0).
   2. What are we looking for? The ***x-coordinates of two points*** on the circle that have the same y-value.
   3. The ***y***-coordinate is the second number in a point (***x***,***y***). So we need those ***y***’s to be the same. Look at the graph. The first two points that jump out at us are (0,0) and (8,0). Those both have the same ***y***-value, meaning they have the same height. The ***x*** -values are 0 and 8. Unfortunately, that’s not an answer choice.
   4. Mark up the drawing. Try the different answers. Remember, these are the ***x*** -values of our points.

***(4,0)***

***(8,0)***

***(0,0)***

***(4,0)***

***(8,0)***

***(0,0)***

***(4,0)***

***(8,0)***

***(0,0)***

***(4,0)***

***(8,0)***

***(0,0)***

***(4,0)***

***(8,0)***

***(0,0)***

(C) ***2*** and ***6***

(A) ***1*** and ***6***

(B) ***1*** and ***8***

(E) ***1*** and ***8***

(D) ***2*** and ***8***

* 1. Of all the answers, only one pair of ***x***-values has the same height, or y-value: ***2*** and ***6***.

C is the correct answer.

1. Remainders again. We saw this in the first section of the first test.
   1. Let’s write our givens:
      * + ***p*** is an integer.
        + 2***p*** + 7 divided by ***5*** has a remainder of ***3***.
   2. What are we looking for? A possible ***p***.
   3. First, what is a remainder? It’s the amount left over in division. Here’s an example:

What’s the remainder when we divide 5 by 2?

2 r ***1*** ***1*** is the remainder in this example

2√5

* 1. Back to our problem. Write the skeleton of the problem, in this case just the division sign.

√

* 1. Now let’s fill it in with the given information.

r ***3*** From the statement: “when 2***p*** + 7 is divided by ***5*** the

***5***√2***p*** + 7 remainder is ***3***.”

That looks a little more manageable.

* 1. Having ***3*** left over means that the number under the division sign needs to be ***3*** greater than a multiple of ***5***. What are some multiples of 5? 10 is a pretty easy one to work with. Now add ***3*** to that.
     + - 10 + ***3*** = ***13***. That’s our number we need to put under the division sign. What’s the remainder when we divide ***13*** by 5?
       - 2 r ***3*** So we’ve shown that ***13*** under the division sign gives us a remainder of ***3***. Now set 13 equal to 2***p*** + 7and solve for ***p***.
       - 2***p*** + 7= ***13*** Subtract 7 from both sides.
       - 2***p*** = ***6*** Divide both sides by 2.
       - 2***p***  = 6
       - ***p***  = ***3***

B is the correct answer.

1. Sequence problem.
   1. Let’s write our given: Stacy is the 12th tallest and 12th shortest in the class.
   2. What are we looking for? The number of ***total students*** in the class.
   3. Break down the given.
      * + Stacy is the 12th tallest. How many people are taller than her? 11.
        + Stacy is the 12th shortest. How many people are shorter than her? 11.
        + That’s 22 students in the class, right? Nope. We left out Stacy.
        + Shorter students + Stacy + Taller students = ***total students***
        + 11 + 1 + 11 = ***23***

B is the correct answer.

1. Quadratic function graph. Quadratic just means it’s some sort of parabola.
   1. Let’s write our givens:
      * + ***g***(x) = ***a***x2 + bx + ***c***
        + ***a*** and ***c*** are ***negative constants***.
   2. What are we looking for? A possible graph of ***g***.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfFirst let’s talk about a line equation, y = mx + b. In a line equation, the m tells us our slope. The b tells us the y-intercept (where the graph crosses the y-axis). Similarly, in our quadratic equation here, the ***a*** tells us the shape and whether or not the parabola opens up (***a*** is positive) or down (***a*** is negative). Also, in a parabola, the ***c*** tells us where it crosses the y-axis.
   4. We know that ***a*** and ***c*** are both negative. If ***a*** is negative the parabola opens down. If ***c*** is negative, it means it crosses the y-axis in negative territory. The only graph that satisfies this is (***A***).

A is the correct answer.

1. Geometry Problem. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + ABCD is a rectangle.
        + BC = 4
        + AB = 6
        + P, Q and R are on a line (draw it in).
        + P and Q are symmetric about line AB.
        + Q and R are symmetric about line CD.
   2. What are we looking for? The ***length of PR***.
   3. To the picture. If P and Q are symmetric around AB, that means that the length from P to the edge of the rectangle is the same as the length from Q to the edge of the rectangle (so, the segments are congruent). Same thing goes for Q and R being symmetric about CD. Draw in the line ***PR***, and mark those congruent segments.

***A***

***B***

***C***

***D***

***R***

***Q***

***P***

* 1. We know that BC = 4. So + = ***4***.
  2. We’re looking for the length of ***PR***. Look at the picture. In mark terms that’s + + + . That’s the same thing as 2 × ( + ). Plug in our value of + , which is 4.
     + - ***PR*** = 2 × ( + )
       - ***PR*** = 2 × (***4***)
       - ***PR*** = ***8***

B is the correct answer.

1. Percentage problem.
   1. Let’s write our givens:
      * + Price of a phone was increased by 10%.
        + New price is then decreased by 25%.
   2. What are we looking for? The ***percentage*** that the ***final*** price is of the ***initial*** price.
   3. This is a multi-stepper. There are three prices here, ***initial***, ***intermediate*** and ***final***. Start by picking a number for the ***initial*** price. Since we’re dealing with percentages, let’s say 100.
      * + ***initial*** price = 100.
        + Find the ***intermediate*** price.
        + ***intermediate*** price = ***initial*** + 10% or ***initial*** price × 110%
        + ***intermediate*** price = 100 × 110%
        + ***intermediate*** price = 110
        + Next find the final price.
        + ***final*** price = ***intermediate*** price - 25% or ***intermediate*** price × 75%
        + ***final*** price = 110 × 75%
        + ***final*** price = 82.5
   4. Now we need to find our percentage. We’re looking for the ***final*** price as a ***percentage*** of the ***initial***.
      * + ***final*** price × 100% = ***percentage***

***initial*** price

* + - * 82.5 × 100% = ***percentage***

100

* + - * .825 × 100% = ***percentage***
      * ***82.5%*** = ***percentage***

C is the correct answer.

1. “Math Speak” Problem
   1. Let’s write our given: ***w*** multiplied by 4 is the same as 4 added to ***w***.
   2. What are we looking for? The ***value*** of 3***w.***
   3. Translate the “math speak.”
      * + ***w multiplied by*** 4 is ***the same*** as 4 ***added to*** ***w***
        + ***w ×*** 4 ***=*** 4 ***+*** ***w***
        + 4***w*** ***=*** 4 ***+ w*** Subtract ***w*** from both sides.
        + 3***w*** ***=*** 4 Isn’t this what we’re looking for? Yes! Technically, we’re done.
        + 3***w*** ***=*** 4 But let’s validate and solve for w. Divide both sides by 3.
        + ***w*** ***=*** ***4/3***  But we’re looking for the of 3 ***times*** ***w***.
        + ***value*** = 3***w***
        + ***value*** = 3 × ***4*** The 3’s cancel out.

***3***

* + - * ***value*** = ***4***

E is the correct answer.

1. Pythagorean friends.
   1. Let’s write our givens:
      * + The sides of a right triangle are consecutive ***even*** integers.
        + The shortest side is ***x***.
   2. What are we looking for? The ***expression*** that could be used to find ***x***.
   3. Let’s review our definitions.
      * + 1, 2 and 3 are consecutive integers; 234, 235 and 236 are also consecutive.
        + We’re dealing with consecutive ***even*** integers, so that would be 2, 4, 6, or 234, 236, 238.
   4. If our shortest side is ***x***, what are the next two consecutive ***even*** integers? ***x*** + 2 and ***x*** + 4. All done now; the answer is (D), right? Nope. That’s because this is a right triangle, and to solve for x we need to use Pythagorean Theorem.
   5. The Pythagorean Theorem is a2 + b2 = c2. a and b are the legs, and c is the hypotenuse (longest side).
      * + a2 + b2 = c2 Plug in our ***x*** terms into the Pythagorean Theorem.
        + ***x*** 2 + (***x*** + 2)2 = (***x*** + 4)2 = ***expression***

C is the correct answer.

1. Friends and fractions.
   1. Let’s write our givens:
      * + ***x*** is an integer greater than 1.
        + ***y*** = ***x*** + 1

***x***

* 1. What are we looking for? The ***statements*** that must be true.
  2. Let’s define our friends.
     + - Let ***x*** = ***2*** (it has to be greater than 1). Find ***y***.
       - ***y*** = ***x*** + 1

***x***

* + - * ***y*** = ***2*** + 1

***2***

* + - * ***y*** = ***2*** Convert it to an improper fraction so we can work with it.
      * ***y*** = ***5/2***
  1. Now look at the statements.

I.  ***y*** ≠ ***x***

***5/2*** ≠ ***2***

That’s true. Always? Yes. Even if ***x*** is a really big number, 1/***x*** is going to be something greater than o, so ***x*** + 1/***x*** is always going to be bigger than ***y***. If ***x*** is 1,000,000, ***y*** is 1 + 1/1,000,000. 1/1,000,000 is really small, but adding that to 1 to get ***y*** still makes ***y*** bigger than ***x*** by 1/1,000,000. (***I***) is always true.

***y*** is an integer.

***5/2***

That’s definitely not an integer. So, we proved (II) can be false with our first value of ***x***.

***xy*** > ***x***2

***2***(***5/2***) > ***2***2

5 > 4

That’s true. Always? Yes. We’re given that ***y*** is always going to be bigger than ***x*** (even if it’s only 1/1,000,000 bigger), and ***x***2 is just ***x*** × ***x***. So, ***x*** × ***y*** > ***x*** × ***x***. (***III***) is always true.

D is the correct answer.

# Test 5 Section 2

1. Starting with a friend.
   1. Let’s write our given: 3***x*** + 9 = 5***x*** + 1
   2. What are we looking for? The ***value*** of ***x***.
   3. Simplify to find x.
      * + 3***x*** + 9 = 5***x*** + 1 Subtract 3***x*** from both sides.
        + 9 = 2***x*** + 1 Subtract 1 from both sides.
        + 8 = 2***x*** Divide by 2.
        + ***4*** = ***x***

D is the correct answer.

1. Sequence problem.
   1. Let’s write our givens.
      * + Sequence is 7, 15, 31, 63
        + First term is 7.
        + Each subsequent term is the proceeding term multiplied by ***m*** and then adding ***p***.
   2. What are we looking for? The value of ***m***.
   3. Translate the ‘math speak” of the expression.
      * + Each ***subsequent term*** ***is*** the ***proceeding term*** ***multiplied*** by ***m*** and then ***adding*** ***p***.
        + ***subsequent term =***  ***proceeding term*** ***×*** ***m*** ***+*** ***p***
        + Remember PEMDAS. Multiplication happens before addition.
   4. Look at the first two numbers, ***7*** and ***15***, and plug them into our equation.
      * + Each ***subsequent term*** ***=***  the ***proceeding term*** ***×*** ***m*** ***+*** ***p***
        + ***15*** ***=***  ***7*** ***×*** ***m*** ***+*** ***p***
   5. How do we get from 7 to 15? Well, ***7*** x ***2*** is 14, and if we add ***p,*** it gets us to ***15***. So let’s say ***m*** = ***2*** and ***p*** = ***1***.
      * + ***15*** ***=***  ***7*** ***×*** ***m*** + ***p***
        + ***15*** ***=***  ***7*** ***×*** ***2*** + ***1***
        + Now try the next sequence terms, 31 and 63, to see if it works for the whole sequence.
        + ***63*** ***=***  ***31*** ***×*** ***2*** + ***1***
        + ***63*** ***=***  ***62*** + ***1***
        + ***63*** ***=***  ***63*** That works. So ***m*** = ***2***

B is the correct answer.

1. Combinations.
   1. Let’s write our givens:
      * + Shirts can be red, white or blue.
        + Sizes are small, medium, large and x-large.
   2. What are we looking for? The ***total combinations*** of different colors and sizes that are possible.
   3. Start with red. How many combos do we have if we have a red shirt?
      * + Red small
        + Red medium
        + Red large
        + Red x-large
        + That’s ***4*** combinations of red shirts.
   4. How many different color shirts do we have? ***3***: red, white and blue. Multiply the number of shirt colors by the sizes.
      * + ***Number of shirt colors*** x ***sizes*** = ***total combinations***
        + ***3*** x ***4*** = ***12***

B is the correct answer

1. Functions and an inequality.
   1. What are we looking for? The ***function*** in which f(***-3***) > f(***3***).
   2. A function f(x) has an input of (x). In this problem, we need to put in ***-3*** and ***3*** for x and see where the output for ***-3*** is bigger than the output for ***3***. Let’s go through the answers.
      1. f(x) = 4x2
         * f(***-3***) = 4(***-3***)2 f(***3***) = 4(***3***)2
         * f(***-3***) = 4×9f(***3***) = 4×9
         * f(***-3***) = 36f(***3***) = 36 Those are equal. So (A) is out.
      2. f(x) = 4
         * f(***-3***) = 4f(***3***) = 4 Those are equal. (B) is out. Don’t let this one fool

us, it doesn’t matter what x is, the output is always 4.

* + 1. f(x) = 4/x
       - f(***-3***) = 4/(***-3***)f(***3***) = 4/***3***
       - f(***-3***) = -4/3f(***3***) = 4/3 Nope. -4/3 < 4/3, so f(***-3***) < f(***3***). (C) is out.
    2. f(x) = 4 - x3
       - f(***-3***) = 4 - (***-3***)3 f(***3***) = 4 - (***3***)3
       - f(***-3***) = 4 - (-27)f(***3***) = 4 - (27)
       - f(***-3***) = 4 + 27f(***3***) = 4 - 27
       - f(***-3***) = 31f(***3***) = -23 That works. 31 > -23. f(***-3***) > f(***3***). Try (E) to make sure.
    3. f(x) = x4 + 4
       - f(***-3***) = (***-3***)4 + 4f(***3***) = (***3***)4 + 4
       - f(***-3***) = 81 + 4f(***3***) = 81 + 4
       - f(***-3***) = 85f(***3***) = 85 Those are equal. (E) is out.

D is the correct answer.

1. Proportions Problem
   1. Let’s write our givens:
      * + The force required to stretch a spring beyond its natural length is ***proportional*** to how far the spring is being stretched.
        + Force of ***15*** pounds stretches a spring ***8*** centimeters beyond its natural length.
   2. What are we looking for? What ***force*** is needed to stretch the spring ***20*** cm beyond its natural length?
   3. Write the proportion.
      * + Force (pounds)

Length (cm)

* + - * ***15*** (pounds) = ***force*** (pounds)Cross-multiply and solve for ***force***. (pounds) and (cm)

***8***  (cm) ***20*** (cm) are written to show that the units of measure correspond.

* + - * ***force*** × ***8*** = ***15*** × ***20***
      * ***8***(***force***)= 300 Divide both sides by 8.
      * ***force*** = ***37.5***

E is the correct answer.

1. Friends and a line. DRAW THE LINE!
   1. Let’s write our given: ***Y*** is the midpoint of ***XZ***.
   2. What are we looking for The ***true*** statements.
   3. Draw the line.

***X***

***Y***

***Z***

* 1. Since ***Y*** is the midpoint, ***XY*** has to equal ***YZ***. Give those friends some values. Let ***XY*** and ***YZ*** = 1. That makes ***XZ*** = 2.

***X***

***Y***

***Z***

***1***

***1***

* 1. Go to the answer choices and plug and chug.

***YZ*** = ½(***XZ***)

1 = ½(2)

1 = 1 (***I***) is ***true***.

½(***XZ***) = 2***XY***

½(2) = 2(1)

1 = 2 (II) is false.

2***XY*** = ***XZ***

2(1) = 2

2 = 2 (***III***) is ***true***.

E is the correct answer.

1. Friends.
   1. Let’s write our givens:
      * + 2***r*** = 5***s***
        + 5***s*** = 6***t***
   2. What are we looking for? ***r*** in terms of ***t***.
   3. We have 5***s*** in both equations, so 2***r*** = 5***s*** = 6***t***. Get rid of the 5***s*** and 2***r*** = 6***t***. Simplify that.
      * + 2***r*** = 6***t*** Divide both sides by 2.
        + ***r*** = 3***t***

C is the correct answer.

1. Friendly word problem.
   1. Write the givens:
      * + ***k*** passengers went on a trip.
        + ***n*** buses were used.
        + Each bus can seat a maximum of ***x*** persons.
   2. What are we looking for? The ***expression*** for ***k***, ***n*** and ***x*** that describes the situation when ***3*** seats are empty on the bus trip.
   3. We’re looking for an expression for passengers, so ***k*** is going to be by itself.
      * + ***k*** = something
   4. Now find the total seats on the buses (***n***) using the max number of people (***x***) that can fit on each bus. That is just ***n*** × ***x***.
      * + Total seats = ***nx***
   5. Now the expression is:
      * + ***k*** = ***nx*** + something
   6. What’s that last “something”? In the given it says 3 seats are empty. So that’s a ***-3***. So, our ***expression*** is:
      * + ***k*** = ***nx*** + (-3)
        + ***k*** = ***nx*** - 3

A is the correct answer.

1. Lines. MARK UP THE PICTURE!
   1. Let’s write our given: Line l is parallel to line m.
   2. What are we looking for? The value of ***x***.
   3. To the picture. Since we have two parallel lines cut by a third line (k), we can use our rules (Same Side Interior, Alternating Interior, etc.) Mark angles that have the same measure.

***80°***

***l***

***m***

***50°***

***x°***

***k***

***80°***

***l***

***m***

***50°***

***x°***

***k***

* 1. The red angles are alternate interior angles, so they are equal. Same thing for the blue angles. Look at ***m***. A straight line has ***180°***, so ***x°*** + ***blue angle*** = 180***°***. To find the blue angle, we have to first go through the red angles.
     + - ***red angle*** = ***red angle***

***30°***

***80°***

***l***

***m***

***50°***

***x°***

***k***

***30°***

* + - * ***80°*** = ***50°*** + ***blue angle***
      * -50° = -50°
      * ***30°*** =  ***blue angle***
      * Now go back to our equation to find ***x***.
      * ***180°*** = ***30°*** + ***x°***. Subtract 30°.
      * ***150°*** = ***x°***
      * ***150*** = ***x***

A is the correct answer.

1. Inequality problem.
   1. Let’s write our given: 3***x***2 < (3***x***)2
   2. What are we looking for? The ***value*** of ***x*** that makes the statement ***false***.
   3. Simple plug and chug. Put in the values of the answers and see which one is not true.
      1. ***x*** = ***-3***
         * 3 × (***-3***)2 < (3×***-3***)2
         * 3 × 9 < (-9)2
         * 27 < 81 True. (A) is out.
      2. ***x*** = ***0***
         * 3 × (***0***)2 < (3×***0***)2
         * 3 × 0 < (0)2
         * 0 < 0 False. (***B***) looks pretty good. Check the other ones.
      3. ***x*** = ***1/3***
         * 3 × (***1/3***)2 < (3×***1/3***)2
         * 3 × 1/9 < (1)2
         * 1/3 < 1 True. (C) is out.
      4. ***x*** = ***1***
         * 3 × (***1***)2 < (3×***1***)2
         * 3 × 1 < (3)2
         * 3 < 9 True. (D) is out.
      5. We’ve proven in (B) that we can get a ***false*** statement, so (E) is out.

B is the correct answer.

1. Geometry word problem. DRAW A PICTURE!!
   1. Let’s write our given: The ***front*** wheel has half the diameter of the ***back*** wheel.
   2. What are we looking for? The number of ***front wheel revolutions*** for ***one*** revolution of the back wheel.
   3. Draw a picture of the wheels, noting the diameters. The diameter is the line that goes through the center of the circle and touches both edges. We’re given no numbers, so let the ***front*** wheel diameter be ***1*** and the ***back*** wheel diameter be ***2***.

***Back*** ***Front***

***2***

***1***

* 1. Circumference = 2πr, or πd, where r is the radius and d is the diameter. We have our diameters, so let’s use C = πd. Find the circumferences of the ***front*** and ***back*** wheels.
     + - ***Front*** Circumference = πd = ***1***π
       - ***Back*** Circumference = πd = ***2***π
       - ***1***π is half of ***2***π, so the ***front*** wheel revolves ***2 times*** for every one revolution of the ***back*** wheel.

C is the correct answer.

1. Probability Problem
   1. Let’s write our givens:
      * + ***p*** = the number of positive numbers
        + ***n*** = the number of negative numbers
        + When a number is picked at random, the probability that it is positive is 3/5.
   2. What are we looking for? The ***value*** of ***n***/***p***.
   3. Start with the only numbers that are given. 3/5 is the probability of picking a positive number. A probability can be written like:
      * + Outcome .

Total potential outcomes

* + - * In our problem, the outcome is picking a positive number, and the “total potential outcomes” is all the numbers in the list.
      * Positive Number . = ***3*** = ***p*** therefore ***p*** = ***3***

All Numbers 5 5

* + - * If we have 5 total potential outcomes, we have 5 total numbers. Remember the first given; the list only has positive and negative numbers. We can subtract to find ***n***, the number of negative numbers.
      * Positive + Negative = Total numbers
      * ***n*** = Total - ***p***
      * ***n*** = 5 - ***3***
      * ***n*** = ***2***
  1. Now we have ***n*** and ***p***. Plug them in to find the ***value*** of ***n***/***p***.
     + - ***n***/***p*** =  ***2***/***3***

C is the correct answer.

1. Function Problem
   1. Let’s write our givens:
      * + ***c(x)*** = total daily cost
        + ***x*** = units produced
        + ***x*** ≤ 100
        + ***k*** is a constant.
        + ***20*** units were produced yesterday for ***$640***.
   2. What are we looking for? The ***value*** of ***k***.
   3. In our function, ***x*** is the input and ***c(x)*** is the output. Yesterday, ***x*** was ***20*** and ***c(x)*** was ***$640***. Plug those into our formula to get us started.
      * + ***c(x)*** = 600***x*** - 200 + ***k***

***x***

* + - * ***640*** = 600(***20***) - 200 + ***k*** ***k*** is the only variable left. Simplify and finish up.

***20***

* + - * ***640*** = 12,000 - 200 + ***k***

***20***

* + - * ***640*** = 11,800 + ***k***

***20***

* + - * ***640*** = 590 + ***k***
      * -590 = -590
      * ***50*** = ***k***

B is the correct answer.

1. Friends and inequalities. A common theme in this section.
   1. Let’s write our given:
      * + An ordered pair of positive integers is (x, y).
   2. What are we looking for? ***The number of*** ordered pairs of positive integers that work for 2x + 3y < 6.
   3. We’re limited here by 3 things:
      * + The numbers are integers.
        + The numbers are positive.
        + 2x + 3y < 6
   4. We don’t have a lot of room to work with. What’s the lowest possible positive ordered pair? (1,1). Plug that into the inequality.
      * + 2x + 3y < 6
        + 2(1) + 3(1) < 6
        + 2 + 3 < 6
        + 5 < 6 That’s true. So we have 1 ordered pair that works. Try (1,2).
   5. Ordered pair (1,2).
      * + 2x + 3y < 6
        + 2(1) + 3(2) < 6
        + 2 + 6 < 6
        + 8 < 6 That’s false. So with a 2 in there, it doesn’t work.
   6. Try (2,1) as well, and we’ll see that x = 2 also produces a false statement. If 2 doesn’t work, is there any way that numbers greater than 2 will work? No, because our result must be less than 6.

We only have ***1 ordered pair*** that works.

A is the correct answer.

1. Triangle Problem. MARK UP THE DRAWING!
   1. Let’s write our givens:
      * + ***y*** = ***60***
        + Figure ***NOT*** drawn to scale.
   2. What are we looking for? How much ***greater*** the perimeter of ∆***ABC*** is than the perimeter of ∆***DEF***.
   3. We need to find the perimeters of both triangles. Perimeter is all the sides added together. Let’s start with our given info, ***y*** = ***60*** and ∆***DEF***.

***5***

***D***

***F***

***E***

***5***

***60°***

***60°***

***5***

***60°***

***5***

***y°***

***y°***

***D***

***F***

***E***

***D***

***F***

***E***

***5***

***60°***

***60°***

* 1. ∆***DEF***
     + - Since a triangle has 180° and 2 angles are ***60***°, the other angle must be ***60***°. So, our triangle is equiangular. Equiangular triangles are equilateral triangles, so all the sides are the same length. In this case, they are all 5.
       - Add up the sides of ∆***DEF*** to get the perimeter.
       - 5 + 5 + 5 = 15
       - Perimeter of ∆***DEF*** is 15.

***5***

***x°***

***x°***

***A***

***C***

***B***

***8***

***5***

***x°***

***x°***

***A***

***C***

***B***

***8***

***8***

* 1. ∆***ABC***
     + - We have two sides of different lengths, but two of our angles are the same. What kind of triangle is this? Isosceles. When 2 base angles are the same, the 2 sides are also the same. AC is our base, and the two legs AB and BC are the sides. AB = 8, so BC = 8.
       - Add up the sides of ∆***ABC*** to get the perimeter.
       - 5 + 8 + 8 = 21
       - Perimeter of ∆***ABC*** is 21.
  2. How much ***greater*** is the perimeter of ∆***ABC*** than the perimeter of ∆***DEF***? Subtract to find out.
     + - Perimeter of ∆***ABC*** - perimeter of ∆***DEF***
       - 21 - 15 = ***6***

C is the correct answer.

1. Integer friends.
   1. Let’s write our givens:
      * + ***y*** and ***x*** are positive, ***consecutive, odd*** integers.
        + ***y*** > ***x***
   2. What are we looking for? The ***expression*** that is equal to ***y***2 - ***x***2.
   3. Pick some numbers for ***x*** and ***y***. Make sure they follow the rules in the given. How about ***1*** and ***3?***
      * + Let ***x*** = ***1*** and ***y*** = ***3***. Plug them into ***y***2 and ***x***2 to find a value.
        + ***y***2 - ***x***2
        + ***3***2 - ***1***2
        + 9 - 1 = ***8***
   4. Now look at our answers. They all have an ***x***, so we need to use our ***x*** = ***1*** to see which one works. Plug ***1*** into our answers to see which one equals ***8***.
      1. 2***x***

2(***1***)

2 2 doesn’t equal ***8***. (A) is out.

* + 1. 4***x***

4(***1***)

4 4 doesn’t equal ***8***. (B) is out.

* + 1. 2***x*** + 2

2(***1***) + 2

2 + 2

4 4 doesn’t equal ***8***. (C) is out.

* + 1. 2***x*** + 4

2(***1***) + 4

2 + 4

6 6 doesn’t equal ***8***. (D) is out. It must be (E). Check to make sure it works.

* + 1. 4***x*** + 4

4(***1***) + 4

4 + 4

***8*** ***8*** equals ***8***. (E) it is.

E is the correct answer.

1. Coordinate Plane Problem
   1. Let’s write our givens:
      * + Line l passes through the origin (which is point 0,0).
        + Line l is perpendicular to the line 4x + y = k.
        + k is a constant.
        + The two lines intersect at a point with coordinates (t, t+1).
   2. What are we looking for? The value of ***t***.
   3. To get the value of ***t***, we need to get line ***l*** into equation form: y = ***m***x + ***b***. We know our ***b*** (y-intercept) because line ***l*** goes through the origin (0,***0***). That means our ***b*** = ***0***. So our line ***l*** equation is:
      * + y = ***m***x + ***0***
        + We still need to find ***m***, the slope of line ***l***. We have to use the second given to find this.
   4. The second line is 4x + y = k. It’s in a weird form, rearrange it into y = mx + b form.
      * + 4x + y = k Subtract 4x from both sides.

-4x -4x

* + - * + y = -4x + k
      * Our second line is y = -4x + k.
  1. What is the relationship between our second line and line ***l***? They are ***perpendicular***. That means that their slopes are negative reciprocals.
     + - Slope of second line = -4, which is the same as -4/1.
       - The negative reciprocal of -4/1 is ***positive*** ***¼***.
  2. So, ***m,*** the ***slope*** of line ***l*** = ***¼***. Now we have all the pieces of our line ***l*** equation:
     + - Line ***l*** is: y = ***¼***x + ***0***
  3. Now what? We need to get to ***t***. We know the point ***(t***, ***t*** + 1) is on line l, because the given told us that *that* point is the point where the two lines intersect.
  4. Plug in our point (***t***, ***t*** +1) into the equation of line ***l***. A point is in the form (***x***,y). In our problem, ***x*** = ***t***, and y = (***t*** + 1). So, plug those into our equation for line ***l***.
     + - y = ¼***x*** + 0
       - ***t*** + 1 = ¼(***t***) Subtract ***t*** from both sides.
       - 1 = ¼(***t***) - ***t***
       - 1 = -¾(***t***) Isolate ***t***. Multiply by -4/3.
       - -4/3 × 1 = -¾(***t***) × (-4/3)
       - ***-4/3*** = ***t***

A is the correct answer.

1. Average friends.
   1. Let’s write our given: The average of ***x*** and ***y*** is ***k***.
   2. What are we looking for? The ***expression*** for the ***average*** of ***x***, ***y*** and ***z***. An average is the sum of the terms divided by the number of terms. So:
      * + ***x*** + ***y*** + ***z*** = ***average***

3

* + - * That’s ugly. We need to do something with our first given.
  1. From our first given we have this as the average expression:
     + - ***x*** + ***y*** = ***k***

2

* + - * Notice any similarities between the two? They both have ***x*** + ***y***. Isolate ***x*** + ***y*** in the 2-term average. Then we can put that into the 3-term average.
      * ***x*** + ***y*** (×2)= ***k*** (×2)

2

* + - * ***x*** + ***y*** = 2***k***
  1. Plug the value of ***x*** + ***y*** into the 3-term average.
     + - ***x*** + ***y*** + ***z*** = ***average***

3

* + - * 2***k*** + ***z*** = ***average***

3

A is the correct answer.

1. Geometry Problem. MARK UP THE PICTURE!
   1. Let’s write our givens.
      * + ∆***XYZ*** is equilateral.
        + The sides of ∆***XYZ*** are all ***2***.
        + ***WY*** is the diameter of the circle with center ***O***.
   2. What are we looking for? The ***area*** of the circle.
   3. What do we need for the area of a circle? The formula is ***area*** = π***r***2, so we need ***r***, the ***radius***. ***r*** in this picture is either ***OY*** or ***O***W. W***Y*** is a diameter, and half of the diameter is the ***radius***. If we find the length of W***Y*** we can take ½ of it, and we will know the ***radius*** of the circle. We have to go through the triangle first.
   4. Mark up the picture.
      * + The sides are 2.
        + ∆***XYZ*** is equilateral, so all the angles have to be the same, ***60°***.

***2***

***2***

***X***

***Y***

***O***

***60°***

***Z***

***2***

***60°***

***W***

***60°***

* 1. Look at ∆***XYZ***. We know two angles, ***60°*** and the right angle ***90°***. That means the other angle has to be ***30°***. Special triangle bells should be ringing in our head. Pull out ∆***WYZ*** and draw it.

***Y***

***Y***

***W***

***Z***

***60°***

***30°***

***2***

***O***

***60°***

***30°***

***2x***

***x***

***x√3***

short leg = ***x***

long leg = ***x√3***

hypotenuse = 2 ***x***

***W***

***Z***

* + - * Again, if we find the length of W***Y*** and take half of it, we’ll have the radius of our circle. ∆***WYZ*** is a ***30°***/***60°***/***90°*** triangle. The formulas are above on the right. So our side W***Y*** (the diameter of the circle) = x√3 in the triangle. We have to do some manipulating here. Look at side ***Y***Z. ***Y***Z = ***2***, and it also is the “***2x***” in the ***30°***/***60°***/***90°*** triangle.
      * 2x = 2 That’s ***Y***Z.
      * x = 1 That’s WZ.
      * W***Y*** = x√3
      * W***Y*** = 1√3
      * W***Y*** = √3 Divide it by 2, and we’ll have our radius (***OY*** or OW).
      * ***OY*** = W***Y***/2
      * ***OY*** = ***√3/2*** Plug that into the formula for area and we’re done.
      * ***area*** = π***r***2.
      * ***area*** = π(***√3/2***)2. (***√3***)2 = 3 and 22 = 4
      * ***area*** = π(3/4)
      * ***area*** = ***3π/4***

C is the correct answer.

1. Remainder problem. We’ve seen this before.
   1. Let’s write our givens:
      * + ***k*** is a positive integer.
        + When 15 is divided by ***k***, the remainder is ***3***.
   2. What are we looking for? The number of ***different values*** of ***k*** for which the given statement is true.
   3. First, what is a remainder? It’s the amount left over in division. Here’s an example:

What’s the remainder when we divide 5 by 2?

2 r ***1*** ***1*** is the remainder in this example.

2√5

* 1. Back to our problem. Write the skeleton of the problem, in this case just the division sign.

√

* 1. Now let’s fill it in with the given information.

r ***3*** From the statement: “when 15 is divided by ***k***, the remainder is ***3***.”

***k***√15

* 1. ***k*** has to be a positive integer, and we need a remainder of 3. Start trying numbers for ***k***. How about ***2?*** We’re not trying ***1***, because 15/***1*** = 15 and there is no remainder.
     + - ***k*** = ***2***

7 r 1 Remainder of 1. That doesn’t equal ***3***. 2 is not a value of ***k***. Try ***3***.

***2***√15

* + - * ***k*** = ***3***

5 r 0 Remainder of 0. That doesn’t equal ***3***. 3 is not a value of ***k***. Try ***4***.

***3***√15

* + - * ***k*** = ***4***

3 r ***3*** Remainder of ***3***. That works. ***4*** is one value of ***k***. Try ***5***.

***4***√15

* + - * ***k*** = ***5***

3 r 0 Remainder of 0. That doesn’t equal ***3***. ***5*** is not a value of ***k***. Try ***6***.

***5***√15

* + - * ***k*** = ***6***

2 r ***3*** Remainder of ***3***. That works. ***6*** is another value of ***k***. We’ve got 2. Try ***7***.

***6***√15

* + - * ***k*** = ***7***

2 r 1 Remainder of 1. That doesn’t equal ***3***. ***7*** is not a value of ***k***. Try ***8***.

***7***√15

* + - * ***k*** = ***8***

1 r 7 Remainder of 7. That doesn’t equal ***3***. ***8*** is not a value of ***k***. Try ***9***.

***8***√15

* ***k*** = ***9***

1 r 6 Remainder of 6. That doesn’t equal ***3***. ***9*** is not a value of ***k***. Try ***10***.

***9***√15

* + - * ***k*** = ***10***

1 r 5 Remainder of 5. That doesn’t equal ***3***. ***10*** is not a value of ***k***. Try ***11***.

***10***√15

* + - * ***k*** = ***11***

1 r 4 Remainder of 5. That doesn’t equal ***3***. ***11*** is not a value of ***k***. Try ***12***.

***11***√15

* + - * ***k*** = ***12***

1 r ***3*** Remainder of ***3***. That works. ***12*** is another value of ***k***. We’ve got ***3***.

***12***√15

* + - * ***k*** = ***13***

1 r 2 Remainder of 2. That doesn’t equal ***3***. ***13*** is not a value of ***k***.

***13***√15

* + - * The pattern continues for k = 14 and k = 15, with remainders of 1 and 0, respectively. Anything greater than 15 is not going to work, because we’ll have no remainder, so we have ***3*** ***k***’s.

C is the correct answer.

# Test 5 Section 4

1. Easy friends to start.
   1. Let’s write our given: s + t = 3
   2. What are we looking for? The ***value*** of s + t - 6.
   3. Let’s put some parentheses around the s + t.
      * + (s + t) = 3, and we’re looking for (s + t) - 6.
        + That’s just like having one variable. Plug the value of (s +t) into (s + t) - 6.
        + (s + t) - 6
        + 3 - 6
        + ***-3***

A is the correct answer.

1. Geometry Problem. Mark up the drawing.
   1. Let’s write our given: The figure is a cube. That means all the edges are the same length.
   2. What are we looking for? The ***point*** that is ***not*** the same distance from points P and Q.
   3. To the drawing. Even though it doesn’t look like it in our picture below, all the edges are the same length.

***A***

***E***

***C***

***B***

***Q***

***P***

***D***

***A***

***E***

***C***

***B***

***Q***

***P***

***D***

***A***

***E***

***C***

***B***

***Q***

***P***

***D***

* 1. Start going through the answers.
     1. Point A. Look at edges AP and AQ. They are both edges of the cube so they are the same length. So, A is the same distance from P and Q. (A) is out. Same thing goes for (E), where the edges are PE and EQ. (E) is out.
     2. Point B. A little trickier, but look at the ***red dashed lines*** BP and BQ, drawn on the second cube. Those are just two diagonals of two faces. Faces of a cube are all equal just like the edges, so these diagonals are the same length. Therefore (B) is the same distance from P and Q. (B) is out. So is (D) for the same reason.
     3. Point C. Look at the third cube. CQ and CP are the ***blue dashed lines***. CQ forms one edge. But CP cuts all the way across the cube. Besides the fact that this is our only choice left, the two distances are not the same. ***Point C*** is ***not*** the same distance from P and Q.
     4. Same reasoning as (B).
     5. Same reasoning as (A).

C is the correct answer.

1. Data problem with a graph and a pie chart.
   1. What are we looking for? The ***pie chart*** that best reflects the bar graph.
   2. Think about a circle. A circle has 360°. Looking at the Web Sites category, we have 78% of the fraud. That’s not a friendly number, but it’s a little more than 75%. What’s 75% of a circle? 75% x 360° = 270°. That’s ¾ of the circle. So 78% is a little more than ¾ of the circle.
   3. So now let’s look at the answers.
      * + (A) and (B) are eliminated because the Web Sites are less than ¾ of the circle.
        + (E) is gone because Web Sites in that pie graph are a lot more than ¾ of the circle.
        + In (C), there are only 3 categories. The pie graph leaves out the “Other” category. (C) is out.
        + Only (D) is left, and this one looks good because News Groups and Other are the same (and small).

D is the correct answer.

1. Fractions and “Math Speak”.
   1. Let’s write our givens:
      * + The ***numerator*** is 5 less than the ***denominator***.
        + The fraction equals ¾.
   2. What are we looking for? The ***denominator*** of the fraction.
   3. Look at the second given. It means:
      * + ***numerator***  ***=*** *3*

***denominator*** *4*

* 1. In our first given, we know the ***numerator*** is 5. We need a variable for the ***denominator***, so let that be ***x***. Now translate the “math speak” of the first given.
     + - ***numerator*** ***is*** 5 less than than the ***denominator***.
       - ***numerator*** ***=*** -5 + ***x***
       - ***numerator*** ***= x - 5***
       - ***x*** - 5 ***= 3***   This is our fraction.Cr*o*ss multiply and find ***x***.

***x*** *4*

* + - * (***x*** - 5) × 4 ***=*** 3***x***
      * 4***x*** - 20  ***=*** 3***x*** Subtract 4***x*** from both sides.

-4***x*** -4***x***

* + - * - 20  ***=*** -***x***
      * ***20***  ***= x***

D is the correct answer.

1. Triangle and a coordinate plane. Awesome. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + The scales on the x- and y-axis are different.
        + The area of ∆ABC is 18.
   2. What are we looking for? The ***value*** of ***k***.
   3. What do we know about the area of a triangle? The area = ½(***base*** x ***height***). Can we get to ***k*** from that equation? Indeed.
   4. To the picture! We can figure out our ***base*** and ***height*** by looking at the coordinate plane.

***B***

***C***

***A***

***6***

***2***

***2k***

***5k***

***B***

***C***

***A***

***6***

***2***

***2k***

***5k***

* + - * The ***height*** of this triangle is the ***red line***. The length of that ***red line*** is 6 – 2, so our

***height*** = ***4***. What about the base? That’s where our ***k*** joins the party.

* + - * The length of the ***base*** of this triangle is 5***k*** – 2***k***.
  1. Plug our values into the area formula.
     + - area = ½(***base*** x ***height***)
       - 18 = ½(5***k*** – 2***k***)(***4***) Simplify and solve for ***k***.
       - 18 = 2(3***k***)
       - 18 = 6***k*** Divide both sides by 6.
       - ***3*** = ***k***

E is the correct answer.

1. Friends and negative exponents. Yuck.
   1. Let’s write our givens:
      * + m and ***k*** are positive integers.
        + 10m2 ***k***-1 = 100m
   2. What are we looking for? The ***value*** of ***m-1*** in terms of ***k***.
   3. First thing we need to do is get those m’s together. Throw the 10 in there, too.
      * + 10m2***k*** -1 = 100m Divide by 10m2.

10m2 10m2

* + - * The two 10m2 expressions cancel each other out on the left side of the equation. The numbers on the right side are easy. 100/10 is just 10. So we’re left with:
      * ***k*** -1 = 10m

m2

* + - * Look at the m’s. At least they’re on the same side now. What happens when we take an exponent in the denominator and flip it up to the numerator? The exponent ***switches signs***. So, in doing that, our m2 on the bottom becomes m-2 on the top. 1/m2 = m-2. Back to the equation.
      * ***k*** -1 = 10m

m2

* + - * ***k*** -1 = 10m×m-2
      * When we multiply like bases (m) we add the exponents. Our exponents are 1 and -2.
      * m 1×m-2 = ***m-1*** because 1 + -2 = -1. Now we have:
      * ***k*** -1 = 10***m-1*** Get ***m-1*** by itself. Divide by 10.
      * ***k*** -1 = ***m-1*** Flip ***k*** -1 into the denominator and we’re done. Remember to ***switch the***

10 ***sign*** of the exponent.

* + - * ***1*** = ***m-1*** Anything to the power of 1 is just the number. ***k***1  = ***k***.

***10k***

D is the correct answer.

1. Word problem with rates.
   1. Let’s write our givens:
      * + ***Edna*** and ***Nancy*** leave from the same place and both walk for ***4 hours***.
        + ***Edna*** walks due east at ***4*** km/h.
        + ***Nancy*** walks due north at ***3*** km/h.
   2. What are we looking for? The ***straight line*** ***distance*** between ***Edna*** and ***Nancy*** at the end of 4 hours
   3. We need to draw a picture. Look at the nice triangle. We know it’s a right triangle because the North and East lines form a 90° angle. Pythagorean Theorem anyone?

***Nancy***

***Edna***

* 1. Now the rates. How do we figure out distance? Distance = rate × ***time***. We have rates for both women, and we also have the ***time*** (***4 hours***). Figure out their total distances.
     + - ***Edna*** walks due east at ***4*** km/h.

***Nancy***

***Edna***

***12***

***16***

* + - * Distance = ***rate*** × ***time***
      * Distance = ***4km/h*** × ***4 hours***
      * ***Edna’s Distance*** = ***16 km***
      * ***Nancy*** walks due north at ***3*** km/h.
      * Distance = ***3 km/h*** × ***4 hours***
      * ***Nancy’s*** Distance = ***12 km***
      * Mark those distances on the picture.
  1. Use the Pythagorean Theorem to find the ***straight line*** ***distance***.
     + - ***a***2 + ***b***2 = ***c***2
       - ***16***2 + ***12***2 = ***c***2
       - 256 + 144 = ***c***2
       - 400 = ***c***2
       - ***20*** = ***c***

C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfE is the correct answer.

* 1. The other way to find the distance is to notice that 16 and 12 are both multiples of 4. The Pythagorean triplet 3, 4, 5 can be used here where 3 and 4 are the legs and 5 is the hypotenuse. We just have to multiply all sides by 4. 3×4 = ***12***, 4×4 = ***16***, and 5×4 = ***20***. Gets us to the same place, but it’s faster.

1. A function without the formula. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + The function ***f*** has a minimum at (***1***,***1***).
        + ***f***(***b***) = ***f***(***3***)
   2. What are we looking for? A possible ***value*** of ***b***.
   3. A terminology refresher. A function ***f***(***x***) has an input of x. ***f***(***x***) is the output. ***f***(***x***) is the same thing as ***y***. We’re given a graph, so we can see the inputs (***x***) and output (***y***) for the function. We know each box on the graph is one, because our minimum is (***1***,***1***). Mark that.

***(1,1)***

***(1,1)***

***(1,1)***

***(-3,5)***

***(3,5)***

***(3,5)***

* 1. Look at the second given, more specifically ***f***(***3***). That’s an ***x*** in a point (***x***,***y***). What’s the ***y***-value? Look at the second graph above. Count the blocks on the graph. When ***x*** = ***3***, ***y*** = ***5***. So ***f***(***3***) = ***5***.
  2. We’re looking for ***b***. Back to the given info.
     + - ***f***(***b***) = ***f***(***3***)
       - ***f***(***3***) = ***5***
       - ***f***(***b***) = ***5***
       - So ***5*** is our output (***y***-value) and ***b*** is the input (***x*** -value). If we find the ***x*** value that gives us a value of ***5***, we’ll know our ***b***.
       - Look at the third graph. There are two points with ***5*** as the y-value: (***3***, ***5***) and (***-3***, ***5***). So ***b*** can be ***-3*** or ***3***.
       - Look at the answers. ***3*** isn’t there. So ***b*** = ***-3***

C is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. Word Problem
   1. Let’s write our givens:
      * + Family of ***5***
        + ***4***-day camping trip
        + Each person needs ***1*** bottle of water a day.
        + Water is sold only in ***3-bottle packages***.
   2. What are we looking for? The number of packages the family has to buy for the trip.
   3. First figure out how much water the family needs.
      * + ***# of people*** x ***days camping*** x ***bottles per person per day = total bottles needed***
        + ***5*** x ***4*** x ***1*** ***= 20***
   4. The family needs ***20*** bottles. Bottles come in ***3-bottle packages***, so we can buy bottles in multiples of 3. Is 20 a multiple of 3? Nope. We need to buy 21 bottles. How many ***3-bottle packages*** are needed for 21 bottles? Divide to find out.
      * + 21/ 3 = ***7*** ***3-bottle packages***. There’s an extra bottle for their dog.

***7***  is the correct answer.

1. Absolute Value Problem
   1. Let’s write our givens:
      * + |10 – ***k***| = 3
        + | ***k*** – 5| = 8
   2. What are we looking for? The value of ***k*** that works for ***both*** equations.
   3. Solve both equations for ***k*** and see what we get. Remember, keep the expression inside the absolute value bars the same, and put a + and a – in front of the number on the other side of the equation (e.g., 3 in the first given).
   4. |10 – ***k***| = 3
      * + 10 – ***k*** = 3 10 – ***k*** = -3
        + -10 -10 -10 -10
        + -***k*** = -7 -***k*** = -13
        + ***k*** = 7 ***k*** = ***13***
   5. | ***k*** – 5| = 8
      * + ***k*** – 5 = 8 ***k*** – 5 = -8

+5 +5 +5 +5

* + - * ***k*** = ***13*** ***k*** = -3
  1. ***k*** = ***13*** works for both absolute value equations, so that’s our answer.

***13*** is the correct answer.

1. Lines and angles. MARK UP THE PICTURE!
   1. What are we looking for? The value of ***x***.
   2. To the picture!

***20***°

***x***°

***65***°

***l***

***m***

***20***°

***x***°

***65***°

***l***

***m***

***25°***

***25°***

* 1. Look at line ***m***. By definition, a line has 180°. So all the angles above line m add up to 180°, and so do all the angles below it. There are 3 angles above line ***m***, a right angle (90°), 65° and a ***no-name*** angle. Those must add up to 180°.
     + - Right angle + 65° + ***no-name*** ° = 180°
       - 90° + 65° + ***no-name***° = 180°
       - 155° + ***no-name***° = 180° Subtract 155° from both sides.
       - ***no-name***° = ***25***°
  2. Remember ***vertical angles***? They are opposite angles formed by two intersecting lines, like our ***no-name angles*** formed by lines ***l*** and ***m*** in this problem. They are always equal. So both of our ***no-name angles*** = ***25***°. That’s marked on the drawing on the right.
  3. Back to line ***m***. Now we have 3 angles on the bottom that must add up to 180°: ***25***°, ***20***° and ***x***°. Solve it.
     + - ***x***° + ***25***° + ***20***° = 180°
       - ***x***° + 45° = 180° Subtract 45° from both sides.
       - ***x***° = ***135***°
       - ***x*** = ***135***

***135*** is the correct answer. Don’t write the ° sign in the answer.

1. Median Problem
   1. Let’s write our givens:
      * + Set of 9 ***consecutive*** integers
        + ***Median*** is ***42***.
   2. What are we looking for? The ***greatest number*** in the set.
   3. A median is the middle number in a list of numbers. We know these are consecutive integers, so they are whole numbers and separated by 1. Start in the middle and keep adding numbers to both sides until our set has 9 numbers.
      * + ***42***
        + 41 ***42*** 43
        + 40 41 ***42*** 43 44
        + 39 40 41 ***42*** 43 44 45
        + 38 39 40 41 ***42*** 43 44 45 ***46***
   4. So, we have a set of 9 consecutive numbers. ***42*** is in the middle, so ***42*** is the ***median***. What’s the biggest number in our set? ***46***

***46*** is the correct answer.

1. Ugly function problem with two friends.
   1. Let’s write our givens:
      * + f(***x***) = ***x*** + 1
        + 2f(***p***) = 20
   2. What are we looking for? The ***value*** of f(3***p***).
   3. Remember, the friend inside the ( ) of a function is the ***input***, whether it’s ***x***, ***p***, 523 or qwerty. The ***output*** is the value of f(***x***), f(***p***), f(523) or f(qwerty). This function takes an input and adds 1 to it.
      * + f(***x***) = ***x*** + 1 Let’s say ***x*** is 1. Then f(***x***) = 2.
   4. Look at the second given, 2f(***p***) = 20. Treat f(***p***) just like a regular friend, an x, a y, or any other letter. We can then get f(***p***) by itself by dividing both sides by 2.
      * + 2f(***p***) = 20
        + f(***p***) = 10 That’s easier to work with now. When we use ***p*** as an input, the output is 10.
   5. Back to the first given, f(***x***) = ***x*** + 1. We need to get a ***p***, though. Can we input ***p*** instead of ***x***? Of course.
      * + f(***x***) = ***x*** + 1
        + f(***p***) = ***p*** + 1 Don’t we know a value for f(***p***)? Yes. It equals 10. Plug it in and find ***p***.
        + 10 = ***p*** + 1
        + ***9*** = ***p***
   6. Almost done. We’re looking for the ***value*** of f(3***p***). Use 3***p*** as the input (***x***) in our function f(***x***) = ***x*** + 1.
      * + f(***x***) = ***x*** + 1
        + f(3***p***) = 3***p*** + 1
        + We know ***p*** = ***9***, so plug that in to find 3***p***.
        + 3***p*** = ***?***
        + 3(***9***) = ***27*** Back to our equation.
        + f(3***p***) = 3***p*** + 1
        + f(3***p***) = ***27*** + 1
        + f(3***p***) = ***28***

***28*** is the correct answer.

1. Lines, angles, triangles. Pure joy. Mark up the picture.
   1. Let’s write our givens:
      * + KN ⊥ JL
        + ***LM*** ⊥ JL
        + ***LM*** = ***LN***
   2. What are we looking for? The ***value*** of ***x***.
   3. To the picture! If KN and ***LM*** are both perpendicular to JL, doesn’t that mean that KN ∥***LM***? Indeed. Mark that. Also mark ***LM*** = ***LN***.

***J***

***K***

***L***

***l***

***N***

***M***

***t***

***x°***

***125°***

***J***

***K***

***L***

***l***

***N***

***M***

***t***

***x°***

***125°***

***55°***

* + - * Start with the only number we have: 125°. That’s part of a straight line, t. A straight line has 180°, so we can figure out ∠***NML***.
      * ∠***NML*** + 125° = 180° Subtract 125° from both sides.
      * ∠***NML*** = ***55°*** Mark that (picture on the right above).
  1. Look at ∆LNM. We have two sides that are the same. That’s an isosceles triangle. The two legs (***LM*** and ***LN***) are the same, so the base angles (∠***MNL*** and ∠***NML***) are equal. Mark that (left picture below).
     + - ∠***MNL*** = ∠***NML*** = ***55°***

***J***

***K***

***L***

***l***

***N***

***M***

***t***

***x°***

***125°***

***55°***

***55°***

***J***

***K***

***L***

***l***

***N***

***M***

***t***

***x°***

***125°***

***55°***

***55°***

***70°***

* + - * We have two angles of ∆NLM. Find the third angle, ∠***NLM***, maybe it will help us. The angles of a triangle add up to 180°.
      * ∠***MNL*** + ∠***NML*** + ∠***NLM = 180°***
      * ***55°*** + ***55°*** + ∠***NLM = 180°***
      * ***110°*** + ∠***NLM = 180°*** Subtract 110° from both sides.
      * ∠***NLM = 70°*** Mark that (right figure above).
  1. Now we need to use our parallel line rules. Remember, KN ∥ ***LM***. The line ***LN*** is the transversal that cuts across both of them. Look at ∠***NLM*** and ∠***x*** in the picture below (we’ve gotten rid of all the extra parts). Those angles are ***opposite interior angles***, so they have to be equal. We found ∠***NLM*** above, so now we know ***x***.

***x°***

***K***

***L***

***N***

***M***

***70°***

* + - * ∠***NLM*** = ***70***° = ***x***°. ***x*** = ***70***

***70*** is the correct answer.

1. Word problem. Draw a picture.
   1. Let’s write our givens:
      * + A measuring cup contains ***1/5*** cup of ***orange juice***.
        + It’s then filled to the ***1 cup*** with equal amounts of ***orange***, ***grapefruit*** and ***pineapple*** juice.
   2. What are we looking for? The ***fraction*** of the final mixture that is ***orange juice***
   3. Draw a picture. We know ***OJ*** is ***1/5***.  ***Orange***, ***grapefruit*** and ***pineapple*** juice make up the rest. How much is the rest? We have ***1 cup total***, and the first ***1/5*** is ***orange juice***. Subtract to get the amount of the mix.
      * + ***1*** = ***orange juice*** + (***orange*** + ***grapefruit*** + ***pineapple***)
        + ***1*** = ***1/5*** + (***orange*** + ***grapefruit*** + ***pineapple***)

-1/5 = -1/5

* + - * **4/5** = (***orange*** + ***grapefruit*** + ***pineapple***)

***1/5 OJ***

***4/5 mix***

***1 cup total***

***1/3 of mix is OJ***

***1/3 of mix is pineapple***

***1/3 of mix is grapefruit***

* 1. Now the mixture. We know the juices are split into three equal parts. So each juice is 1/3 of the mixture. We know the mixture is 4/5 of the total. Multiply those to get the fraction of ***orange juice*** in the mix.
     + - 4/5 × ***1/3*** = ***4/15*** That’s our “second ***orange juice***.” Now add them up.
       - ***1*** + ***4*** = ***Fraction of orange juice*** in final mixture.

***5*** ***15***

* + - * Get a common denominator. Multiply ***1/5*** by 3/3.
      * ***1*** × 3 + ***3***

***5*** 3 ***15***

* + - * ***3*** + ***4*** = ***7*** That’s our fraction of ***orange juice*** in the total mixture.

***15*** ***15*** ***15***

***7/15*** is the correct answer.

1. Friends and percents.
   1. Let’s write our given:
      * + ***a*** + 2***b*** is ***125%*** of 4***b***.
   2. What are we looking for? The ***value*** of ***a***/***b***.
   3. Translate the “math speak.”
      * + ***a*** + 2***b*** ***is*** ***125%*** ***of*** 4***b***
        + ***a*** + 2***b*** ***=*** ***125%*** ***×*** 4***b*** 125% x 4 = 5
        + ***a*** + 2***b*** ***=*** 5***b*** Get the ***b***’s together. Subtract by 2***b***.
        + ***a =*** 3***b*** Divide by ***b***.
        + ***a =*** ***3***

***b***

***3*** is the correct answer.

1. Number line and a square root friend.
   1. Let’s write our givens:
      * + 9 equal intervals between o and 1
        + √***x*** is the 6th mark over.
   2. What are we looking for? The ***value*** of ***x***.
   3. We’re dealing with a fraction here, since the number is between 0 and 1. What if that 6th mark was just x and not the √***x***?
      * + The mark is the 6th mark over. There are 9 equal intervals, so x would be 6/9.
        + In this problem though, 6/9 isn’t the ***x***, it is the √***x***. Translate that ‘math speak.”
        + 6/9 ***is*** √***x***
        + 6/9 ***=*** √***x*** Simplify that fraction. 6/9 = 2/3.
        + 2/3 = √***x*** Square both sides to get rid of the √.
        + (2/3) 2 = (√***x***)2
        + ***4/9*** = ***x***

***4/9*** is the correct answer.

1. Coordinate plane to finish up.
   1. Let’s write our givens:
      * + 2 points: A (***x***,***3***) and B (***10***,***18***)
        + The distance between the points is ***17***.
   2. What are we looking for? A possible ***value*** of ***x***.
   3. This is a distance formula problem. This formula is ***not*** given on this test, so we need to know it by heart before test day. Write the formula and plug and chug.
      * + Distance = √[(***xA*** - ***xB***)2 + (***yA*** - ***yB***)2]
        + ***17*** = √[(***x - 10***)2 + (***3*** -***18***)2]
        + ***17*** = √[(***x - 10***)2 + (-***15***)2] Still ugly. Square both sides.
        + (***17***)2 = (√[(***x - 10***)2 + (-***15***)2])2
        + 289 = (***x - 10***)2 + (-***15***)2 A little better. FOIL the (***x - 10***)2. Don’t forget (-***15***)2.
        + 289 = ***x***2 - 20***x*** + 100 + (-***15***)2 Don’t forget (-***15***)2, simplify that.
        + 289 = ***x***2 - 20***x*** + 100 + 225 Simplify.
        + 289 = ***x***2 - 20***x*** + 325 Subtract 289 from both sides so we can factor.
        + 0 = ***x***2 - 20***x*** + 36
        + 0 = ***x***2 - 20***x*** + 36 Factors of 36 are: (1,36), (***2***,***18***), (3, 12), (4, 9)
        + 0 = (***x*** - 18)(***x*** – 2)
        + ***x*** = ***18*** or ***2***

***18*** or ***2*** is the correct answer.

# Test 5 Section 8

1. Sets.
   1. Let’s write our givens:
      * + ***E*** is the set of even integers.
        + ***P*** is the set of positive integers.
        + ***F*** is the set of integers less than 5.
   2. What are we looking for? Which ***integer*** is in sets E, P and F.
   3. Draw a number line and mark the sets.

***5***

***P***

***E***

***0***

***-2***

***F***

***4***

* + - * P and E constrain us to numbers that are between 0 and 5. E further constrains us to only the even numbers between o and 5. There’s only 1 number in our answers that meets this: ***4***.

B is the correct answer.

1. Root friend.
   1. Let’s write our given: 8 + √***k*** = 15
   2. What are we looking for? The ***value*** of ***k***.
   3. Simplify and solve.
      * + 8 + √***k*** = 15 Subtract 8 from both sides.
        + √***k*** = 7 Square both sides.
        + (√***k***)2 = 72
        + ***k*** = ***49***

B is the correct answer.

1. Polling question.
   1. Let’s write our givens:
      * + 35 people in favor
        + 14 people against
        + 1 person had no opinion
   2. What are we looking for? The ***fraction*** of the people polled that are in favor of the new library.
   3. For fractions we need the ***total*** number of people, so we can figure out the ***part***/***total***.
      * + 35 + 14 + 1 = ***50*** ***total*** people
        + In this case our “***part***” is people in favor, which is ***35***. So our fraction is:
        + ***part***/***total*** = ***35***/***50*** Now reduce that fraction. Divide by 5.
        + ***35***/***50*** = ***7/10***

A is the correct answer.

1. Triangles and vertical angles. MARK UP THE PICTURE!
   1. What are we looking for? The ***value*** of t + u.
   2. To the picture! Look at the two unnamed angles. Aren’t they vertical angles? Yes! They have to be the same. Mark them as ***x***°.

***70°***

***30°***

***t°***

***u°***

***x°***

***x°***

***70°***

***30°***

***t°***

***80°***

***80°***

***u°***

* 1. Figure out the ***x***° in the top triangle. A triangle has 180°, so:
     + - 70° + 30° + ***x***° = 180°
       - 100° + ***x***° = 180° Subtract 100° from both sides.
       - ***x***° = ***80***°
  2. Look at the second figure with the ***80***° marked. We know that a triangle’s angles add up to 180°. So the equation for the angles of the bottom triangle is:
     + - t° + u° + ***80***° = 180° Get those numbers together. Subtract 80° from both sides.
       - t° + u° = ***100***° That’s the ***value*** we’re looking for.
       - We don’t need to know what t and u are individually, we just need the sum.
       - t + u = ***100***

C is the correct answer.

1. Data analysis.
   1. What are we looking for? The ***consecutive years*** between which there was the greatest change in price.
   2. For each answer choice, write down the change between the years. Remember, change can be positive or negative.
      1. 1981 and 1982. A decrease of ***$0.25***. Not very big.
      2. 1982 and 1983. An increase of ***$0.75***. Right now, (B) is the greatest change. Keep going.
      3. 1983 and 1984. An increase of ***$0.50***. (B) is still winning.
      4. ***1984 and 1985***. A decrease of ***$1.00***. Bigger change. (D) is now the greatest change. Check (E).
      5. 1985 and 1986. An increase of ***$0.75***. Nope. (***D***) is the greatest change.

D is the correct answer.

1. Graph of a function. Don’t let the terminology throw us off.
   1. Let’s write our givens:
      * + Graph represents y = g(x).
        + g(k) = 1
   2. What are we looking for? One possible value of ***k***.
   3. First look at g(x). The x value is the ***input***, and g(x) is the ***output***. This is the general form of this particular function. Put in an x and it spits out a y.
   4. Now look at g(k) = 1. That just means that when k is the input, the output (y-value) is 1. Where does y = 1 on our graph? It’s in ***red*** on the graph.

***-1***

***O***

***1***

* + - * The points on the function with a y-value of 1 have x-values between -1 and 0. Which one of our answer choices falls in that range? ***-0.5***

B is the correct answer.

1. Exponent friends.
   1. Let’s write our givens:
      * + ***a***, ***b*** and ***c*** are ***different positive*** integers.
        + 2***a*** × 2***b*** × 2***c*** = 64
   2. What are we looking for? The ***value*** of 2***a*** + 2***b*** + 2***c***.
   3. Exponent rules. When we multiply like bases (like 2 here) we add the exponents. So:
      * + 2***a*** × 2***b*** × 2***c*** = 64
        + 2(***a***+ ***b***+***c***) = 64 Now get 64 in an exponent form with 2 as the base.
        + 64 = 2×2×2×2×2×2 That’s 6 2’s. So 64 = 26.
        + 2(***a***+ ***b***+***c***) = 26 Both sides have like bases, so the exponents must equal each other.
        + ***a*** + ***b*** + ***c*** = 6
   4. Remember the first given. ***a***, ***b*** and ***c*** are ***different, positive*** integers. The only numbers that work are 1, 2 and 3.
      * + ***a*** + ***b*** + ***c*** = 6
        + ***1*** + ***2*** + ***3*** = 6
   5. Plug the values of ***a***, ***b*** and ***c*** into 2***a*** + 2***b*** + 2***c*** to find the ***value*** we’re looking for.
      * + 2***a*** + 2***b*** + 2***c***
        + 2***1*** + 2***2*** + 2***3***
        + 2 + 4 + 8
        + ***14***

A is the correct answer.

1. Circle on the coordinate plane.
   1. Let’s write our givens:
      * + Center of circle is (3,7).
        + One endpoint of the diameter is (-2,-7).
   2. What are we looking for? The ***coordinates (x,y) of*** the other endpoint of the diameter.

***(x, y)***

***(3,-7)***

***(-2,-7)***

* 1. Draw a picture!
  2. This diameter is horizontal, so all the length comes from the ***x***-values.
     + - From the center to the point (-2,-7) the distance is:
       - 3 – (-2) = 5
       - Our x has to be the same distance from the center.
       - 3 + 5 = 8. So our ***x***-value is ***8***.
  3. The ***y***-values are all the same on a horizontal line.
     + - ***y*** = ***7***
       - Our coordinates are (***8***, ***-7***).

E is the correct answer.

1. Inequalities and Absolute Value.
   1. Let’s write our given: A child’s height, ***h***, must be greater than 30 inches and less than 50 inches.
   2. What are we looking for? The ***expression*** that satisfies this requirement.
   3. The given says ***h*** must be greater than 30 and must be less than 50. It doesn’t say ***h*** must be greater than ***or equal to*** 30 or less than ***or equal to*** 50. So at the very least, if ***h*** were ***equal to*** 30 or ***h*** were ***equal*** 50 and the inequality were true, then that inequality choice won’t work. First go through the answers using ***h*** = 30 to see which inequalities are true, so we can eliminate them.
      1. |***h*** – 10| < 50

|***30*** – 10| < 50

|20| < 50

20 < 50 This is true. A 30 inch tall child could get on the ride using this equation. (A) is

out.

* + 1. |***h*** – 20| < 40

|***30*** – 20| < 40

|10| < 40

10 < 40 Also true. (B) is out.

* + 1. |***h*** – 30| < 20

|***30*** – 30| < 20

|0| < 20

0 < 20 Also true. (C) is out.

* + 1. |***h*** – 40| < 10

|***30*** – 40| < 10

|-10| < 10

10 < 10 ***Not*** a true statement. This keeps the ***30*** inch tall kid off the ride. (D) is possible.

* + 1. |***h*** – 45| < 5

|***30*** – 45| < 5

|-15| < 5

15 < 5 ***Not*** a true statement. This keeps the ***30*** inch tall kid off the ride. (E) is possible.

* 1. We’ve narrowed our answers down to (D) and (E). What about the ***50*** inch tall kid?
     1. |***h*** – 40| < 10

|***50*** – 40| < 10

|10| < 10

10 < 10 ***Not*** a true statement. This keeps the ***50*** inch tall kid off the ride. (D) is possible.

* + 1. |***h*** – 45| < 5

|***50*** – 45| < 5

|5| < 5

5 < 5 ***Not*** a true statement. This keeps the ***50*** inch tall kid off the ride. (E) is possible.

* 1. That didn’t help. What now? We didn’t check the value of a kid that was in the good range. Try ***h*** = ***40*** in (D) and (E). Now we *want* the statement to be true since a ***40*** inch kid is allowed on the ride.
     1. |***h*** – 40| < 10

|***40*** – 40| < 10

0 < 10

0 < 10 ***True***. We want this to be true for a ***40*** inch kid. (D) looks pretty good. Check (E).

* + 1. |***h*** – 45| < 5

|***40*** – 45| < 5

|5| < 5

5 < 5 ***Not*** a true statement. This keeps the ***40*** inch tall kid off the ride. (E) is out.

D is the correct answer.

1. Geometry problem. DRAW A PICTURE!
   1. Let’s write our given:
      * + Right circular cylinder with radius of ***5*** and ***height*** of ***4*** has volume ***v***.
   2. What are we looking for?
      * + The ***volume***, in terms of ***v***, of a right circular cylinder with a radius of ***5*** and ***height*** of ***8***.
   3. Definition first. A right circular cylinder means the base of the cylinder is a circle, and that it goes straight up and down.
   4. Now to the formula for the volume of a cylinder. ***Volume*** = (***area of base***) × ***height***. The base in this case is a circle, and the area of a circle = πr2, where r is the radius. We have the numbers; let’s plug and chug.
   5. Draw a picture.

***Cylinder 1***

***Cylinder 2***

***4***

***8***

* + - * Notice we didn’t even put the base numbers in there. It doesn’t matter, because the radius is the same for both cylinders, so the bases are the same. The only thing that changes is the height.
      * v1 = πr2 × 4
      * v2 = πr2 × 8
      * 8 is 2 times 4, so that means that the height ***and*** volume of cylinder 2 are both ***twice*** the height and volume of cylinder 1.
      * v2 = ***2*** × v1. We know v1 is just v, so:
      * v2 = ***2v***
      * Or we could have just looked at the picture and seen that the second cylinder looks twice as big, so that volume must be ***2v***.
  1. If drawing the picture isn’t your thing, here’s the algebraic way to figure it out:
  2. Volume of cylinder 1:
     + - ***Volume*** = (***area of base***) × ***height***
       - ***v*** = (***π52***) × ***4***
       - ***v*** = (***25π***) × ***4***
       - ***v*** = ***100π***
  3. Volume of cylinder 2:
     + - ***Volume*** = (***area of base***) × ***height***
       - ***v*** = (***π52***) × ***8***
       - ***v*** = (***25π***) × ***8***
       - ***v2*** = ***200π***
  4. If ***v*** is ***100π***, and ***v2*** is ***200π***, what’s ***v2*** in terms of ***v***? 200/100 is 2. So ***v2*** = ***2v***

B is the correct answer.

1. What is this?
   1. Let’s write our givens:
      * + ***k***, ***n*** and ***r*** are integers.
        + ***k*** ◆ (***n***, ***r***) is ***true*** only if ***n*** < ***k*** < ***r*** .
        + ***-2*** ◆ (***n***, ***0***) is ***true***.
   2. What are we looking for? Possible value(s) of ***n***.
   3. Weird symbols, parentheses, blah. The key to this problem is keeping the friends in the right order and seeing if any of the answer choices make the statement true.
   4. Start with the given formulas, keeping everything in line.
      * + ***-2*** ◆ (***n***, ***0***)

***k*** ***n***  ***r***

* + - * ***n*** < ***k*** < ***r*** (Given)
      * ***n*** < ***-2*** < ***0***
      * Anything that is less than -2 is less than 0, so we don’t care about the 0.
      * ***n*** < ***-2***

-3: ***-3*** < ***-2***. True. (***I***) is a good answer.

-1: -1 < ***-2***. False. (II) is out.

3: 3 < ***-2***. False. (III) is out.

A is the correct answer.

1. Percentage “math speak” with friends.
   1. Let’s write our given: 20 ***percent*** of ***x*** ***equals*** 80 ***percent*** of ***y***.
   2. What are we looking for? The ***expression*** that expresses ***y*** in terms of ***x***.
   3. Translate the “math speak.”
      * + 20 ***percent*** of ***x*** ***equals*** 80 ***percent*** of ***y***.
        + 20 ***%*** × ***x*** ***=*** 80 ***%*** × ***y***
        + 20***%***(***x***) ***=*** 80***%***(***y***) Get ***y*** by itself. Divide both sides by 80%.
        + 20***%***(***x***) ***=*** 80***%***(***y***) % signs go away.

80% 80%

* + - * 2 × (***x***) ***=*** ***y***

8

* + - * 1 ***×*** (***x***) ***=*** ***y*** Now translate back into “math speak.”

4

* + - * 1 ***of*** (***x***) ***=*** ***y*** What’s ¼ in percentage terms? 25%.***\****

4

* + - * 25% ***of*** (***x***) ***=*** ***y***

B is the correct answer.

***\* ¼ = 0.25. Move the decimal over two places and add the % sign. 0.25 = 25%.***

1. Odd and even friends.
   1. Let’s write our givens:
      * + ***x***, ***y*** and ***z*** are positive integers.
        + ***x*** + ***y*** is ***even***.
        + (***x*** + ***y***)2 + ***x***+ ***z*** is ***odd***.
   2. What are we looking for? The statement that must be ***true***.
   3. Keep the numbers simple. It doesn’t say the numbers have to be *different,* they just have to be positive. Try to eliminate answers by picking numbers that satisfy the answers, testing them in our given equations and identifying resulting statements that disagree with the given equations. If they’re not true, they’re out.
      1. ***x*** is ***odd***. Remember, keep it simple. Try ***x*** = ***1***, ***y*** = ***1***, and ***z*** = ***1***.
         * ***x*** + ***y*** Our first equation – it’s supposed to be ***even***.

***1*** + ***1*** = 2 That’s ***even***. First equation is true. Try the second equation.

* + - * (***x*** + ***y***)2 + ***x***+ ***z*** This more complex expression is supposed to be ***odd***.

(***1*** + ***1***)2 + ***1***+ ***1***

(2)2 + 2

4 + 2 = 6 That’s ***even***. It’s supposed to be ***odd***.

* + - * In this case ***x*** is ***odd***, but the second equation is not satisfied (it’s even). Answer (A) is out.
    1. ***x*** is ***even***. Try ***x*** = ***2***, ***y*** = ***1***, and ***z*** = ***1***.
       - ***x*** + ***y*** Our first equation – it’s supposed to be ***even***.

***2*** + ***1*** = 3 That’s ***odd***. It’s supposed to be ***even***.

* + - * In this case ***x*** is ***even***, but it doesn’t satisfy the first equation. Go no further. Answer (B) is out.
    1. If ***z*** is ***even***, then ***x*** is ***odd***. Try ***x*** = ***1***, ***y*** = ***1***, and ***z*** = ***2***.
       - ***x*** + ***y*** Our first equation – it’s supposed to be ***even.***

***1*** + ***1*** = 2 That’s ***even***. First equation is ***true***. Try the second equation.

* + - * (***x*** + ***y***)2 + ***x***+ ***z*** Our second equation – it’s supposed to be odd.

(***1*** + ***1***)2 + ***1***+ ***2***

(2)2 + 3

C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmf 4 + 3 = 7 That’s ***odd***. Second equation is ***true***, but we’re not done yet.

* + - * We need to see what happens when ***z*** is ***even*** *and* ***x*** *is* ***even*** because our statement says x must (only) be odd. Let ***x*** = ***2***, ***y*** = ***1*** and ***z*** = ***2***.
      * ***x*** + ***y*** Back to our first equation – it’s supposed to be even.

***2*** + ***1*** = 3 That’s ***odd***. It’s supposed to be ***even***.

* + - * So, when ***z*** is ***even***, ***x*** must be ***odd***, because *an even* ***x*** *doesn’t work* (when ***y*** is ***odd***). (***C***) looks good. Check the rest.
    1. If ***z*** is ***even***, then ***xy*** is ***even***. Why not first check to see if ***z*** is ***even***, ***xy*** is ***odd.*** Because if those equations work, we know (D) is out. So, try ***x*** = ***1***, ***y*** = ***1***, and ***z*** = ***2***. Check ***xy*** first.
       - ***x*** × ***y***

***1*** × ***1*** = 1 That’s ***odd***. Now on to our given equations.

* + - * ***x*** + ***y*** Our first equation – it’s supposed to be ***even.***

***1***  + ***1*** = 2 That’s ***even***. Try the second equation.

* + - * (***x*** + ***y***)2 + ***x*** + ***z*** Our second equation – it’s supposed to be odd.

(***1*** + ***1***)2 + ***1*** + ***2***

(2)2 + 3

4 + 3 = 7 That’s ***odd***. Second equation is true.

* + - * In this case, ***z*** is ***even*** (***2***), ***xy*** is ***odd*** (1), and our equations are true. Our answer states that ***xy*** must only be ***even***. So, (D) is out.
    1. ***xy*** is ***even***. We don’t even need to work out this answer, because we showed in the second part of answer (D) above that ***xy*** can be ***odd*** and both equations can still be true. (E) is out.

C is the correct answer.

1. A fractional friend.
   1. Let’s write our given: 0 < ***x*** < 1
   2. What are we looking for? The statement that must be ***true***.
   3. If ***x*** is between o and 1, it’s got to be a fraction. Keep it easy. Let ***x*** = ***½***. To the statements!

***x***2  > ***x***3

(***½***2) > (***½***)3

***¼*** > ***1/8***

That’s true. Squaring a “1/number” fraction is always going to give a bigger result than cubing the same fraction because the cubed denominator is bigger (like 8 > 4) and thus makes a smaller fraction. This is true for all “1/number” fractions. (**I**) is a good answer.

***x*** > ***x***/2

***½*** > ***½***/2

***½***/***2*** = ***½*** × ***½*** = ***¼*** Dividing by ***2*** is the same as multiplying by ***½***.

***½*** > ***¼***

That’s true. Dividing a fraction by a whole number just means the denominator of the new fraction is going to be bigger, like in this case (4 instead of 2). That means the new fraction is smaller than the original. This is true for all “1/number” fractions. (**II**) is a good answer.

***x*** > ***x***3

***½*** > (***½***)3

***½*** > ***1/8***

That’s true. When a “1/number” fraction is raised to any exponent, the new fraction always has a bigger denominator, and thus makes a smaller fraction. This is true for all “1/number” fractions. (**III**) is a good answer.

E is the correct answer.

1. Data analysis and a function. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Scatterplot of 12 hamsters in a maze
        + ***p*** = number of practices (on the x-axis)
        + ***t*** = time to complete the maze (on the y-axis)
   2. What are we looking for? The ***function*** that best models the relationship.
   3. To. The. Picture!

***40***

***50***

***40***

***50***

***p***

***p***

***t***

***t***

* 1. We’re looking for the best model. Fancy words for the line that best fits the data. Draw it in. Look at the picture above on the right. That’s a horizontal line. What’s the slope of a horizontal line? ***ZERO***!
  2. Now a ***y*** = ***mx*** + ***b*** refresher. When ***x*** is the ***input, y*** is always the ***output***, so ***y*** = ***f*** (***x***) = ***y***(***x***). It doesn’t matter how we write it, the ***output*** is always the ***y*** value. In our case, ***x*** = ***p***, and ***y*** = ***y***(***x***) = ***t***(***p***).
     + - ***t***(***p***) is the ***y***-value in a point (***x***,***y***) on the line.
       - ***p*** is the ***x***-value in a point (***x***,***y***) on the line.
       - ***m*** is the ***slope*** (rise over run) of the line.
       - ***b*** is the y-intercept (where the line crosses the y, or vertical, axis)
       - In our case, ***x*** = ***p*** and ***y*** = ***t***(***p***), so rewrite the ***y*** = ***mx*** + ***b*** equation with ***p*** and ***t***(***p***).
       - ***y*** = ***mx*** + ***b***
       - ***t***(***p***) = ***mp*** + ***b***
  3. Since the ***slope*** of a horizontal line is ***0***, the function for our line above is:
     + - ***t***(***p***) = ***mp*** + ***b***
       - ***t***(***p***) = (***0***)***p*** + ***b***
       - ***t***(***p***) = ***b***
  4. The ***p*** completely falls out of the equation! Answers (B), (C), (D) and (E) all have ***p***’s in them. These are all not possible answers because having a ***p*** means there is a slope and we know it’s zero.
     1. ***t***(***p***) = ***44*** No ***slope***, and the ***y-intercept*** looks good. (***A***) is our answer.
     2. ***t***(***p***) = (***1***)***p*** ***Slope*** is 1, so this line would go up 1 and over 1. Nope.
     3. ***t***(***p***) = (***44***)***p*** ***Slope*** is 44, so this line would go up 44 and over one. Very steep line. Nope.

Also, the ***y-intercept*** is ***0***, and our line crosses somewhere around 44.

* + 1. ***t***(***p***) = (***1/44***)***p*** ***Slope*** is 1/44, so this line would go up 1 and over 44. Very flat line, but not still not

horizontal line with a ***slope*** of ***0***. Nope.

* + 1. ***t***(***p***) = (***1***)***p*** + ***44*** Slope is 1, so this line would go up 1 and over 1. Nope.

A is the correct answer.

1. Geometry to end the section. Mark up the picture.
   1. Let’s write our givens:
      * + The pattern is made up of rectangles.
        + The individual rectangles have lengths of ***L*** and widths of ***W***.
        + The pattern is repeated to cover a large rectangular area 12***L*** ***long*** and 10***L*** wide.
   2. What are we looking for? The ***number*** of rectangles with dimensions of ***L*** × ***W*** needed to cover the large rectangular area.

***W***

***W***

***W***

***W***

***L***

***L***

***L***

***2***

***2***

***2***

***2***

***3***

***3***

***3***

* 1. To the picture.
  2. We can get an equation with ***W*** and ***L*** out of this, look at the 2 sides of the rectangular pattern.
     + - ***L*** + ***L*** = ***W*** + ***W*** + ***W***
       - 2***L*** = 3***W*** Divide both sides by 2.
       - ***L*** = 3***W*** Now give a value to ***W*** to find an ***L*** we can work with. Keep it simple; let

2 ***W*** = ***2***, so the 2 in the denominator falls out.

* + - * ***L*** = 3(***2***)

2

* + - * ***L*** = ***3***
  1. What now? We need the area of ***one*** small ***L*** × ***W*** rectangle and the area of the larger 12***L*** ***long*** by 10***L*** ***wide*** rectangle. We have ***W*** = ***2*** and ***L*** = ***3***. Let’s figure out those areas. Area = length × width.
  2. One small ***L*** × ***W*** rectangle.
     + - Area = length × width
       - Area = ***L*** × ***W***
       - Area = ***3*** × ***2***
       - Area = ***6*** This is the area of a small ***L*** × ***W*** rectangle.
  3. Larger rectangular area. Tricky, because width here is in terms of ***L***.
     + - Area = length × width
       - Area = (12***L***) × (10***L***)
       - Area = (12×***3***) × (10×***3***)
       - Area = 36 × 30
       - Area = ***1,080*** That’s the area of the larger pattern.
  4. We know that the small rectangles must fit together to form the larger one. Divide the two to find out the ***number*** of small ***L*** × ***W*** rectangles needed to cover the 12***L*** long × 10***L*** wide rectangle.
     + - ***number*** of small ***L*** × ***W*** rectangles = area of 12***L*** long × 10***L*** wide rectangle

area of small ***L*** × ***W*** rectangles

* + - * ***number*** of small ***L*** × ***W*** rectangles = ***1,080***

***6***

* + - * ***number*** of small ***L*** × ***W*** rectangles = ***180***

E is the correct answer.

# Test 6 Section 2

1. Hot dogs word problem.
   1. Let’s write our givens:
      * + 5 packages of hot dog rolls
        + 1 package contains 12 rolls.
        + The others contain 8 rolls.
   2. What are we looking for? The number of ***total hot dog rolls*** bought.
   3. How many of 8-roll packages do we have? Subtract.
      * + Total bags – 12-roll packages = 8-roll packages
2. - 1 = ***4*** 8-roll packages
   1. Multiply the number of rolls in each type of package by the number of rolls in the package, and then add.
   2. 1 12-roll package
      * + 1 × 12 = ***12*** rolls
   3. 4 8-roll packages
      * + 4 × 8 = ***32***
   4. ***12*** + ***32*** = ***44***

C is the correct answer.

1. Number line problem. DRAW A PICTURE!
   1. Let’s write our givens:
      * + ***A***, ***B*** and ***C*** are points on a line, ***in that order***.
        + ***AB*** =***30***
        + ***BC*** is ***20 more*** than ***AB.***
   2. What are we looking for? The ***value*** of ***AC***.
   3. If BC is 20 more than BC, and we know AC, what’s BC? Translate the “math speak.”
      * + ***BC*** is ***20 more*** than ***AB***
        + ***BC*** = ***20 +*** ***30***
        + ***BC*** = ***50***
   4. Draw the line and label the lengths.

***50***

***30***

***C***

***A***

***B***

* 1. Now we have numbers for the entire length of ***AC***. Add and finish up.
     + - ***AB*** + ***BC*** = ***AC***
       - ***30 + 50 = AC***
       - ***80 = AC***

D is the correct answer.

1. Friends.
   1. Let’s write our given: ***x*** + 3 = ***a***
   2. What are we looking for? The ***value*** of 2***x*** + 6.
   3. Notice that all of our answers have ***a***’s in them. We aren’t finding a number answer; it has to have an ***a***.
   4. The long way first. Get ***x*** by itself in the given.
      * + ***x*** + 3 = ***a*** Subtract 3 from both sides.
        + ***x*** = ***a*** - 3 Now substitute that into 2***x*** + 6.
        + 2***x*** + 6
        + 2(***a*** - 3) + 6
        + 2***a*** - 6 + 6
        + ***2a*** That’s our value!
   5. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfNow,the short way: Look at the left side of the equation, ***x*** + 3, and the 2***x*** + 6 expression. To get from ***x*** to 2***x***, we multiply by 2. To get from 3 to 6 we multiply by 2. So if we just multiply ***a*** by 2, we’ll have our answer. 2***a***.
      * + ***x*** + 3 = ***a***

×2 ×2 ×2.

* + - * 2***x*** + 6 = ***2a***

C is the correct answer.

1. Data analysis problem.
   1. What are we looking for? The ***student*** who had the greatest change in scores from Test I to Test II.
   2. Best thing to do here is make a table and subtract the scores. Remember this just says change, so it makes no mention of increase or decrease, so we’re just looking for the biggest change in value.

|  |  |  |  |
| --- | --- | --- | --- |
| **Student** | **Test I Score** | **Test II Score** | **Difference** |
| A | 40 | 70 | ***30*** |
| B | 40 | 60 | 20 |
| C | 60 | 70 | 10 |
| D | 80 | 80 | 0 |
| E | 80 | 60 | -20 |

* Student A’s test scores had the greatest change( ***30*** points).

A is the correct answer.

1. Average problem with data from problem #4. Glad we made that table.
   1. What are we looking for? The ***average score*** of the ***5*** students on test II.
   2. What’s an average? Sum of terms / Number of terms. Plug and chug.
      * + ***Average Score on Test II***: Sum of Student Scores on Test II

***Number of Students***

* + - * ***Average Score on Test II***: 70 + 60 + 70 + 80 + 60

***5***

* + - * ***Average Score on Test II***: 340

***5***

* + - * ***Average Score on Test II***: ***68***

C is the correct answer.

1. Friends and a line.
   1. Let’s write our given: ***t***, ***u***, ***v***, ***w***, ***x***, ***y***, ***z*** are points on a line.
   2. What are we looking for? The ***point*** with the ***value closest to*** | ***u*** + ***v*** |.
   3. Draw the line! Let’s focus first on just ***u*** and ***v*** since those will guide us to the answer. Between those points, we only have numbers 0 and -1, and it looks like ***u***, ***v*** and ***w*** are equally spaced. Assuming those spaces are equal, ***u***, ***v*** and ***w*** divide the line into fourths. Label those. -2/4 reduced is -1/2.

***u***

***0***

***-1***

***v***

***w***

***-3/4***

***-1/2***

***-1/4***

* 1. Now we have some values for ***u*** (***-3/4***) and ***v*** (***-1/2***). Plug those in to figure out |***u*** + ***v***|.
     + - | ***u*** + ***v*** |
       - |***-3/4*** + ***-1/2***|
       - | ***-1 ¼*** |
       - ***1 ¼***
       - Now look at the line and see where 1 ¼ is on the line. What point is the closest to this value? We don’t know exactly, but point ***y*** looks pretty good.

***1 ¼***

***y***

***z***

***2***

***1***

D is the correct answer.

1. Fractions and a friend.
   1. Let’s write our given: x = ½
   2. What are we looking for? The ***value*** of 1/x + 1/(x – 1).
   3. Simply plug and chug. We like fractions.
      * + 1 + 1 .

x (x – 1)

* + - * 1 . + 1 .

(1/2) (1/2 – 1)

* + - * 1 . + 1 Dividing a fraction by a fraction just means multiplying the numerator by

½ -½ the reciprocal of the denominator..

* + - * 1 × 2 + 1 × (-2).
      * 2 . + -2
      * ***0***

B is the correct answer.

1. Geometry on the coordinate plane. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + ***RS*** = ***ST***
        + Coordinates of **S** are (***k***,3).
   2. What are we looking for? The ***value*** of ***k***.
   3. To the picture. Mark the givens.

***(k,3)***

***S***

***(1,0)***

***T***

***R***

***x***

***y***

***O***

***(k,3)***

***S***

***(1,0)***

***T***

***R***

***x***

***y***

***O***

* 1. Since ***RS*** = ***ST***, and we also have right angles at ***R*** and ***T***, this figure is a square. All the sides are equal (marked in the picture above on the right). We first need to find the length of the side.
     + - Look at point ***S***. Its coordinates are (***k***,3). Its y-value is 3, so the length ***RS*** from the x-axis (horizontal axis) to point ***S*** is 3. That means every ***side*** of our square is 3.
       - Point R and point S both are the same distance from the y-axis (3). But point R and S are in the negative-x territory, so the x-value of those points is ***-3***. So, point ***S*** is (***-3***,3), which means ***k*** = -3.

A is the correct answer.

1. Quadratic function problem.
   1. What are we looking for? The ***function*** that defines ***f***.
   2. It should say “defines ***f***(x),” but we know what they mean.
   3. Try the answers for our different values of x to see if they give the ***f***(x) in the table. We love ***0*** and ***1***, so start with x = ***0***. Plug and chug.
      1. ***f***(x) = x2 + 1

***f***(***0***) = (***0***)2 + 1

***f***(***0***) = 1 (A) works for x = ***0***. ***f***(***0***) = 1, which matches the table. Try (B).

* + 1. ***f***(x) = x2 + 2

***f***(***0***) = (***0***)2 + 2

***f***(***0***) = 2 (B) doesn’t work. ***f***(***0***) = 2 in this case. It’s supposed to be 1. (B) is out.

* + 1. ***f***(x) = 2x2 - 2

***f***(***0***) = 2(***0***)2 - 2

***f***(***0***) = -2 (C) doesn’t work. (C) is out.

* + 1. ***f***(x) = 2x2 - 1

***f***(***0***) = 2(***0***)2 - 1

***f***(***0***) = -1 (D) doesn’t work. (D) is out.

* + 1. ***f***(x) = 2x2 + 1

***f***(***0***) = 2(***0***)2 + 1

***f***(***0***) = 1 (E) works for 0.

* 1. We’ve eliminated (B), (C) and (D), so now try x = ***1*** in (A) and (E).
     1. ***f***(x) = x2 + 1

***f***(***1***) = (***1***)2 + 1

***f***(***1***) = 2 (***A***) works for x = 1 and x = o. That’s our answer, but check x= ***1*** in (E) to confirm.

* + 1. ***f***(x) = 2x2 + 1

***f***(***1***) = 2(***1***)2 + 1

***f***(***1***) = 3 (E) doesn’t work for 1. (E) is out.

A is the correct answer.

1. Word problem with friends.
   1. Let’s write our given: ***x*** years ago the person was ***y*** years old.
   2. What are we looking for? The ***age*** of the person ***1*** year ago.
   3. Sketch out this problem in a table.
      * + We know that ***x*** years ago the person was ***y*** years old. So if we add those years up we’ll have the person’s age now; it’s ***x*** + ***y***.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Time*** | ***x*** years ago | ***1 year ago*** | Now |
| ***Age*** | ***y*** | ***?*** | ***x*** + ***y*** |

* + - * If we know the person’s age now (***x*** + ***y***), we just subtract 1 to find out how old the person was a year ago.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Time*** | ***x*** years ago | ***1 year ago*** | Now |
| ***Age*** | ***y*** | ***x*** + ***y*** - ***1*** | ***x*** + ***y*** |

* + - * ***x*** + ***y*** – ***1*** is the same as ***y*** + ***x*** – ***1***.

E is the correct answer.

1. Sequencing Problem
   1. Let’s write our givens:
      * + 2 adjacent letters can be switched.
        + The entire sequence can be reversed.
   2. What are we looking for? The ***least number*** of changes needed to put the sequence in alphabetical order.
   3. Let’s start interchanging letters to get this thing in order.
      * + Original: Z***W***YX
        + 1st change: ***W***ZYX Flipping the Z and ***W***
        + 2nd change: ***W***ZXY Flipping the X and Y
        + 3rd change: ***W***XZY Flipping the Z and X
        + 4th change: ***W***XYZ Flipping the Z and Y
        + That makes ***4*** changes, but there’s another given we need to test. Let’s see if it requires fewer changes.
   4. Notice that the sequence is almost in reverse alphabetical order; the ***W*** just needs to be moved all the way to the right. Remember the second given, we can flip the ***entire*** sequence. Get ***W*** all the way to the right.
      * + Original: Z***W***YX
        + 1st change: ZY***W***X Flipping the ***W*** and Y
        + 2nd change: ZYX***W*** Flipping the ***W*** and X
        + 3rd change: ***W***XYZ Flipping the ***entire*** sequence
        + That makes ***3*** changes, and it’s now in alphabetical order.

B is the correct answer.

1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfGeometry Problem
   1. Let’s write our givens:
      * + ***Cube*** has edges with lengths of ***4*** cm.
        + ***Rectangular box*** has dimensions of ***20*** cm × ***24*** cm × ***32*** cm.
   2. What are we looking for? The ***number*** of cubes needed to fill the rectangular box.
   3. This is a volume problem. If we find the volume of both figures, we can divide to find out ***how many*** cubes are needed to fill the rectangular box. The volume formula is the same for both: ***length*** × ***width*** × ***depth***. We’ve left off the cm because all the length measurements are in cm. That is a good spot where they could have tricked us, though.
   4. Volume of ***cube***:
      * + Volume = ***length*** × ***width*** × ***depth***
        + Volume = ***4*** × ***4*** × ***4***
        + Volume = ***64***
   5. Volume of ***rectangular box***:
      * + Volume = ***length*** × ***width*** × ***depth***
        + Volume = ***20*** × ***24*** × ***32*** Calculator time.
        + Volume = ***15,360***
   6. Now divide to find out ***how many*** cubes we need to fill the box.
      * + ***Number of cubes*** = ***Volume of Rectangular Box***

***Volume of Cube***

* + - * ***Number of cubes*** = ***15,360***

***64***

* + - * ***Number of cubes*** = ***240***

D is the correct answer.

1. Fractions, roots, exponents and a friend. Fun.
   1. Let’s write our given: 0 < ***n*** < 1
   2. What are we looking for? The ***correct order*** of ***√n***, ***n*** and ***n2.***
   3. We know ***n*** has to be a positive fraction because it’s between o and 1. Let’s pick one that has a nice perfect square in the denominator so the numbers will be easy to work with. How about ***n***=***¼***. Plug that in to find our different values.
      * + ***n*** = ***¼*** That was easy.
        + ***√n*** = √***¼***

***√n*** = √***1***/√***4***

***√n*** = ***½***

* + - * ***n2*** = (***¼***)***2***

***n2*** = 1***2***/4***2***

***n2*** = ***1/16***

* 1. We have our three values. Now put the fractions in order with the ***n*** expressions below them.
     + - ***1/16*** < ***¼*** < ***½***
       - ***n2*** < ***n*** < ***√n***

E is the correct answer.

1. Medians and slopes.
   1. What are we looking for? The ***median*** of the slopes of OA, OB, OC, OD and OE.
   2. First off, what’s a median? In a list of numbers, it’s the number in the middle. We have 5 numbers in this problem, the 5 slopes.
   3. We don’t need to figure out all the slopes. Just by looking at the picture, we know the slopes are in order from steepest (OA) to flattest (OE). Whatever line is in the middle will have the median slope.
   4. The line in the middle is OC, so the slope of OC is the ***median***. What’s the slope of OC?
      * + Slope is ***rise***/***run***. Point O is (***0***,***0***) and Point C is (***4***,***3***).
        + ***rise*** = ***y2*** - ***y1***

***run*** ***x2*** - ***x1***

* + - * ***rise*** = ***3*** - ***0***

***run*** ***4*** - ***0***

* + - * ***rise*** = ***3***

***run*** ***4***

* + - * The ***slope of OC*** is ***¾***.

C is the correct answer.

1. Time zones problem. Make a table.
   1. Let’s write our givens:
      * + When it’s ***noon*** (***12 p.m.***) in New York City (***EST***), it’s ***9 a.m.*** (***PST***) in San Francisco.
        + A plane took off at ***noon*** (***EST***) from New York.
        + That plane arrived in San Francisco at ***4 p.m.*** (***PST***).
        + There are two plane trips each taking the same amount of total time.
   2. What are we looking for? The ***arrival time in New York (EST)*** of a second plane that took off from San Francisco at noon (12 p.m. PST).
   3. Make a table. We know the flights of both planes take the same amount of total time, so we need to figure out the flight time of plane 1. It leaves at ***12 p.m.*** ***EST*** and arrives at ***4 p.m.*** ***PST***. To get PST in EST we need to add 3 hours. ***4 p.m.*** ***PST*** + ***3 hours*** is ***7 p.m.*** ***EST***.

|  |  |  |  |
| --- | --- | --- | --- |
| Plane | Departure/Arrival | Time San Fran PST | Time New York EST |
| Plane 1 | Departure |  | ***12 p.m.*** |
|  | Arrival | ***4 p.m.*** | ***7 p.m.*** |
|  | ***Total Flight Time*** |  | ***7 hours*** |

* + - * Our total flight time for Plane 1 is ***7 hours***.
      * Plane 2 has the same flight time, ***7 hours***.
      * Figure out the ***PST*** arrival time of plane 2 and add ***3 hours*** to get to ***EST***.

|  |  |  |  |
| --- | --- | --- | --- |
| Plane | Departure/Arrival | Time San Fran PST | Time New York EST |
| Plane 2 | Departure | ***12 p.m.*** |  |
|  | Arrival | ***7 p.m.*** | ***10 p.m.*** |

* + - * Plane 2 has an ***arrival time in New York (EST)*** of ***10 p.m***.

A is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Rectangle PQRS
        + QT and RT are quarter circles with centers at P and S, and have radii of ***1***.
   2. What are we looking for? The ***area*** of the shaded region.
   3. To the picture. If QT and RT are quarter circles with radii of 1, then QP, PT, ST and SR are all ***1***.

***T***

***S***

***R***

***P***

***Q***

***1***

***1***

***1***

***1***

* 1. We first need to find the areas of our ***2 quarter circles*** and the ***rectangle***. Let’s start with the rectangle.
  2. Area of PQRS. Area of a rectangle = length × width. Length: PS = PT + ST = **1** + **1** = **2**. Width: QP = **1**
     + - ***Area of PQRS*** = length × width
       - ***Area of PQRS*** = ***2*** × ***1***
       - ***Area of PQRS*** = ***2***
  3. Area of ***2 quarter circles***. 2 quarter circles have the same area as ½ of a circle. The radii of the 2 quarter circles are 1. The area of a circle = ***πr2***. If we find the area of a circle with a radius of 1, we can divide by 2 and find the area of the ***2 quarter circles***.
     + - ***Area of 2 quarter circles*** = Area of Circle

2

* + - * ***Area of 2 quarter circles*** = ***πr2***

2

* + - * ***Area of 2 quarter circles*** = ***π(1)2***

2

* + - * ***Area of 2 quarter circles*** = ***π/2***
  1. Now subtract the area of the 2 quarter circles from the area of the rectangle, and we’ll find the ***area*** of the shaded region.
     + - ***Area*** of the shaded region =  ***Area of PQRS*** - ***Area of 2 quarter circles***
       - ***Area*** of the shaded region =  ***2*** - ***π/2***

B is the correct answer.

1. Function on a coordinate plane.
   1. Let’s write our given: The graph is ***y*** = f(***x***).
   2. What are we looking for? The ***graph*** of ***y*** = f(***x*** + 2).
   3. This is a tricky problem. Pick one point on the given f(***x***). Let’s use (1,0).
      * + In ***y*** = f(***x***), when ***x*** is 1, ***y*** = 0.
        + When ***y*** = f(***x*** + 2), what are ***x*** and ***y***?
          - ***x*** is going to be two lower, so it’s ***x*** = -1.
          - ***y*** is still ***0.*** The y-value (our output) stays the same.
          - Our input is now (***x*** + 2) not just ***x***. So, -1 + 2 gives us 1. Again, that input gives us the same output, or ***y***-value, of ***0***.
          - So, the point on ***y*** = f(***x*** + 2) is (-1,***0***).
   4. Look at the answers to see if (-1,***0***) is on any of potential ***y*** = f(***x*** + 2) graphs.
      1. At x = -1, the point is (-1, 3), not (-1,***0***). Nope.
      2. At x = -1, the point is (-1, 1), not (-1,***0***). (B) is out.
      3. At x = -1, the point is (-1, ***0***). Looks good. Check the others.
      4. At x = -1, the point is (-1, -1), not (-1,***0***). (D) is out.
      5. At x = -1, the point is (-1, 4), not (-1,***0***). (E) is out.

C is the correct answer.

* 1. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfEasy Way:
     + - Know that with a function in the form f(x+n), if the n is positive, the f(x) shifts to the left. If the function was f(x-n), the graph would shift to the right by n.

1. Triangle joy. MARK UP THE PICTURE!

***E***

***B***

***C***

***A***

***F***

***30°***

***50°***

***?***

***D***

***?***

* 1. Let’s write our givens:
     + - AB = BC
       - DE = EF = DF
       - ∠ABC = 30°
       - ∠BDE = 50°
  2. What are we looking for? The ***measure*** of the ***∠DFA***.
  3. To the picture! There are many ways we can do this. Straight lines add up to 180° and triangles have 180°. Mark all the givens.
  4. If we figure out ***∠BAC*** and ***∠ADF*** we can figure out ***∠DFA***.

***D***

***75°***

***E***

***B***

***C***

***A***

***F***

***30°***

***50°***

***?***

***60°***

***60°***

***60°***

* 1. Let’s start with the ***∠BAC***. Look at the big triangle, ∆ABC. Two sides are the same,

so it’s isosceles. That means those base angles ∠BAC and ∠BCA are equal.

We know ∠ABC = 30° so:

* + - * ∠ABC + ***∠BAC*** + ∠BCA = 180°
      * 30° + ***∠BAC*** + ∠BCA = 180° Subtract 30° from both sides.
      * ***∠BAC*** + ∠BCA = 150°
      * ***∠BAC*** = ∠BCA = 150°/2 They’re equal. Divide by 2.
      * ***∠BAC*** = ***75°*** Mark that.
  1. Now we need to find ***∠ADF***.
     + - Look at the small triangle, ∆DEF. We know all the sides are equal,

***75°***

***E***

***B***

***C***

***A***

***F***

***30°***

***50°***

***?***

***60°***

***60°***

***60°***

***70°***

***D***

so it’s equilateral and equiangular. All those angles must be 60°.

* + - * A straight line (like AB) has 180° so:
      * ***∠ADF*** + ∠EDF + ∠BDE = 180°
      * ***∠ADF*** + 60° + 50° = 180°
      * ***∠ADF*** + 110° = 180° Subtract 110° from both sides.
      * ***∠ADF*** = ***70°*** Mark that.
  1. Look at ∆ADF. We now have ***∠ADF*** and***∠BAC*** (same as ***∠DAF*** ).
     + - ***∠ADF*** + ***∠DAF*** + ***∠DFA*** = 180°
       - ***70°*** + ***75°*** + ***∠DFA*** = 180°
       - 145°+ ***∠DFA*** = 180° Subtract 145° from both sides.
       - ***∠DFA*** = ***35°***

B is the correct answer.

1. Proportions and friends.
   1. Let’s write our given: ***a***, ***b***, ***c*** and ***f*** are non-zero numbers.
   2. What are we looking for? The answer choice that gives us a proportion that is ***not equal*** to all the others.
   3. We can figure this out by cross-multiplying. Whichever result gives us a different equation is our answer.
      1. ***a*** = ***b*** ***ac*** = ***bf***

***f*** ***c***

* + 1. ***f*** = ***b*** ***fa*** = ***cb*** (A) and (B) are different. Which one is like the others? Keep going.

***c*** ***a***

* + 1. ***c*** = ***f*** ***af*** = ***cb*** Isn’t that proportional to (B)?Looks like A is the odd man out. Check the others.

***a*** ***b***

* + 1. ***a*** = ***b*** ***fa*** = ***bc***

***c*** ***f***

* + 1. ***af*** = ***1*** ***af*** = ***bc***

***bc*** ***1***

* 1. When (B), (C), (D) and (E) are cross-multiplied, they all give ***fa*** = ***bc***. (***A***) is different.

A is the correct answer.

1. Ugly friends and a symbol function. Nice way to end the section.
   1. Let’s write our givens:
      * + The operation □ is ***x*** □ ***y*** = ***xy*** – ***y***.
        + ***a*** and ***b*** are positive integers.
   2. What are we looking for? The ***expressions*** that can be equal to ***zero.***
   3. Let’s forget about the ***x*** and ***y***. Let’s work in terms of ***a*** and ***b*** because all the answers are in that form.
      * + ***x*** = ***a*** and ***y*** = ***b***
        + ***x*** □ ***y*** = ***xy*** – ***y***
        + ***a*** □ ***b*** = ***ab*** – ***b***
   4. Pick some easy numbers for ***a*** and ***b***. How about ***1*** and ***1****?* The integers have to be positive, not different.
      * + ***a*** = ***1*** and ***b*** = ***1***

***a*** □ ***b***

***1*** □ ***1*** = ***1***(***1***) – ***1***

***1*** □ ***1*** = 1 – ***1***

***1*** □ ***1*** = ***0*** ***I*** works. Answers (B) and (C) are out.

(***a*** +***b***) □ ***b***

(***1*** +***1***) □ ***1***

2 □ ***1***

2 □ ***1*** = 2(***1***) - 1

2 □ ***1*** = 2 - 1

2 □ ***1*** = 1 ***II*** doesn’t work for 1 and 1. We might need to try more numbers.

But first check III.

***a*** □ (***a*** +***b***)

***1*** □ (***1*** +***1***)

***1*** □ 2

***1*** □ 2 = ***1***(2) - 2

***1*** □ 2 = 2 - 2

***1*** □ 2 = ***0*** That equals zero. ***III*** works. (A) and (D) are eliminated

because they don’t include ***III***.

E is the correct answer.

# Test 6 Section 4

1. Easy friends to start.
   1. Let’s write our givens:
      * + ***x*** - ***y*** = 8
        + ***y*** = 3***z***
        + ***z*** = ***2***
   2. What are we looking for? The ***value*** of ***x***.
   3. Simply plug and chug. Start with the 2nd and 3rd givens.
      * + ***z*** = 2
        + ***y*** = 3***z***
        + ***y*** = 3(***2***)
        + ***y*** = ***6*** Plug that into the 1st given and simplify.
        + ***x*** - ***y*** = 8
        + ***x*** - ***6*** = 8 Add 6 to both sides.
        + ***x*** = ***14***

E is the correct answer.

1. Age word problem with friends.
   1. Let’s write our givens:
      * + ***Todd*** is ***t*** years old.
        + ***Marta*** is ***m*** years old.
        + ***Susan*** is ***s*** years old.
        + ***Todd*** is older than ***Marta*** but younger than ***Susan***.
   2. What are we looking for? The ***true*** statement.
   3. Translate the ‘math speak.” Split it into two sentences.
      * + ***Todd*** is older than ***Marta***.
        + ***t*** > ***m***
        + ***Todd*** is younger than ***Susan***.
        + ***t*** < ***s***
        + Put it all together in one statement. ***t*** is in the middle.
        + ***m*** < ***t*** < ***s***

A is the correct answer.

1. Average problem.
   1. Let’s write our givens:
      * + Areas of ***two*** regions are equal.
        + ***Sum*** of the areas is ***5***.
   2. What are we looking for? The ***average*** of the areas of the two regions.
   3. Sketch a picture.

=

* 1. What’s an average? The ***sum*** of the terms/ the ***number*** of terms. Plug and chug. In this case our “terms” are “areas.”
     + - ***sum*** of the terms

***number*** of terms

* + - * ***5***

***2***

B is the correct answer.

1. Number set.
   1. Let’s write our givens:
      * + ***S*** is the set of all integers that can be written as ***n***2 + 1.
        + ***n*** is a non-zero integer.
   2. What are we looking for? The ***answer*** choice that can be in ***S***.
   3. Plug and chug using the answer choices. Remember, they are possible integer values of ***S***, not ***n***. So, plug in the S and see if ***n*** is an integer. If it is, then we have our answer.
      1. ***S*** = ***16***

***S*** = ***n***2 + 1

***16*** = ***n***2 + 1

***15*** = ***n***2

***√15*** = ***n*** ***√15*** is ***not an integer*** (***15*** is not a perfect square), so ***n*** won’t be an integer. (A) is out.

* + 1. ***S*** = ***28***

***28*** = ***n***2 + 1

***27*** = ***n***2 ***27*** is not a perfect square. (B) is out.

* + 1. ***S*** = ***35***

***35*** = ***n***2 + 1

***34*** = ***n***2 ***34*** is not a perfect square. (C) is out.

* + 1. ***S*** = ***38***

***38*** = ***n***2 + 1

***37*** = ***n***2 ***37*** is not a perfect square. (D) is out.

* + 1. ***S*** = ***50***

***50*** = ***n***2 + 1

***49*** = ***n***2

***7***  = ***n 7*** is an integer. ***S*** can be ***50***. (E) works.

E is the correct answer.

1. Circle problem. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + O is the center of the circle.
        + ***x*** = ***40***
   2. What are we looking for? The ***value*** of ***y***°.
   3. To the picture! Look at the triangle; two of the legs are radii of the circle.
      * + The radii are equal. Mark that.
        + If the two legs of a triangle are equal, that means the two base angles are the same. Mark that.

***40°***

***y°***

***O***

* + - * The sum of the angles of a triangle is 180°. So, ***40***° plus our two ***green*** angles (both ***y***°) add up to 180°.
      * ***40***° + ***y***° + ***y***° = 180°
      * ***40***° + 2***y***° = 180° Subtract 40° from both sides.
      * 2***y***° = 140° Divide both sides by 2.
      * ***y***° = 70°

D is the correct answer.

1. Simple squares.
   1. Let’s write our givens:
      * + A simple square is an integer greater than 1.
        + It only has 3 positive factors: itself, its square root and 1
   2. What are we looking for? The ***answer*** that is a ***simple square***.
   3. Based on our givens, we know that if an answer is divisible by more positive integers than just itself, its square root and 1, then it’s not a simple square. Check the factors of the answer choices.
      1. ***121***:
         * The only factors of ***121*** are: ***1***, ***11*** and ***121***.
         * (A) looks pretty good, because there aren’t any others factors. Check the others.
      2. 100:
         * Square root is 10, so that’s an integer.
         * But there are many factors of 100: 1, 2, 4, 5, 10, 20, 25, 50 and 100. That’s more than 3. (B) is out.
      3. 81:
         * Square root is 9, so that’s an integer.
         * But there are many factors of 81: 1, 3, 9, 27 and 81. That’s more than 3. (C) is out.
      4. 64:
         * Square root is 8, so that’s an integer.
         * But there are many factors of 64: 1, 2, 4, 8, 16, 32 and 64. That’s more than 3. (D) is out.
      5. 33:
         * The square root of 33 isn’t an integer. (E) is out.

A is the correct answer.

1. Triangle and a friend. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + ***XZ*** is 6/7 of ***h***.
        + ***h*** is the altitude of the triangle.

***(6/7)h***

***h***

***Z***

***X***

***Y***

* 1. What are we looking for? The ***area*** of ∆XYZ in terms of ***h***.
  2. Translate the “math speak” for ***XZ***.
     + - ***XZ*** ***is*** 6/7 ***of*** ***h***.
       - ***XZ*** ***=*** 6/7 ***×*** ***h***.
       - ***XZ*** ***=*** 6***h***

7

* 1. Mark up the picture.
  2. The ***area*** of a triangle = ½(***base*** ***×***  ***height***) Plug and chug.
     + - ***area*** of ∆XYZ = ½(***base*** ***×***  ***height***)
       - ***area*** of ∆XYZ = ½( ***XZ ×***  ***h*** )
       - ***area*** of ∆XYZ = 1 6***h ×***  ***h***

2 7

* + - * ***area*** of ∆XYZ = 1 6***h***2

2 7

* + - * ***area*** of ∆XYZ = 6***h***2

14

* + - * ***area*** of ∆XYZ = 3***h***2

7

B is the correct answer.

1. Ugly exponential friends. This is a tough one.
   1. Let’s write our givens:
      * + ***a*** and ***b*** are positive integers.
        + (***a***1/2 ***b***1/3)6 = 432
   2. What are we looking for? The ***value*** of ***ab***.
   3. First, we can distribute that ***6*** on the outside to make it a little easier. Remember the exponent rules. Exponents raised to exponents means we ***multiply*** the exponents.
      * + (***a***1/2 ***b***1/3)***6*** = 432
        + ***a***(1/2)***×6*** ***b***(1/3)***×6*** = 432 In the exponents: ½ × 6 = 3 1/3 × 6 = 2
        + ***a***3***b***2 = 432 That’s a little better.
   4. Next, let’s start trying the answer choices.
      1. ***ab*** = 6 That means ***a*** could be ***2*** and ***b*** could be ***3*** (2 x 3 = 6). Try that in ***a***3***b***2.
         * ***a***3***b***2 = 432
         * ***2***3***3***2 = 432
         * ***4*** × ***9*** = 432
         * 36 ≠ 432 That doesn’t work.
         * That’s way too low. Even if ***a*** were ***3*** and ***b*** were ***2***, our answer would be too small. Move to (B).
      2. ***ab*** = 12 That means ***a*** could be ***3*** and ***b*** could be ***4***. Try that in ***a***3***b***2.
         * ***a***3***b***2 = 432
         * ***3***3***4***2 = 432
         * ***27*** × ***16*** = 432
         * ***432*** = 432 (B) works.

B is the correct answer.

* 1. This is definitely a “feel” question. After getting rid of the fractional exponent, we just need to start plugging in values for ***a*** and ***b*** into ***a***3***b***2 and see what we get. We started at (A), and the value was way too small. If we had started at (E), it would have been a nightmare because there are many factors of 36 and the result of ***a***3***b***2 would have also been way too big.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. Integer Problem
   1. Let’s write our given: We have a 3-digit number that has a factor of 10.
   2. What are we looking for? The ***greatest*** 3-digit number that has a factor of 10.
   3. Pretty straight-forward. What are all the positive 3-digit numbers? They range from 100-999.
      * + 999 is not divisible by 10, but what’s the closest number that is? 998? Nope. 995? Nope. Nothing between 991 and 999 is divisible by 10, but ***990*** is. Check it.
        + ***990*** = 99. It checks out.

10

***990*** is the correct answer.

1. Rate Word Problem
   1. Let’s write our given:
      * + Chili for ***20 people*** requires ***4 pounds of beans***.
   2. What are we looking for? The number of ***pounds*** of ***beans*** (let that be ***x***) that are needed for ***150 people***.
   3. Rate is the same thing as proportion. They’re getting tricky on us.
      * + ***20 people*** .

***4 pounds of beans***

* 1. Set that equal to another proportion with 150 people.
     + - ***20 people*** . = ***150 people*** .

***4 pounds of beans x pounds of beans***

* 1. Cross multiply and solve.
     + - ***20*** × ***x*** = ***150*** × ***4***
       - ***20x*** =  ***600*** Divide both sides by 20.
       - ***x*** =  ***30 pounds of beans***

***30*** is the correct answer.

1. Integer Problem
   1. Let’s write our givens:
      * + ***n*** is a ***positive, even*** integer.
        + When ***n*** is increased by 50% of ***itself***, the result is between 10 and 20.
   2. What are we looking for? A possible ***value of n***.
   3. First translate the 'math speak’ of the second given.
      * + ***n is increased by*** 50% ***of*** ***itself***
        + ***n*** ***+*** (50% ***×*** ***n***)
        + ***n*** ***+*** 50%***n*** What’s 50% in decimal form? .50.
        + ***n*** ***+*** .5***n***
        + 1.5***n***
   4. We need an ***n*** that will make 1.5***n*** be between ***10*** and ***20***. ***n*** has to be positive and even. Try 2.
      * + ***n*** = ***2***

1.5***n***

1.5(***2***) = 3 That’s not between 10 and 20. We need to move higher. What about 8?

* + - * ***n*** = ***8***

1.5***n***

1.5(***8***) = ***12*** 12 is between 10 and 20. ***n*** = ***8*** works. (So do ***10*** and ***12.***)

***8*** is a correct answer.

1. Rectangle word problem. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + The ***perimeter*** of the rectangle is ***250*** meters.
        + The ***length*** of one side of the plot is ***40*** meters.
   2. What are we looking for? The ***area*** of the rectangular plot.
   3. ***Area*** is ***length*** × ***width***. We know one ***side*** of the rectangle is ***40***. That means two sides are ***40***. We don’t know the other sides. But we do know they are equal, so let the other sides be ***x***.

***Perimeter = 250***

***40***

***40***

***x***

***x***

* 1. We have all the sides, so write an equation for the ***perimeter***.
     + - ***perimeter*** = ***side*** + ***side*** + ***side*** + ***side***
       - ***250*** = ***40*** + ***x*** + ***40*** + ***x***
       - ***250*** = ***80*** + 2***x*** Subtract 80 from both sides.
       - 170 = 2***x*** Divide both sides by 2.
       - 170 = 2***x***

2 2

* + - * ***85*** = ***x***
  1. Back to ***area***.
     + - ***Area*** = ***length*** × ***width***
       - ***Area*** = ***40*** × ***85***
       - ***Area*** = ***3400***

***3400*** is the correct answer.

1. Tricky system of equations word problem.
   1. Let’s write our givens:
      * + School ordered $600 worth of light bulbs.
        + Bulbs cost $1 and $2.
        + Twice as many ***$1 light bulbs*** were ordered than ***$2 light bulbs***.
   2. What are we looking for? The ***total number*** of light bulbs ordered.
   3. We need to define some friends so we can write some equations for our system.
      * + Let ***$1 light bulbs*** = ***x.***
        + Let ***$2 light bulbs*** = ***y.***
   4. Write some equations by translating the ‘math speak.”
      * + School ordered $600 worth of light bulbs.
        + $600 = $1(***x***) +$2(***y***)
        + 600 = ***x*** + 2***y*** ***Equation 1***
        + ***Twice as many*** ***$1 light bulbs*** were ordered ***than*** ***$2 light bulbs***.
        + ***x*** ***=*** ***2×*** ***y***
        + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfThis isn’t a straight translation, but think about it. If ***10*** ***$1 light bulbs*** were bought, and that is twice as many bulbs as ***$2 light bulbs***, there would be ***5*** ***$2 light bulbs***. ***10*** = ***2× 5***, so the ***2×*** has to go with the ***y***.
        + ***x*** ***=*** ***2y*** ***Equation 2***
   5. Solve the system. ***x*** is already by itself, so substitute Equation 2 into Equation 1.
      * + ***x*** ***=*** ***2y***
        + 600 = ***x*** + 2***y***
        + 600 = (***2y***) + 2***y***
        + 600 = 4***y*** Divide both sides by 4.
        + ***150*** = ***y*** Plug that back into ***Equation 2*** to find ***x***.
        + ***x*** ***=*** ***2y***
        + ***x*** ***=*** ***2***(***150***)
        + ***x*** ***=*** ***300*** Add ***x*** and ***y*** to get the ***total bulbs***.
        + ***x*** + ***y*** = ***total bulbs***
        + ***300*** + ***150*** = ***total bulbs***
        + ***450*** = ***total bulbs***

***450*** is the correct answer.

1. Friends and factoring.
   1. Let’s write our givens:
      * + 4(***x + y***)(***x - y***) = 40
        + ***x - y*** = 20
   2. What are we looking for? The ***value*** of ***x + y***.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfIt looks awful tempting to foil this thing, but that will lead us down the wrong path. Look at the second given. ***x - y*** appears in both givens, and it equals ***20***. We can just plug that into the first given!
      * + 4(***x + y***)(***x - y***) = 40
        + 4(***x + y***)(***20***) = 40
        + 80(***x + y***) = 40 Divide by 80 and we’re done.
        + ***x + y*** = ***½***

***½***  is the correct answer. (***.5***  works as well.)

1. Circle on a coordinate plane. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + The center of the circle is (***5***,***12***).
        + The circle touches the x-axis at one point only.
   2. What are we looking for? The ***radius*** of the circle.
   3. To the picture! Label the x-axis and y-axis, and draw a circle that touches the x-axis ***at only one point***.

***x***

***y***

***(5,12)***

***12***

***x***

***y***

***(5,12)***

* + - * Our center point, (***5***,***12***), is to the right ***5*** and up ***12***. Since the circle touches the x-axis at one point only, that means that the x-axis is tangent to the circle and forms a right angle with the vertical linethat goes from the x-axis to the center. The distance of that vertical line is our ***radius***.
      * The center of circle is ***12*** above the x-axis, so our ***radius*** = ***12***.

***12*** is the correct answer.

1. Data Analysis. Add a row to the table.
   1. Let’s write our givens:
      * + ***40 percent*** of the ***total voting-age population*** voted.
        + ***Turnout*** is (***number who actually voted***) / (***number of registered voters***).
   2. What are we looking for? The ***turnout*** for the election.
   3. Add a row to the table for “***total***”. Sum the columns.

|  |  |  |
| --- | --- | --- |
|  | Voting-Age Population | ***Number of Registered Voters*** |
| Men | 1,200 | 1,000 |
| Women | 1,300 | 1,200 |
| ***Total*** | ***2,500*** | ***2,200*** |

* 1. Find out how many people voted. ***40 percent*** of the voting-age population voted.
     + - ***Total voting age population*** = ***2,500***
       - ***40 percent*** of the ***total voting age population*** = ***number who actually voted***
       - ***40 percent*** × ***2,500*** = ***number who actually voted***
       - ***1,000*** = ***number who actually voted***
  2. Plug our values into the ***turnout*** fraction. We know that there are ***2,200 total registered voters***.
     + - ***turnout*** = ***number who actually voted***

***number of registered voters***

* + - * ***turnout*** = ***1,000***

***2,200***

* + - * ***turnout*** = ***.455***

***.455*** is the correct answer. We divided and rounded. ***5/11*** and ***.454*** also are acceptable.

1. Geometry. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + The figure has 2 bases that are parallel and 18 edges.
        + Line segments are drawn between V and the other 11 vertices.
   2. What are we looking for? The ***number*** of these segments that ***are not on an edge*** of the figure.
   3. First, we need to know how many segments we are dealing with. There are lines between V and 11 other vertices, so there are ***11 total line segments***.
   4. Quick definition refresher.
      * + An edge is a line. All the solid and dotted lines are edges.
        + A face is the top, the bottom and the 6 sides of the figure.
        + If a segment cuts across a face, it ***is not*** on an edge.
   5. Let’s go backwards, and try to see how many segments ***are*** on edges. Look at the picture below, we’ve taken just a small piece of the entire figure.

***V***

* + - * Those ***3 segments*** are the only ones that are on edges, two on the top of the figure, and one on the back. If we have ***11 total line segments*** from point V, we can subtract.
      * ***Total line segments*** = ***segments on edges*** + ***segments not on an edge***
      * ***11*** = ***3*** + ***segments not on an edge***

-3 -3

* + - * ***8*** = ***segments not on an edge***

***8*** is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + ABCD is a rectangle.
        + Points A and C are also on the function ***y*** = ***px***3.
        + ***p*** is a constant.
        + The ***area*** of ABCD is ***4***.
   2. What are we looking for? The ***value*** of ***p***.
   3. First, look at points A (***-½***,***a***) and B(***-½***,***b***). They both have ***-½*** as their ***x-value***, so that means that this rectangle is straight up and down. Now look at points A (***-½***,***a***) and D(***-½***,***d***). Since this rectangle is straight up and down, we know that ***a*** and ***b*** have to be the same number, with ***a*** being negative and ***b*** being positive. We don’t know what they are yet, but that’s okay. We can find the length of AB (and CD) from the points.
   4. To find the length of AB, we first need to find the width of the rectangle. If we look at the drawing, the first thing we realize is that the rectangle has a width of ***1***. Let’s confirm it.
      * + AD = (***x-value*** of point D) – (***x-value*** of point A)
        + AD = (***½***) – (***-½***)
        + AD = ***½*** + ***½***
        + ***AD*** = ***1*** (and BC = 1) This confirms our width.
   5. Now use that to find the length of AB. How? We know the area of ABCD.
      * + Area of ABCD = length × width
        + Area of ABCD = AB × ***AD***
        + ***4*** = AB × ***1***
        + 4 = AB = CD
   6. Now the function comes into play. In all functions in the form y = x3, as the value of x goes *right* (positive), and the value of y goes *up* (positive), the value of -x will travel the same distance *left* (negative)*,* and the value of -y will travel the same distance *down (negative)*. Huh? Look at the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| y = x3 | x | x3 | y |
|  | 1 | (1)3 | 1 |
|  | -1 | (-1)3 | -1 |
|  | 2 | (2)3 | 8 |
|  | -2 | (-2)3 | -8 |

* 1. This also holds true for our problem. That helps us, because A and C both have ***x-values*** of ***½***, but A’s is negative. That means the ***y-value*** for C, which is positive, is going to be the same value as the ***y-value*** of point A, which is negative. You can see that two points like these have the same distance above and below the x- axis. So, with our rectangle centered about the origin (0,0), exactly half of the length (AB) is going to be above the x-axis, and the rest will be below.
  2. Now we can find the y-value of point C (***½***, ***c***). We know CD = AB = 4.
     + - CD = 4
       - Half of CD is above the x-axis, so that’s ***2*** above the x-axis. The ***y-value*** of C is ***2***.
       - Point C is (***½***, ***2***).
  3. Back to the function. Use point C to find ***p***. The coordinates of C are (***½***, ***2***). Plug and chug to find ***p***.
     + - ***y*** = ***px***3
       - ***2*** = ***p***(***½***)3
       - ***2*** = ***p*** (1/8)
       - ***2*** = ***p***

8

* + - * ***2*** × 8 = ***p*** × 8

8

* + - * ***16*** = ***p***

***16*** is the correct answer.

# Test 6 Section 8

1. Simple friend.
   1. Let’s write our given: 3(***n*** – 4) = 18
   2. What are we looking for? The ***value*** of ***n***.
   3. Simplify the equation.
      * + 3(***n*** – 4) = 18 Divide both sides by 3.
        + ***n*** – 4 = 6 Add 4 to both sides.
        + ***n*** = ***10***

D is the correct answer.

1. Combination Problem
   1. Let’s write our givens:
      * + Ring can be one of ***4 stone types***.
        + Ring can be one of ***3 metal types***.
   2. What are we looking for? The number of ***combinations*** of ***stones*** and ***metals***.
   3. We can write symbols for every type of ***stone*** and ***metal***, but we know each ***stone*** can go with each ***metal***. Just multiply the total ***stones*** by the total ***metals*** to get the total ***combinations***.
      * + ***4*** × ***3*** = ***12*** ***combinations***

D is the correct answer.

1. “Math Speak” Problem
   1. Let’s write our given: The sum of 3***a*** and the square root of ***b*** is equal to the square of sum of ***a*** and ***b***.
   2. What are we looking for? The ***expression*** that reflects the statement.
   3. Translate the“math speak.”
      * + The ***sum*** of 3***a*** and the square root of ***b*** ***is equal to*** the ***square of the sum*** of ***a*** and ***b***.
        + 3***a*** + √***b*** ***=*** (***a*** ***+*** ***b***)2
        + 3***a*** + √***b*** =(***a*** ***+*** ***b***)2. That’s a match for (B).

C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\QLBV975J\MC900441892[1].wmfB is the correct answer.

* 1. Be careful with “the square of the sum” statement. That means add the ***a*** and ***b*** ***before*** squaring the whole term, it doesn’t mean ***a***2 + ***b***2. That statement would be “the sum of ***a*** squared and ***b*** squared.”

1. Triangle problem. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + Phone has range of 1,000 feet.
        + Kerry walks 800 feet due north.
        + From 800 feet due north, Kerry then walks due east to the max range of the phone.
   2. What are we looking for? The ***direction***(***s***) that Kerry can walk and still be in range.
   3. Draw a picture and label it. A range forms a circle, with the range being the radius.

***Base***

***800***

***1,000 ft range***

* + - * Now look at the answer choices.

Due north. That doesn’t work, because Kerry will be out above the circle. (I) is out.

Due south. That works. Still in the circle. (***II***) is good.

Due west. That also works. Still in the circle. (***III***) is good.

E is the correct answer.

1. Friends and fractions.
   1. Let’s write our givens:
      * + ***x*** = 2***x***

4 ***a***

* + - * ***x*** ≠ 0
  1. What are we looking for? The ***value*** of ***a***.
  2. 2 variables, one equation. We usually need two equations. Let’s cross-multiply and see what happens.
     + - ***x*** = 2***x***

4 ***a***

* + - * ***ax*** = 4(2***x***)
      * ***ax*** = 8***x*** Divide both sides by ***x***.
      * ***a*** = ***8***

A is the correct answer.

1. Vertical angles. MARK UP THE PICTURE!
   1. Let’s write our givens.
      * + l ∥m
        + ***r*** = ***50***
   2. What are we looking for? The ***value*** of ***s*** + ***t*** + ***u***.
   3. To the picture. Mark the parallel lines, ***l*** and ***m,*** and ***r***° = ***50***°. Notice that angles ***r***° and ***u***° are corresponding angles. Since corresponding angles are equal, we know that angles ***r***° and ***u***° are equal. Mark those, too.

***l***

***m***

***r***°

***u***°

***s***°

***t***°

***l***

***m***

***50***°

***50***°

***s***°

***t***°

* 1. ***s***° and ***t***° form a straight line, the transversal that cuts across ***l*** and ***m***.
     + - ***s***° + ***t***° = 180°
  2. Plug and chug for our answer.
     + - (***s*** + ***t***) + ***u***
       - 180 + ***50***
       - ***230***

A is the correct answer.

1. Coordinate plane problem. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + Line ***l*** is perpendicular to the ***y-axis***.
        + Line ***l*** passes through the point (5,-3).
   2. What are we looking for? An ***equation*** of line ***l***.
   3. Draw the picture, labeling the x-axis and y-axis. Draw ***l***. It’s perpendicular to the y-axis, so it forms a ***right angle*** with the y-axis. That means ***l*** is a horizontal line.

***y***

***(5,-3)***

***x***

* 1. We know that a horizontal line has only one variable in the equation, but is it x or y? Look at the answer choices.
     1. x= 0 If x = 0, it means that x never varies. It doesn’t matter what y would be. The point could be (0, -1,228) or (0,2); the x would always be 0. That’s a *vertical* line (in fact, it’s the y-axis). Our line is horizontal. (A) is out.
     2. x = 5 Also a vertical line. (B) is out.
     3. ***y = -3*** y is -3 for every value of x. (1,-3) and (800,-3) are points on this line. This looks good.
     4. x and y are both in this equation. That means this line has a slope. (Just plug in values for x: if x=0, y=2; x=1, y=3. See the slope?) (D) is out.
     5. Same reasoning as (D). (E) is out.

C is the correct answer.

1. Word problem with a couple friends.
   1. Let’s write our givens:
      * + ***p*** = daily ***profit***
        + ***x*** = ***units*** sold and produced
        + The profit function is ***p***(***x***) = 17***x*** – (10***x*** + ***b***).
        + ***300*** ***units*** were sold for a ***profit*** of ***$1,900***.
   2. What are we looking for? The ***value*** of ***b***.
   3. This is a pure plug and chug. Put in the ***units*** and ***profit*** numbers given to figure out ***b***.
      * + ***p***(***x***) = 17***x*** - (10***x*** + ***b***) Distribute the negative.
        + ***p***(***x***) = 17***x***  -10***x*** - ***b*** Combine like terms (the ***x***’s).
        + ***p***(***x***) = 7***x*** - ***b***
        + ***1,900*** = 7(***300***) - ***b***
        + ***1,900*** = 2,100 - ***b*** Subtract 2,100 from both sides.
        + -200 = - ***b*** Multiply both sides by -1.
        + ***200*** = ***b***

E is the correct answer.

1. Word Problem
   1. Let’s write our given: An integer is multiplied by itself. That is the same thing as ***squaring*** that integer.
   2. What are we looking for? The ***digit*** that the squared number ***cannot end in***.
   3. We’re looking at the ***last digit*** of a number that results from squaring an integer. For example, 102 = 10***0***, and ***0*** is the last digit. Let’s look at our answer choices, and see if we can square a number to have that answer as the last ***digit***.
      1. 1 12 = ***1*** (A) works, ***1*** is the last digit when 1 is squared. (A) is out.
      2. 4 22 = ***4*** (B) works, ***4*** is the last digit when 2 is squared. (B) is out.
      3. 5 52 = 2***5*** (C) works, ***5*** is the last digit when 5 is squared. (C) is out.
      4. 6 62 = 3***6*** (D) works, ***6*** is the last digit when 6 is squared. (D) is out.
      5. ***8*** By elimination, ***8*** is the answer that doesn’t work. Instead of going through every possible number that ends in ***8*** (because we’d have to check to see if it were a perfect square), be confident in the work we did in (A) through (D).

E is the correct answer.

1. Probability Problem
   1. Let’s write our givens:
      * + A bag contains ***red***, ***blue*** and ***yellow*** marbles.
        + Probability of selecting a ***red*** marble is ***¼***.
        + Probability of selecting a ***blue*** marble is ***1/6***.
   2. What are we looking for? A possible ***total number*** of marbles in the bag.
   3. Sketch out a table.

|  |  |  |  |
| --- | --- | --- | --- |
|  | ***red*** | ***blue*** | ***yellow*** |
| Probability of pulling one marble | ***¼*** | ***1/6*** | ***?*** |

* 1. We don’t know the probability of pulling one ***yellow*** marble, but don’t worry about that. Get the ***red*** and ***blue*** probabilities into fractions with ***common denominators***. ***4*** and ***6*** both go into ***12***.
     + - ***red: 1*** = ***3*** ***blue***: ***1*** = ***2***

***4*** ***12*** ***6*** ***12***

* + - * Probability = chance of pulling 1 marble

***total number of marbles***

* + - * Using the rewritten probability for ***red***, that’s ***3 .***

***12***

* + - * ***12*** is a possible total number of marbles.
      * The total number of marbles must be divisible by 4 & 6. None of the other answers can be divided evenly into 12.

B is the correct answer.

1. Averages.
   1. Let’s write our given: When the ***sum*** of a list of prices is divided by the ***average*** of the prices, the result is ***k***.
   2. What are we looking for? What ***k*** represents.
   3. Write the basic definition of an average.
      * + ***sum*** = ***average*** Let’s call this the general equation.

***number***

* 1. Translate the “math speak” of the given.
     + - When the ***sum*** of a list of prices is ***divided*** by the ***average*** of the prices, the result ***is*** ***k***.
       - ***sum***  ***=*** ***k*** Let’s call this the “math speak” equation***.***

***average***

* 1. The only difference between the first and second equations is that ***average*** and ***number*** have switched places. If we multiply the general ***average*** equation by ***number*** and then divide by the ***average***, we will get the same form as the equation from the given.
     + - ***sum*** = ***average***

***number***

* + - * ***sum***  × ***number*** = ***average*** × ***number*** Simplify the equation.

***number***

* + - * ***sum*** ×= ***average*** × ***number*** Now divide by ***average***.
      * ***sum***  = ***average*** × ***number*** Simplify again.

***average average***

* + - * ***sum*** = ***number***

***average***

* 1. Compare that to the “math speak” equation.
     + - ***sum of the prices*** ***=*** ***number of prices***

***average of prices***

* + - * ***k*** *represents* the ***number of prices***.

D is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Area of the square is 81.
        + ***Perimeter*** of ***each*** of the four triangles is ***30***.
   2. What are we looking for? The ***perimeter*** of the figure (***the solid line***).
   3. To the picture! Mark the sides of the square first. Area = side × side, and in a square all the sides are equal, so side2 = area.

***9***

***9***

***9***

***9***

***y***

***x***

***9***

***9***

***9***

***9***

* + - * area = 81
      * side2 = 81
      * √(side2) = √81
      * side = 9
  1. Now let’s look at the triangles. The perimeter of each triangle is 30. We know one side = 9. Let’s name two friends, ***x*** and ***y***, to be the other sides. Mark that (picture on the right above). Write an equation for the ***perimeter***.
     + - ***Perimeter*** of triangle = ***x*** + ***y*** + ***9***
       - ***30*** = ***x*** + ***y*** + ***9*** Subtract 9 from both sides.
       - ***21*** = ***x*** + ***y***
       - C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfThat helps us because now we know the two outer legs of the triangle, which make up the ***perimeter*** of the whole figure. We don’t need to know what ***x*** and ***y*** are individually. The sum of ***x*** + ***y*** (or ***21***) is going to be the sum of the two outer legs on all the triangles, because all the triangles have a perimeter of ***30*** with one side of length ***9***.
  2. We have ***4*** triangles, and the ***x*** + ***y*** from all of those triangles will give us the ***perimeter*** of the figure. Multiply to find the perimeter.
     + - ***4*** × (***x*** + ***y***) = ***perimeter***
       - ***4*** × (***21***) = ***perimeter***
       - ***84*** = ***perimeter***

D is the correct answer.

1. Function problem. We need to keep our inputs and outputs straight and MARK UP THE GRAPH!
   1. Let’s write our givens:
      * + ***y*** = ***g***(***x***)
        + ***g***(***2***) = ***k***
   2. What are we looking for? The ***value*** of ***g***(***k***).
   3. Function review.
      * + ***x*** is an ***x-value*** input that gives an output of ***y*** (***y-value***). When we input ***x*** into ***g***(***x***) we get a ***y***.
   4. To the picture. Look at the second given, ***g***(***2***) = ***k***. That means when our ***x*** is ***2***, our ***y-value*** is ***k***. Look at the graph to find the ***y-value*** (***k***) when ***x*** = ***2***.

***x***

***y***

***x***

***y***

***5***

***2***

***2.5***

***5***

* + - * When ***x*** = ***2***, ***y*** = ***5***, so ***k*** = ***5***.
  1. Now we need to find ***g***(***k***). This time, ***k*** is our input (the ***x-value***). We’re looking for ***g***(***k***), and ***k*** = ***5,*** so find the point on the graph where ***x*** = ***5***. This is in the picture on the right above.
     + - where ***x*** = ***5***, our ***g***(***x***), in this case ***g***(***5***) is ***y*** = ***2.5*** (or somewhere between 2 and 3). The only answer choice that’s between 2 and 3 is 2.5. ***g***(***k***) = ***2.5***.

B is the correct answer.

1. Inequalities and multiplied friends.
   1. Let’s write our givens: We have two sets:
      * + 0 ≤ ***x*** ≤ 8
        + -1 ≤ ***y*** ≤ 3
   2. What are we looking for? The ***set of possible values*** for ***xy***.
   3. We’ve got two ranges here. Start by multiplying the ends of each range by each other to get some possible values of in our set of ***xy***. Using ***x*** = ***8***, find ***xy*** when ***y*** = ***-1*** and when ***y*** = ***3***. (We’re not starting with ***x*** = ***0*** because when ***x*** = ***0***, it doesn’t matter what ***y*** is; ***xy*** is always going to be ***0***. And we already know ***0*** must be in our set of ***xy***.)
      * + ***x*** = ***8*** and ***y*** = ***-1***
        + ***xy***
        + (***8***)(***-1***)
        + ***-8 -8*** must be in our ***set***.
        + ***x*** = ***8*** and ***y*** = ***3***
        + ***xy***
        + (***8***)(***3***)
        + ***24*** ***24*** must be in our ***set***.
   4. We know ***-8***, ***0***, and ***24*** are in our set for ***xy***. It seems like ***-8*** ≤ ***xy*** ≤ ***24***, but check ***x*** = ***7*** to make sure.
      * + ***x*** = ***7*** and let ***y*** = ***-1***
        + ***xy***
        + (***7***)(***-1***)
        + ***-7 -7*** is in the ***set*** ***-8*** ≤ ***xy*** ≤ ***24***. ***-7*** is a possible value of ***xy***.
   5. Try a different value for ***y***. Let ***y*** = ***2***.
      * + ***x*** = ***7*** and ***y*** = ***2***
        + ***xy***
        + (***7***)(***2***)
        + ***14*** ***14*** is in the ***set*** ***-8*** ≤ ***xy*** ≤ ***24***. Our range is good.

E is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. What are we looking for? The ***sum, in terms of n,*** of the ***four marked angles***.
   2. To the picture!

***n°***

***(180 – n)°***

***(180 – n)°***

* 1. A straight line has 180°. ***n°*** is part of a straight line, so the ***angle*** inside the ***left*** triangle is ***(180 – n)°***. Mark that. The other ***purple angle*** marked is also ***(180 – n)°***, because the two ***purple angles*** are ***vertical angles***. Vertical angles are ***always*** equal. We’re getting closer.
  2. Now to the triangles. There are 180***°*** in the triangles, and the only angles we don’t know are the ***red angles***, which we are trying to find ***in terms of n***. Write an equation for the sum of the angles in ***one*** of the triangles.
     + - 180° = sum of the angles of a triangle
       - 180° = ***(180 – n)°*** + (***2 red angles***)
       - -(180 – n)° -(180 – n)°
       - 180° -(180 – n)° = ***2 red angles***
       - 180° - 180° + n° = ***2 red angles***
       - n° = ***2 red angles***
       - That’s the ***2 red angles in terms of n***. We have ***2*** triangles though, so we need to multiply this expression by ***2*** to get the measure of all ***4 red angles in terms of n***.
       - n° × ***2***= ***4 red angles***
       - ***2n*** = ***4 red angles***

B is the correct answer.

1. Sequence problem with some “math speak” to finish up.
   1. Let’s write our givens:
      * + After the first term, the next term is 3 greater than 1/3 of the previous term.
        + ***t*** is the first term of the sequence.
        + ***t*** ≠ 0
   2. What are we looking for? The ***ratio*** of the 1st term to the 2nd term.
   3. A ***ratio*** is a fraction. Ours is going to be:
      * + ***2nd term*** = ***2nd term***

***1st term*** = ***t*** .

* 1. First, let’s figure out our second term. Translate the “math speak.” Remember the order of operations.
     + - the ***next term*** ***is*** ***3 greater than*** 1/3 ***of*** the ***previous term***
       - ***2nd term*** ***=*** ***3 +***  1/3 ***×*** ***1st term***
       - ***2nd term*** ***=*** ***3 +***  1/3 ***×*** ***t***
       - ***2nd term*** ***=*** ***3 +***  (1/3)***t***
  2. We now have our ***1st term*** and our ***2nd term.*** Put them in a ratio and simplify.
     + - ***ratio*** = ***2nd term***

***t***

* + - * ***ratio*** = 3 +(1/3)***t*** .

***t***

* + - * We don’t like fractions within fractions. We can get rid of the 1/3 in the top by multiplying both terms in the numerator by 3. Remember that we also have to multiply the bottom by 3, because that is just like multiplying the whole ***ratio*** by 1 (which is 3/3) and that does not change the value of the ***ratio***.
      * ***ratio*** = { 3 +(1/3)***t*** } × 3 .

{ ***t*** } × 3

* + - * ***ratio*** = 9 +(3/3)***t*** .

3***t***

* + - * ***ratio*** = 9 +***t***

3***t***

C is the correct answer.

# Test 7 Section 3

1. Rate word problem.
   1. Let’s write our given:
      * + ***25*** ***pounds*** of flour are needed to make ***300 rolls***.
   2. What are we looking for? The number of ***pounds*** are needed to make ***12 rolls***.
   3. Set up the proportion. Let the ***pounds*** for ***12 rolls*** be ***x***.
      * + ***25*** ***pounds*** = ***x pounds*** Cross-multiply to find ***x***. We dropped the labels.

***300 rolls*** = ***12 rolls***

* + - * ***25*** × ***12*** = ***x*** × ***300*** Simplify and solve for ***x***.
      * ***300*** = ***300x*** Divide both sides by 300.
      * ***1*** = ***x***

A is the correct answer.

1. The friends arrive.
   1. Let’s write our given: ***xy*** = ***10***
   2. What are we looking for? The ***value*** of 2 × ***x***/***y*** × ***y***2.
   3. Simplify the expression.
      * + 2 × ***x*** × ***y***2 The ***y*** on the bottom and one ***y*** on the top cancel out. We’re left with one ***y*** on top.

***y***

* + - * 2 × ***xy*** Plug in the value of ***xy***.
      * 2 × ***10***
      * ***20***

E is the correct answer.

* 1. We could also plug in values for ***x*** and ***y*** and solve. Since ***xy*** = ***10***, say ***x*** = ***5*** and ***y*** = ***2***. Plug those into the expression.
     + - 2 × ***5*** × ***2***2 The left 2 and the ***2*** underneath go away.

***2***

* + - * ***5*** × ***2***2
      * ***5*** × ***4***
      * ***20*** Same answer.

1. Inequality Problem
   1. Let’s write our givens:
      * + ***x*** + ***y*** = 30
        + ***x*** > 8
   2. What are we looking for? The ***statement*** in the answer choices that must be ***true***.
   3. First solve the equation for ***y***. We know ***x*** > 8, but solve it for ***x*** = ***8***.
      * + ***x*** + ***y*** = 30
        + ***8*** + ***y*** = 30
        + ***8*** + ***y*** = 30 Subtract 8 from both sides.
        + ***y*** = ***22*** We’re not done yet.
   4. We know that ***x*** can’t be ***8***. It has to be greater than ***8***. If that’s true, then ***y*** can’t be ***22.*** Given that ***x*** + ***y*** = 30, ***y*** must be ***less than*** ***22***.
      * + ***y*** < ***22***.

B is the correct answer.

1. Points (not snakes) on a plane.
   1. Let’s write our givens:
      * + Point ***P*** is (***3***,***2***).
        + Point ***Q*** is (***7***,***2***).
        + Point ***R*** is (***7***,***4***).
        + The points are in the xy-plane (so it forms a flat 2-D triangle).
   2. What are we looking for? The ***perimeter*** of ∆***PQR***.
   3. ***Perimeter*** is the sum of the lengths of the sides. We need to find those lengths: ***PQ***, ***QR*** and ***PR***
   4. ***PQ***
      * + Pretty straight forward. The y-values are the same (2), so the entire length comes from the x-values. Subtract those and we will have the length of ***PQ***.
        + ***7*** – ***3*** = 4
        + ***PQ*** = ***4***
   5. ***QR***
      * + Similar to ***PQ***, but the x-values are the same (7), so the entire length comes from the y-values. Subtract those and we will have the length of ***QR***.
        + ***4*** – ***2*** = 2
        + ***QR*** = ***2***
   6. ***PR*** is a little trickier. We need to use the distance formula.
      * + The general distance formula is:
        + Distance = √[(***x2*** – ***x1***)2 + (***y2*** – ***y1***)2].
        + For this problem, replace the ***2*** with a ***R***, and the ***1*** with ***P*** as those are the names of the points.
        + ***PR*** = √[(***xR*** – ***xP***)2 + (***yR*** – ***yP***)2] Plug and Chug. Be careful to put in the right numbers.
        + ***PR*** = √[(***7***– ***3*** )2 + (***4***– ***2*** )2]
        + ***PR*** = √[ (4)2 + (2)2]
        + ***PR*** = √[ 16 + 4]
        + ***PR*** = ***√20***
   7. We now have all three sides. Add them up to find the ***perimeter***.
      * + ***perimeter*** = ***PQ*** + ***QR***+ ***PR***
        + ***perimeter*** = ***4*** + ***2*** + ***√20***
        + ***perimeter*** = ***6*** + ***√20***

C is the correct answer.

1. Sequence problem.
   1. Let’s write our givens:
      * + First 5 terms are ***8***, ***17***, ***26***, ***35***, ***44***.
        + Each term is found by adding 9 to the term immediately preceding it.
   2. What are we looking for? The ***term*** that is equal to 8 + (26 – 1)9.
   3. First, look at the sequence and find an algebraic function to represent it. Let the ***place value*** of the term be ***n***. So for the term (8), ***n*** = ***1***.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***Place value*** (or ***n***) | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
| ***Term*** | ***8*** | ***17*** | ***26*** | ***35*** | ***44*** |

* 1. We know ***9*** has to come in here somewhere; let’s look at the first term.
     + - When ***n*** = ***1***, the ***term*** is 8. What is 8 in terms of ***9***? It’s ***9*** – 1. But we need an ***n*** in our equation to make it work for all the ***terms***, not just the first one. 1 x 9 is 9, and ***1*** = ***n***, so our expression for the sequence could be 9***n*** – 1. Try that for ***n*** = ***2*** to see if our equation works.
       - 9***n*** – 1
       - 9(***2***)– 1
       - 18– 1 = ***17***. Our equation works. Now we have to find the ***term*** our expression is equal to.
  2. Simplify the expression that is equal to our given ***term***.
     + - 8 + (26 – 1)9
       - 8 + (25)9
       - 8 + 225
       - ***233***
  3. Set our equation for the sequence equal to 233 and solve for n.
     + - 9***n*** – 1 = ***233*** Add 1 to both sides.
       - 9***n*** = 234
       - 9***n*** = 234 Divide both sides by 9.
       - ***n*** = ***26*** The ***26th term*** in our sequence equals the expression given.

D is the correct answer.

1. Vertical angles. Mark up the picture.
   1. Let’s write our given: The lines all intersect at one point.
   2. What are we looking for? The ***pair of angles*** that, if known, would ***not*** enable us to find all the angles in the figure.
   3. To the picture!

***x°***

***r°***

***s°***

***z°***

***y°***

***t°***

* 1. 3 consecutive (right beside each other) angles in this picture form a straight line (180°) so if we know two of those consecutive angles, we can find the 3rd angle. Also, opposite angles (like t and z) are vertical angles, so they are equal. With that information, we can find all of the angles in the figure. Let’s check our answer choices.
     1. t and z (***marked in red above)***. We know those two angles are equal, but we can’t find any more of the angles because ***t and z*** aren’t part of the same line. (***A***) looks like our answer.
     2. t and y (***marked in blue above)***. If we know those, we can find x (3 consecutive angles form a straight line-180°), and each of the vertical angles, so we can find all the angles of the figure. (B) is out.
     3. s and x. For the same reasoning as (B), we can find all the angles. (C) is out.
     4. r and t. For the same reasoning as (B), we can find all the angles. (D) is out.
     5. r and s. For the same reasoning as (B), we can find all the angles. (E) is out.

A is the correct answer.

1. Word problem with a friend.
   1. Let’s write our given:
      * + The sum of two numbers that ***differ by 1*** is ***t***.
   2. What are we looking for? The ***value*** of the ***greater number*** in terms of ***t***.
   3. We need to name another friend. Let the ***greater number*** be ***x***. The numbers ***differ by 1***, so the lower number is going to be ***x –*** ***1***. The sum of those two is ***t***. Write that equation.
      * + ***x*** +(***x - 1***) = ***t*** Simplify.
        + 2***x - 1*** = ***t*** Solve for ***x***, which is our ***greater number***. Add 1 to both sides.
        + 2***x***  = ***t*** + 1Divide both sides by 2.
        + ***x***  = ***t*** + 1That’s the value of our ***greater number*** in terms of ***t***.

2

C is the correct answer.

1. Tough average problem.
   1. Let’s write our givens:
      * + 12 preschoolers with either 0, 1, 2 or 3 siblings.
        + When a new student is added, the average number of siblings equals the median number of siblings.
   2. What are we looking for? The ***number of siblings that*** the new student has.
   3. First we need to calculate the ***median*** and ***average*** of the current 12-student class to get a starting point. Look at the table and write out the total number of siblings in order from least to greatest. The ***median*** is the number exactly in the middle of the ordered list. In our case, the ***median*** is the average of the middle two terms because we have an ***even*** number of terms.
      * + 0, 0, 0, 1, 1, ***1***, ***1***, 1, 1, 2, 2, 3
        + ***1*** and ***1*** are the middle two terms. The average of ***1*** + ***1*** = ***1***. ***1*** is the ***median*** of our current class.
        + The ***average*** is the ***total number*** of siblings/the ***total number*** of students. Add a column to the table and also a total at the bottom of the number of students. In our table below, the third column, siblings per each type of student, is the total siblings for each group of students (0, 1, 2, or 3 siblings). To get that number, we multiply number of siblings × number of students.

|  |  |  |
| --- | --- | --- |
| number of siblings | number of students | siblings per each type of student |
| 0 | 3 | 0 |
| 1 | 6 | 6 |
| 2 | 2 | 4 |
| 3 | 1 | 3 |
| ***Total:*** | ***12*** | ***13*** |

* + - * Our current ***average*** = ***total number*** of siblings .

***total number*** of students

* + - * Our current ***average*** = ***13***

***12***

* + - * That’s a weird fraction, but what are we trying to do? Get the ***median*** and ***average*** to be the same. Our ***median*** = ***1***. If we could get our denominator in our average to be ***13*** instead of ***12***, we’d have a new ***average*** of ***13***/***13*** = ***1***. That would make the ***average*** = ***median***, which syncs with our given. So we need to increase the total number of students from 12 to 13 (remember, a new student joined the class). We do not need to increase the total number of siblings. Sounds like the new student has ***0*** siblings. The table below shows the class with the new student.

|  |  |  |
| --- | --- | --- |
| number of siblings | number of students | siblings per each type of student |
| 0 | ***4*** | 0 |
| 1 | 6 | 6 |
| 2 | 2 | 4 |
| 3 | 1 | 3 |
| ***Total:*** | ***13*** | ***13*** |

* + - * Our new ***average*** = ***13*** = ***1***

***13***

* + - * Now check the new ***median***. Line up the siblings counts again with the new student.
      * ***0***, 0, 0, 0, 1, 1, ***1***, 1, 1, 1, 2, 2, 3
      * Our new ***median*** is still ***1***, and our new ***average*** = ***1*** so the new student has ***0*** siblings.

A is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. Problem with one friend.
   1. Let’s write our given: 2(***x*** – 3) = 8
   2. What are we looking for? The ***value*** of (***x*** – 3)/(***x*** + 3).
   3. Solve for ***x*** in the given.
      * + 2(***x*** – 3) = 8
        + 2***x*** -6 = 8 Add 6 to both sides.
        + 2***x*** = 14 Divide both sides by 2.
        + ***x*** = ***7***
   4. Plugin ***7*** for ***x*** into the expression we are solving.
      * + (***x*** – 3)/(***x*** + 3)
        + (***7*** – 3)/(***7*** + 3)
        + 4/10 Reduce the fraction.
        + ***2/5***

***2/5*** is the correct answer.

1. “Math Speak” Problem
   1. Let’s write our given: When twice a ***number*** is decreased by 3, the result is 253.
   2. What are we looking for? The ***number***.
   3. Let the ***number*** be ***x***. Translate the “math speak.”
      * + When ***twice*** a ***number*** is ***decreased by 3***, the ***result is*** 253
        + ***2×*** ***x -3*** ***=*** 253
        + ***2x*** ***-3*** ***=*** 253 Simplify and solve for ***x***. Add 3 to both sides.
        + ***2x*** ***=*** 256 Divide both sides by 2.
        + ***x*** ***=*** ***128***

***128*** is the correct answer.

1. Data Analysis. Fill in the table.
   1. What are we looking for? The number of ***black sneakers*** that were produced in July.
   2. Fill in the table. We’re looking for ***total black sneakers*** in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | High-Tops | Low-Tops | ***Total*** |
| White | 3,600 |  |  |
| Black |  | 1,500 | ***total black sneakers*** |
| ***Total*** |  | ***5,500*** | ***10,000*** |

* + - * We need to take a couple steps to get to ***total black sneakers***. Look at the ***bottom*** Total row.
      * ***Total High-Tops*** + ***Total Low-Tops*** = ***Total***
      * ***Total High-Tops*** + ***5,500*** = ***10,000*** Subtract 5,500 from both sides.
      * ***Total High-Tops*** = ***4,500***

|  |  |  |  |
| --- | --- | --- | --- |
|  | High-Tops | Low-Tops | ***Total*** |
| White | 3,600 |  |  |
| Black |  | 1,500 | ***total black sneakers*** |
| ***Total*** | ***4,500*** | ***5,500*** | ***10,000*** |

* 1. Now find the number of black High-Tops.
     + - White High-Tops + Black High-Tops = ***Total High-Tops***
       - 3,600 + Black High-Tops = ***4,500*** Subtract 3,600 from both sides.

-3,600 -3,600

* + - * ***Black High-Tops*** = ***900***

|  |  |  |  |
| --- | --- | --- | --- |
|  | High-Tops | Low-Tops | ***Total*** |
| White | 3,600 |  |  |
| Black | ***900*** | 1,500 | ***total black sneakers*** |
| ***Total*** | ***4,500*** | ***5,500*** | ***10,000*** |

* 1. Now we can find the number of ***total black sneakers***.
     + - ***Black High-Tops*** + Black Low-Tops = ***total black sneakers***
       - ***900*** + 1,500 = ***total black sneakers***
       - ***2,400*** = ***total black sneakers***

***2,400*** is the correct answer.

1. A rectangle and a function. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + PQ***R***S is a rectangle with a perimeter of 10.
        + Points Q and ***R*** are on the graph of ***y*** = ***ax***2.
        + ***a*** is a constant.
   2. What are we looking for? The ***value*** of ***a***.
   3. We need a point on the graph to plug into our function. Let’s pick ***R***, so we are using positive numbers. If we find the coordinates of ***R***, we can plug those in and find out what ***a*** is. To the picture!

***P (-1,0)***

***y***

***R***

***Q***

***S (1,0)***

***x***

***2***

***2***

***(1,3)***

***P (-1,0)***

***y***

***R***

***Q***

***S (1,0)***

***x***

***P (-1,0)***

***y***

***R***

***Q***

***S (1,0)***

***x***

***2***

***2***

* 1. PQ***R***S is a rectangle. We need to find the length of side ***R***S, so we can get the y-value of point ***R***. We have to take a detour through side ***PS*** first.
     + - ***PS*** is a side of the rectangle, and we have the coordinates of both points. The ***y-values*** are ***zero***, so all the length comes from the ***x-values***. Subtract them to get the length of ***PS***.
       - ***PS*** = (***x-value*** of S) – (***x-value*** of P)
       - ***PS*** = 1 – (-1)
       - ***PS*** = ***2***
       - Since ***PS*** = ***2***, we know Q***R*** = 2. Mark that (middle picture above).
  2. PQ***R***S has a perimeter of 10. We have the value of 2 of the sides. QP and RS are equal. So we can plug in.
     + - Perimeter = QP + Q***R*** + ***R***S + PS Substitute ***R***S for QP since they are equal.
       - Perimeter = ***R***S + Q***R*** + ***R***S + PS Simplify.
       - 10 = 2***R***S + Q***R*** + PS Plug in 10 for perimeter and 2 for PS and Q***R***.
       - 10 = 2***R***S + 2 + 2
       - 10 = 2***R***S + 4 Solve for ***R***S. Subtract 4 from both sides.
       - 6 = 2***R***S
       - 6 = 2***R***S Divide both sides by 2.
       - ***3*** = ***R***S
       - Now we have the length of ***R***S, so let’s find the coordinates of ***R***.
  3. The coordinates of ***R*** are (***1***,***3***). The ***x-value*** is ***1*** because ***R*** is directly above S, so it has the same ***x-value*** as S. The ***y-value*** is ***3*** because it starts at the x-axis (0) and goes up ***3*** (the ***R***S length of the rectangle). Mark that (right picture above).
  4. Now to the function. Plug in the coordinates of ***R*** (***1***,***3***) to find ***a***.
     + - ***y*** = ***ax***2
       - ***3*** = ***a1***2
       - ***3*** = ***a***(***1***)
       - ***3*** = ***a***

***3*** is the correct answer.

1. Friends.
   1. Let’s write our givens:
      * + ***ab*** + ***b*** = ***a*** + 2***c***
        + ***a*** = ***2***
        + ***c*** = ***3***
   2. What are we looking for? The ***value*** of ***b***.
   3. Plug and chug.
      * + ***ab*** + ***b*** = ***a*** + 2***c***
        + ***2b*** + ***b*** = ***2*** + 2(***3***)
        + ***2b*** + ***b*** = ***2*** + 6
        + 3***b*** = ***2*** + 6
        + 3***b*** = 8 Divide both sides by 3.
        + ***b*** =  ***8/3***

***8/3*** the correct answer.

1. More parallel lines and vertical angles. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + ***m*** ∥ ***n***
        + ***l*** bisects ∠ABC
        + 45 < ***y*** < 55
   2. What are we looking for? A possible ***value*** of ***x***.
   3. Let’s make it easy on ourselves. Pick a value for ***y***. Let’s say ***y*** = ***50***.
   4. To the picture! Mark the parallel lines ***m*** and ***n***. ***l*** bisects ∠ABC, that means ***l*** cuts ∠ABC in half. We’ve put another point, ***D***, on line l so ∠***ABD*** = ∠***DBC***. Don’t forget to label ***y***° as ***50***°.

***A***

***C***

***B***

***m***

***n***

***y***°

***x***°

***l***

***x***°

***C***

***k***

***A***

***B***

***l***

***m***

***n***

***50***°

***x***°

***D***

***50°***

***C***

***k***

***A***

***B***

***m***

***n***

***50***°

***l***

***D***

***C***

***B***

***n***

***x***°

***25***°

***m***

***k***

* 1. Now let’s use some parallel line rules. Forget about line ***l*** for a second; just look at line ***k*** and our parallel lines ***m*** and ***n***. With line ***l*** as the transveral ∠ABC and ***50***° are ***corresponding angles***, so ∠ABC = ***50***°. (Marked in left picture below.)
  2. Now let’s focus on line ***l*** since that will lead us to ***x***. ∠***DBC*** and ***x***° are ***alternate exterior angles***. That means ∠***DBC*** = ***x***°. If ∠ABC = ***50***°, and ∠***DBC*** is ½ of ∠ABC, ∠***DBC*** = ***25***°. (Marked in right picture above.)
     + - ∠***DBC*** = ***25***°
       - ∠***DBC*** = ***x***°
       - ***25*** = ***x***

***25*** is a correct answer. Don’t put the ° sign in the answer.

1. Combination problem.
   1. Let’s write our givens:
      * + 3 plumbers are sent to a job.
        + Company has 4 experienced plumbers and 4 trainees.
        + A team consists of 1 experienced plumber and 2 trainees.
   2. What are we looking for? The ***number*** of ***combination of teams*** that can be formed.
   3. Let’s name our plumbers. Let the 4 experienced plumbers be A, B, C and D. Let the trainees be 1, 2, 3 and 4. If we find out the number of team combinations for 1 experienced plumber, we can multiply that by 4 to get the total team combinations. Make a table and write out the teams.

|  |  |  |
| --- | --- | --- |
| Experienced Plumber | Trainee 1 | Trainee 2 |
| A | 1 | 2 |
| A | 1 | 3 |
| A | 1 | 4 |
| A | 2 | 3 |
| A | 2 | 4 |
| A | 3 | 4 |

* + - * There are 6 teams possible with experienced plumber A and two trainees. Multiply by 4 to get the total team combinations for all 4 experienced plumbers.
      * 6 x 4 = ***24*** total team combinations.

***24*** is the correct answer.

1. Circle in a Circle. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Shaded ***area*** is ***64π***.
        + Little circle has a ***radius*** of ***6***.
   2. What are we looking for? The ***radius*** (***r***) of the ***larger*** circle.
   3. To the picture! Mark the ***radius*** of the smaller circle (***6***) and draw in the ***radius*** of the larger circle (***r***). We can find this ***radius*** through the area formula. Area of a circle = πr2. We have two circles here.

* The area of the shaded circle equals the area of big circle minus the ***area*** of the little circle.

***6***

***r***

* 1. Find the area of the little circle.
     + - ***Area*** = π***r***2
       - ***Area*** = π(***6***)2
       - ***Area*** = ***36π***
  2. Back to the equation. Translate “math speak.”
     + - Area of shaded circle = area of big circle - ***area*** of little circle.
       - ***64π*** = π***r***2 - ***36π*** Add 36π to both sides.
       - 100π = π***r***2 0 The π’s go away.
       - 100 = ***r***2
       - √100 = √(***r***2)
       - ***10*** = ***r***

***10*** is the correct answer.

1. Prime number problem.
   1. Let’s write our givens:
      * + ***p***, ***r*** and ***s*** are different prime numbers ***greater than 2***.
        + ***n*** = ***p*** × ***r*** × ***s***
   2. What are we looking for? The ***number*** of ***positive factors*** of ***n***, including ***n*** and ***1***.
   3. A prime number is only divisible by itself and 1. Pick 3 primes greater than 2. How about ***3***, ***5*** and ***7?*** (4 and 6 are out because they have factors besides themselves and 1.)
      * + ***p*** = ***3***
        + ***r*** = ***5***
        + ***s*** = ***7***
   4. Since we know that ***n*** = ***p*** × ***r*** × ***s*** , then ***n*** = 105, and we have 5 factors: ***3***, ***5***, ***7***, 1, and n (***105***).
   5. If we find different products of 2 numbers using ***p***, ***r*** and ***s***, we’ll get more factors.
      * + ***pr*** = ***3*** × ***5***
        + ***pr*** = ***15*** That’s 6 factors. Keep going.
        + ***ps*** = ***3*** × ***7***
        + ***ps*** = ***21*** That’s 7 factors. There’s another one.
        + ***rs*** = ***5*** × ***7***
        + ***rs*** = ***35*** That’s ***8*** factors. We’ve captured them all.

***8*** is the correct answer.

1. A tough function problem to end the section.
   1. Let’s write our givens:
      * + ***h***(***t***) = ***c*** – (***d*** – 4***t***)2
        + ***Height*** is ***h***(***t***) .
        + ***Time*** is ***t***.
        + ***c*** and ***d*** are ***positive constants***.
        + At ***t*** = ***0***, the ***height*** is ***6*** feet.
        + The maximum ***height*** is ***106*** feet which is reached at ***t*** = ***2.5***.
   2. What are we looking for? The ***height*** when ***t*** = ***1***.
   3. There’s a lot going on here. We need to find the values of ***c*** and ***d***. Start with ***t*** = ***0*** when the ***height*** is ***6***.
      * + ***h***(***t***) = ***c*** - (***d*** – 4***t*** )2
        + ***6*** = ***c*** - (***d*** – 4{***0***})2
        + ***6*** = ***c*** - ***d***2 Get ***c*** by itself. Add ***d***2 to both sides.
        + ***6*** + ***d***2= ***c***
        + ***c*** = ***6*** + ***d***2
   4. Plug that back into the original function.
      * + ***h***(***t***) = ***c*** - (***d*** – 4***t*** )2
        + ***h***(***t***) =(***6*** + ***d***2) - (***d*** – 4***t*** )2
   5. Now we can use the maximum ***height*** of ***106*** feet at ***t*** = ***2.5*** to solve for ***d***. This is ugly.
      * + ***h***(***t***) =(***6*** + ***d***2) - (***d*** – 4***t*** )2
        + ***106*** =(***6*** + ***d***2) - (***d*** – 4{***2.5***})2
        + ***106*** =(***6*** + ***d***2) - (***d*** – 10)2 FOIL the (***d***-10) 2 expression.
        + ***106*** =(***6*** + ***d***2) - (***d***2 -10***d*** – 10***d*** + 100)
        + ***106*** =(***6*** + ***d***2) - (***d***2 - 20***d*** + 100) Distribute the negative sign.
        + ***106*** = ***6*** + ***d***2 - ***d***2 + 20***d*** - 100 Combine like terms. d2 and -d2 cancel out. Nice.
        + ***106*** = 20***d*** - 94 Add 94 to both sides.
        + 200 = 20***d*** Divide both sides by 20.
        + ***10*** = ***d***
   6. Plug ***d*** = ***10*** into the equation for ***c***.
      * + ***c*** = ***6*** + ***d***2
        + ***c*** = ***6*** + (***10***)2
        + ***c*** = ***6*** + ***100***
        + ***c*** = ***106***
   7. Rewrite the original function with our values of ***c*** and ***d***.
      * + ***h***(***t***) = ***c*** – ( ***d*** – 4***t***)2
        + ***h***(***t***) = ***106*** – (***10*** – 4***t***)2
   8. We now have only one friend in our function, ***t***. What are we looking for again? The ***height*** when ***t*** = ***1***.
      * + ***h***(***t***) = ***106*** – (***10*** – 4***t*** )2 Plug in ***1*** for ***t*** in our rewritten function.
        + ***h***(***1***) = ***106*** – (***10*** – 4{***1***})2
        + ***h***(***1***) = ***106*** – (***10*** – 4 )2
        + ***h***(***1***) = ***106*** – (6)2
        + ***h***(***1***) = ***106*** – 36
        + ***h***(***1***) = 70

***70*** is the correct answer.

# Test 7 Section 7

1. Inequality with a friend.
   1. Let’s write our givens:
      * + ***k*** is a positive integer divisible by 3.
        + ***k*** < 60
   2. What are we looking for? The ***highest*** possible ***value of k***.
   3. Since ***k*** < 60, start with 59 and count backwards to find a number divisible by 3.
      * + 59: Not divisible by 3; there’s a remainder of 2.
        + 58: Not divisible by 3; there’s a remainder of 1.
        + 57: There we go. 57/3 = 19 ***57*** is our ***highest value of k***.

C is the correct answer.

1. Symmetry Problem
   1. Let’s write our given: ***H*** is symmetric with respect to two lines.
   2. What are we looking for? The ***letter*** that is also symmetric with respect to two lines.
   3. Draw the dotted vertical and horizontal lines on all of the answer choices, and check to see if:
      * + the sides divided by the horizontal line are the same, AND
        + the sides divided by the vertical lines are the same.
   4. ***X*** is the only possible answer here. The top of the ***X*** folds down over the bottom (symmetrical) and the left side of the ***X*** folds onto the right side of the ***X*** (symmetrical).

D is the correct answer.

1. Word Problem
   1. Let’s write our givens:
      * + Bobby receives $2 for each chore.
        + ***n*** = # of ***chores*** done
        + Bobby also receives a weekly allowance of $10.
   2. What are we looking for? The ***expression*** that describes Bobby’s ***weekly pay***.
   3. We’ve got two parts of Bobby’s pay, chores and allowance. We need to find them and add them. Translate the “math speak.”
      * + Bobby receives $***2*** for each ***chore***.

***2*** × ***n***

***2n***

* + - * But he also receives a weekly allowance of **$10** (even if no chores are done).
  1. Add those two expressions together.
     + - Allowance + Chores = ***weekly pay***
       - ***10*** + ***2n*** = ***weekly pay***

D is the correct answer.

1. Box Problem
   1. Let’s write our givens:
      * + The small squares in A and B all have the same areas.
        + ***Area*** of ***Figure*** ***A*** = ***26***
   2. What are we looking for? The ***area*** of Figure ***B***.
   3. First count the smaller squares in A and B.
      * + There are 13 squares in ***A***.
        + There are 8 squares in ***B***.
   4. The small squares are all equal, and we know the area of A, so we can find the area of one small square.
      * + Area/(# of small squares) = area of one small square.
        + (***Area*** of ***A***)/13 = area of small square
        + ***26***/13 = area of small square
        + ***2*** = area of small square
   5. Rearrange the above equation to find the area of ***B***.
      * + Area/(# of small squares) = area of small square
        + Area ***B***/(# of small squares) = area of small square
        + Area ***B*** =(# of small squares) × (area of small square)
        + Area ***B*** = 8 × ***2***
        + Area ***B*** = ***16***

C is the correct answer.

1. Data Analysis Problem
   1. What are we looking for? The ***salesperson*** with the ***greatest increase*** in units sold from 1998 to 1999.
   2. We need an *increase* here. (A) Albert and (D) Smith are out as their sales went down from 1998 to 1999.
   3. We’re left with (B) Goldberg, (C) Patel and (E) Wang. Goldberg’s bars have the biggest difference, so

(***B***) ***Goldberg*** had the biggest increase.

B is the correct answer.

1. Average problem with a friend.
   1. Let’s write our given: The average of 6, 6, 12, 16 and ***x*** is ***x***.
   2. What are we looking for? The ***value*** of ***x***.
   3. Set up the equation for an average.
      * + Sum of the Terms = Average

Number of Terms

* + - * 6 + 6 + 12 +16 + ***x*** = ***x***

5

* + - * 40+ ***x*** = ***x*** Multiply both sides by 5.

5

* + - * 40+ ***x*** = 5***x*** Combine like terms. Subtract ***x*** from both sides.
      * 40 = 4***x*** Divide both sides by 4.
      * ***10*** = ***x***

D is the correct answer.

1. Triangle problem. MARK UP THE PICTURE!
   1. What are we looking for? The ***value*** of ***z***.
   2. To the picture. If we find a value of ***y,*** we can use it and the right angle (90°) to find out ***z***. Start with the triangle on the right. We know 2 of those angles, the right angle (90°) and 40°. We can find 2***y*** because the angles of a triangle add up to 180°.

***y°***

***40°***

***z°***

***2y°***

***2y°***

***50°***

***25°***

***90°***

***90°***

***z°***

***40°***

* + - * 40° + 90° + 2***y*** = 180°
      * 130° + 2***y*** = 180° Subtract 130° from both sides.
      * 2***y*** = 50° Divide both sides by 2.
      * ***y*** = ***25***°
  1. Now we move to the triangle on the left. The bottom right angle is a right angle (90°) because a straight line has 180°. Write the equation for the angles of the left triangle.
     + - ***z***° + ***25***° + 90° = 180°
       - ***z***° + 115° = 180° Subtract 115° from both sides.
       - ***z***° = ***65***°

C is the correct answer.

1. Integer Word Problem
   1. Let’s write our givens:
      * + Computer randomly selects a positive 2-digit number.
        + If integer selected is odd, ***twice*** that integer is printed.
        + If the integer selected is even, that integer is printed.
   2. What are we looking for? The ***possible integer selected*** if the number printed is ***26***.
   3. Figure out which answer choices follow the rules and give us what we’re looking for. For example, say 1 is selected, then 2 would be printed. If 2 is selected, then 2 would be printed.

13 13 is odd, so ***twice*** the number (26) would be printed. ***I*** works.

26 26 is even, so the number (26) would be printed. ***II*** works.

52 52 is even, so the number (52) would be printed. ***III*** does not work.

C is the correct answer.

1. Friends and time.
   1. Let’s write our givens:
      * + ***m*** minutes
        + ***s*** seconds
   2. What are we looking for? The ***expression*** that shows how many seconds there are in ***m*** minutes and ***s*** seconds.
   3. First, convert the minutes to seconds. For every minute, there are 60 seconds, so multiply the number of minutes by 60 to get the number of seconds. 1 minute = 60 (1) = 60 seconds. The expression for number of seconds in ***m*** minutes is:
      * + ***s*** = 60***m***
   4. We have two parts of the expression, the minutes and the seconds, or minutes + seconds.
      * + ***seconds*** + ***minutes***
        + ***s*** + 60***m***

A is the correct answer.

1. Factoring Problem
   1. Let’s write our given: (2x – 2)(2 – x) = 0
   2. What are we looking for? All the ***possible values*** of ***x***.
   3. Anything multiplied by 0 is zero, so set both parentheses equal to 0 and solve for ***x*** in both of them.
      * + (2***x*** – 2) = 0
        + 2***x*** – 2 = 0
        + 2***x*** = 2
        + ***x*** = ***1*** Now set the other expression to zero.
        + (2 – ***x***) = 0
        + 2 – ***x*** = 0
        + - ***x*** = -2
        + ***x*** = ***2***

D is the correct answer.

1. Exponents Problem
   1. Let’s write our given: ***x***3 = ***y***9
   2. What are we looking for? ***x*** in terms of ***y***.
   3. First we need to get ***x*** by itself. Take the 3rd root.
      * + = (The 3 and 9 under the radicals are exponents.) That can be written as:
        + ***x***3(1/3)= ***y***9(1/3)
        + ***x***3/3 = ***y***9/3
        + ***x***1 = ***y***3
        + ***x***= ***y***3

C is the correct answer.

1. Slope Problem
   1. What are we looking for? The ***line segment*** that has a ***slope*** of ***-1***.
   2. Slope is rise/run, the change in the y-values over the change in the x-values. We know our slope is negative, which means it slopes down from left to right. Let’s check the answer choices, plugging in the point coordinates.
      1. OA. The points are (0,0) and (1, -3).
         * Slope = rise = (-3 – 0) = ***-3*** That’s not ***-1***. (A) is out.

run (1 – 0)

* + 1. OB. The points are (0,0) and (3, -1).
       - Slope = rise = (-1 – 0) = ***-1/3*** That’s not ***-1***. (B) is out.

run (3 – 0)

* + 1. OC. The points are (0,0) and (3, 1).
       - Slope = rise = (1 – 0) = ***1/3*** That’s not ***-1***. (C) is out.

run (3 – 0)

* + 1. OD. The points are (0,0) and (1, 3).
       - Slope = rise = (3 – 0) = ***3*** That’s not ***-1***. (D) is out.

run (1 – 0)

* + 1. DC. The points are (1,3) and (3, 1).
       - Slope = rise = (1 – 3) = ***-2 = -1*** That’s our answer.

run (3 – 1) ***2***

E is the correct answer.

1. Integer Problem
   1. Let’s write our givens:
      * + Locker combo is 3 2-digit numbers.
        + One number is odd.
        + One number is a multiple of 5.
        + One number is the day of the month of Kyle’s birthday.
        + Each number satisfies ***only one*** of the conditions.
   2. What are we looking for? A ***possible combo*** for Kyle’s lock.
   3. To the answers. We know that a birthday date has to be 31 or less, because no month has more than 31 days.
      1. 14 - 20 - 13
         * 14 : birthday
         * 20: multiple of 5
         * 13: odd
      2. 14 - 25 - 13 25 and 13 are **both** odd. (B) is out.
      3. 15 - 18 - 16 15 is odd ***and*** a multiple of 5. (C) is out.
      4. 20 - 15 - 20 Two 20’s are **both** multiples of 5. (D) is out.
      5. 34 - 30 - 21
         * 21 : odd
         * 30: multiple of 5
         * 34: That can’t be a birthday. (E) is out.

A is the correct answer.

1. Friends and roots.
   1. Let’s write our given: = x - 3
   2. What are we looking for? For x > 3, the ***equation*** that is equal to the given.
   3. First, is there any way (A) could be correct? No. x can’t equal x2 unless x = 0 or 1. That does not work, given that x > 3.
   4. Manipulate the given:
      * + = x – 3 Square both sides.
        + ()2 = (x – 3)2
        + x + 9 = (x – 3)2 FOIL the right side of the equation.
        + x + 9 = x2 – 6x + 9 The 9’s go away.
        + x = x2 – 6x That looks like (C).

C is the correct answer.

1. Integers and squares.
   1. Let’s write our given: Set of integers from 1 to 100, *inclusive*.
   2. What are we looking for? The ***number of integers*** in our setthat are ***not*** the square of an integer.
   3. First, how many numbers are there between 1 and 100, inclusive? 100. So we are dealing with 100 total numbers. Instead of trying every integer between 1 and 100 to see if it’s not a square, let’s try to find out how many integers ***are*** perfect squares.
   4. A perfect square is easy to define. 12 = 1 and 102 = 100. So right there, we have 2 of our integers in the set that are the squares of another integer. Look at 1. That’s the lowest possible integer in our set, and the square root of that is 1. Look at 100. That’s the highest possible integer in our set, and the square root of that is 10. That means when we square all the numbers between 1 and 10, the resulting integer will be in our set. How many of those numbers are there? 10.
   5. Subtract that 10 from our total integers in the set; we’ll have the number of integers that are not squares.
      * + 100 – 10 = ***90***.

E is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. What are we looking for? How much ***shorter***, in miles, the direct trip from A to D is than the original trip.
   2. To the picture. First, let’s find the distance from A to D as drawn through points B and C: 16 + 15 + 4 = 35

***D***

***C***

***B***

***A***

***15***

***4***

***16***

***D***

***C***

***B***

***A***

***15***

***4***

***4***

***15***

***E***

***16***

* + - * Now we need to to know the straight shot length of AD. Look at the picture above on the right. If we find the length of AD, the ***red dashed*** side, we can find the difference, or how much shorter the direct route is than the original route.
      * We added in the black dashed lines to make rectangle BCDE. We know that opposite sides of rectangles are equal, so CB = DE = 15. Also, CD = BE = 4. Mark those.
      * Our direct route, AD, is the hypotenuse of the right triangle ADE. We know two sides, so we can find the third using the Pythagorean Theorem.
        + AE = AB + BE

AE = 16 + 4

AE = 20

* + - * + DE = 15
      * a2 + b2 = c2
      * DE2 + AE2 = AD2
      * 152 + 202 = AD2
      * 225 + 400 = AD2
      * 625 = AD2 Take the square root of both sides.
      * ***25*** = AD
  1. Now that we know our two distances, just subtract.
     + - Original Route - AD
       - 35 - ***25*** = ***10***

C is the correct answer.

1. Circles Problem
   1. Let’s write our givens:
      * + Radius of one ***circle*** is ***1/2***.
        + Radius of another ***circle*** is ***1***.
   2. What are we looking for? The ***ratio*** of the area of the ***larger circle*** to the ***smaller circle***.
   3. First, the formula for area of a circle is A = πr2. We’ll use that for our areas.
   4. Write out the ratio and fill it in.
      * + Area of ***larger circle***

Area of ***smaller circle***

* + - * π***r***2

π***r***2

* + - * π***1***2

π(***1/2***)2

* + - * π . π’s go away.

π(1/4)

* + - * 1 . Take the reciprocal to get the ¼ out of the denominator.

(1/4)

* + - * ***4*** (Same as ***4***: ***1.***)

***1***

D is the correct answer.

1. Integers.
   1. Let’s write our given: Sum of consecutive integers from -22 to ***x***, inclusive, is 72.
   2. What are we looking for? The value of ***x***.
   3. Start with a number line. We know we have integers, so we’re dealing with nice round numbers.

***-22***

***22***

***0***

***x***

* + - * First, look at all of the numbers between (and including) -22 and 22. They add up to 0 because every pair of a positive and negative number [(-1,1), (-22,22), etc.] add up to zero. We know our sum has to be 72, so we need to add numbers greater than, or to the right of, 22. Start with 23.
      * 23 doesn’t equal 72.
      * 23 + 24 = 47. Keep going. Add 25.
      * 23 + 24 + ***25*** = 72. There it is.
  1. We’re looking for ***x*** in the sequence from -22 to ***x*** that adds up to 72. ***25*** is our last number, so ***x*** = ***25***.

B is the correct answer.

1. Exponents and friends.
   1. Let’s write our givens:
      * + ***k***, ***n***, ***x*** and ***y*** are positive numbers.
        + ***x*** -4/3 = ***k***-2
        + ***y***4/3 = ***n***2
   2. What are we looking for? (***xy***)-2/3 ***in terms of*** ***n*** and ***k***.
   3. We don’t have an ***x*** or ***y*** in our answer choices, so we need to get rid of those. Start with the ***x***. We need to know our exponent rules cold.
      * + ***x*** -4/3 = ***k***-2 Raise both sides by (-3/4) to get the x exponent to be 1.
        + (***x*** -4/3)-3/4 = (***k***-2)-3/4 When raising exponents by exponents, we multiply.
        + ***x*** 12/12 = ***k***6/4 = ***k***3/2 Now we have a ***k***, expression to plug into ***x***.
        + ***x*** = ***k***3/2
   4. Now do the same with the ***y*** given.
      * + ***y***4/3 = ***n***2
        + (***y***4/3)3/4 = (***n***2)3/4
        + ***y*** = ***n***3/2
   5. Plug in the ***n*** and ***k*** expressions for ***x*** and ***y*** in the (***xy***)-2/3 expression.
      * + (***xy***)-2/3
        + (***k***3/2 ***n***3/2)-2/3 Distribute the -2/3 exponent.
        + (***k***3/2)-2/3  ×(***n***3/2)-2/3
        + ***k***-1  × ***n***-1 Get the exponents in positive form.
        + 1 ×1 Multiply across.

***k n***

* + - * ***1*** . Same as ***1/nk***.

***kn***

A is the correct answer.

1. Shape-Shifting Function Problem
   1. Let’s write our givens:
      * + f(x) = x3 – 4x
        + g(x) = f(x +***h***) + ***k***
   2. What are we looking for? The ***value*** of ***hk***.
   3. We could figure this out by plugging in the function f(x) into g(x), but that would get really ugly, really quickly. Instead, let’s go back to our geometry, where we learned about difficult polynomial functions and how they shift. In most cases, they were in the form f(x) = (x-***h***)3 + ***k***, or some derivation. *Let’s call this form our generic equation.* The ***h***’s and the ***k***’s in these types of problems give us the amount of shift from an original function to a new function.
   4. We have two functions here. Let’s pick a point in the first one, f(x), and find its corresponding point in g(x).
      * + Keep it simple. Pick (1,-3) in f(x). It’s easy to spot as it’s the dip in the graph.
        + Now find the corresponding point (dip) in g(x). That’s (4, -5). So, the original point moved ***down 2*** ( -5 – (-3)) and to the ***right 3*** ( 4 – 1). Since it moved down 2, that means -2.
   5. The ***h*** tells us how far left or right the graph moves, and the ***k*** tells us how far up and down it moves.
      * + We said our graph moved to the ***right 3.*** But look at the ***h*** in our g(x). It’s f(x + ***h***) + ***k***. The ***h*** is ***added, not subtracted*** like it would be in the generic equation referenced in (iii). If this were a generic problem, and ***h*** were 3, it would be (x - 3). But our equation is *(****x + h****),* so ***h*** in our problem here is ***-3***. On to ***k***.
        + ***k*** is easier, as the formula in this problem is the same as in our generic equation. That is, it’s + ***k*** at the end of the function. The graph moves down 2, so ***k*** = ***-2***.
        + We have ***h*** and ***k***, so multiply them out and we’re done.
        + ***hk*** = (***-3***)(***-2***) = ***6***

E is the correct answer.

# Test 7 Section 9

1. Probability to start.
   1. Let’s write our given: 6 out of 10 cars are red.
   2. What are we looking for? The ***probability*** that a car selected at random will be red.
   3. Probability = part/whole. In this case that means:
      * + Red cars/total cars
        + 6/10 Now reduce.
        + ***3/5***

B is the correct answer.

1. Triangles. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + AB = BC
        + BD bisects AC
   2. What are we looking for? The statement in the answer choicesthat ***cannot*** be concluded.
   3. To the picture. Mark the givens.

***A***

***B***

***D***

***x°***

***w°***

***y°***

***z°***

***C***

* 1. Since BD is in both small triangles, and the other two sides are corresponding (AB and BC, AD and CD), we know the triangles are equilateral. That means they are equiangular. Start eliminating answers.

1. w = x There’s a case where this could be true, where w = 45 and x = 45, but that’s not the only triangle that could be formed. We could have w = 60 and x = 30. In that case, w ≠ x, yet all of the rules would still hold. This looks like our answer, but check the others.
2. w = z This must be true. They are corresponding angles of equilateral triangles. (B) is out.

(C) x = y Same reasoning as (B). (C) is out.

(D) AD = DC This is the definition of “bisect”, as in the given BD bisects AC. (D) is out.

(E) BD ⊥ AC BD has to be a perpendicular bisector of AC, because AB = BC. (If AB ≠ BC, BD could still bisect AC, it just wouldn’t necessarily form a 90° angle at ∠ADB.) (E) is out.

A is the correct answer.

1. Friendly percentage.
   1. Let’s write our given: 30% of ***m*** is 40.
   2. What are we looking for? The ***value*** of 15% of ***m***.
   3. Translate the “math speak.”
      * + 30% of ***m*** is 40
        + 30% × ***m*** = 40
        + .3***m*** = 40 Divide both sides by .3.
        + ***m*** = ***133.33***
        + Now find 15% of ***m***.
        + .15 × ***m***
        + .15 × ***133.33***
        + ***20***
   4. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfA faster way to solve this is to notice that 15% is ½ of 30%. We know 30% of ***m*** = 40, so if we take half of 40, we will have 15% of ***m***. 40/2 = 20. Same answer, much faster.

B is the correct answer.

1. Negatives and positives.
   1. Let’s write our given: n is any negative number.
   2. What are we looking for? The ***answer*** that must be ***positive***.
   3. To the answers:

(A) n/2 Dividing a negative number doesn’t change the sign of a number. If n = -2, 1/-2 is still negative. (A) is out.

(B) 2n Multiplying a negative number by 2 doesn’t make it positive. 2 x -1 is -2. (B) is out.

(C) n + 2 This is a little more interesting, because if n is between o and -2, then n + 2 will be positive. But we’re looking for the case where n + 2 is ***always*** positive. If n = -5, the result is still negative. (C) is out.

(D) n - 2 Think of a number line. If we start negative and subtract, then we go more negative. That’s not positive. (D) is out.

(E) 2 – n Besides being the only one left, (***E***) works. When we subtract a negative number, it’s just like adding. In all cases of negative n’s, 2 – n has to be positive.

E is the correct answer.

1. Ratios Problem
   1. What are we looking for? The ***ratio*** that is equal to 1.2 to 1.
   2. Rewrite the given as a fraction. 1.2/1. That just equals 1.2.
   3. Look at the answers. Take the ratios, turn them into fractions and divide. See which one gives us 1.2.

(A) 1 to 2 = ½ = .5 That is not 1.2. (A) is out.

(B) 12 to 1 = 12/1 = 12 (B) is out.

(C) 5 to 6 = 5/6 = .83 (C) is out.

(D) ***6 to 5*** = 6/5 = 1.2 (D) works.

(E) 6 to 50 = 6/50 = .12 (E) is out.

D is the correct answer.

1. Data Analysis Problem
   1. Let’s write our given: Each house symbol = 5 million new homes.
   2. What are we looking for? ***The number of new homes*** that are represented by 3 and a half house symbols.
   3. Multiply the number of houses by 5 million.
      * + (3.5 house symbols) × (5 million)
        + ***17.5 million new homes***

D is the correct answer.

1. Friends and squares.
   1. Let’s write our givens:
      * + ***a*** and b are positive integers.
        + ***a***2 - ***b***2 = 7
   2. What are we looking for? The ***value*** of ***a***.
   3. We know we have two squares, and they are probably going to be pretty close to each other. If we rearrange the equation to get ***b***2 by itself, we can plug in for ***a***, subtract 7, and see if the result is a perfect square. ***a*** and ***b*** must be integers, so ***b***2 has to be a perfect square. If it’s not, we have a bad ***a***.
      * + ***a***2 - 7 = ***b***2
   4. Let’s start going down the answers and plugging in a value for ***a***.

(A) ***a*** = ***3***

* + - * ***3***2 - 7 = ***b***2
      * 9 - 7 = ***b***2
      * 2 = ***b***2 That doesn’t work. 2 is not a perfect square. (A) is out.

(B) ***a*** = ***4***

* ***4***2 – 7 = ***b***2
* 16 - 7 = ***b***2
  + - * 9 = ***b***2 That works. ***b*** = 3. (B) looks good. Check the others.

(C) ***a*** = ***5***

* + - * ***5***2 - 7 = ***b***2
      * 25 - 7 = ***b***2
      * 18 = ***b***2 18 is not a perfect square. (C) is out.

(D) ***a*** = ***6***

* + - * ***6***2 – 7 = ***b***2
      * 36 - 7 = ***b***2
      * 29 = ***b***2 29 is not a perfect square. (D) is out.

(E) ***a*** = ***7***

* + - * ***7***2 – 7 = ***b***2
      * 49 - 7 = ***b***2
      * 42 = ***b***2 42 is not a perfect square. (E) is out.

B is the correct answer.

1. Number Line Problem
   1. What are we looking for? The ***answer*** that corresponds to |***u*** – ***w***|.
   2. We need to give those letters some values. Label them on the number line. It looks like ***w*** is halfway between o and -1, so let ***w*** = ***-½***.

***u***

***v***

***0***

***-1***

***w***

***-1/2***

* 1. There are 3 equal spaces between -1/2 and -1, so to find out ***u*** we need to find the value of 2 spaces. One space is:
  2. (-1/2) /3 = -1/6. Each of those small spaces = 1/6.
     + - ***u*** = -1/2 – 2 small spaces (Subtract because we’re in negative territory.)
       - ***u*** = -1/2 – 2(1/6)
       - ***u*** = -3/6 – 2/6
       - ***u*** = ***-5/6***
  3. Now we have a value for ***u*** and ***w***. Find |***u*** – ***w***|.
     + - |***u*** – ***w***|
       - |***-5/6*** – ***-½***|
       - |-5/6 + ½|
       - |-5/6 + 3/6|
       - |-2/6|
       - |-1/3| = 1/3
  4. Find the point that corresponds to 1/3 on the number line. There’s only one, and it’s ***x***.

C is the correct answer.

1. “Math Speak” Problem
   1. Let’s write our givens:
      1. ***n*** is increased by 5.
      2. That result is multiplied by 5.
      3. That result is decreased by 5.
      4. That result is divided by 5.
   2. What are we looking for? The ***final result*** in terms of ***n***.
   3. Follow the steps.
      1. ***n*** is increased by 5.
         * ***n*** + 5
      2. That is multiplied by 5.
         * 5(***n*** + 5)
      3. That is decreased by 5.
         * 5(***n*** + 5) – 5
      4. That is divided by 5.
         * 5(***n*** + 5) – 5 Simplify this thing.

5

* + - * 5***n*** + 25 – 5

5

* + - * 5***n*** + 20 = 5***n*** + 20 Simplify.

5 5 5

* + - * ***n + 4***

D is the correct answer.

1. Word problem.
   1. Let’s write our givens:
      * + 4 pieces of tape per poster
        + Each piece of tape is 6 inches long.
        + Roll of tape is 300 feet.
        + No tape is wasted.
        + ***n*** posters are hung.
   2. What are we looking for? The answer that represents the number of ***feet*** **left on the roll** after ***n*** posters are hung
   3. First, get from inches to feet per poster.
      * + 4 pieces x 6 inches = 24 inches of tape per poster
        + 24 inches = 2 feet
        + So, each poster requires 2 feet of tape to be hung.
        + We have ***n*** posters and 2 feet of tape per poster; that expression can be written as 2***n***.
        + We start with 300 feet of tape, and we have to subtract 2***n***.
        + ***300 - 2n***.

B is the correct answer.

1. Coordinate plane word problem. Draw a picture.
   1. Let’s write our givens:
      * + Line m is the reflection of line ***l*** about the x-axis.
        + Slope of line ***m*** is - 4/5.
   2. What are we looking for? The ***slope*** of line ***l***.
   3. Slope is rise over run, so line ***m*** goes down 4 and to the right 5. Let’s draw a line by picking two easy points.
      * + We don’t know where line ***m*** is, so pick the first point as (0,0).
        + Now go down 4 and to the right 5 to point (5,-4). Now we can draw line ***m***.
   4. To the picture. We have 2 points of line ***m***. ***l*** is a reflection over the x-axis, so just flip ***m*** over the x-axis to get ***l***.

***y***

***m***

***(5,-4)***

***(5,4)***

***l***

***(0,0)***

***x***

* + - * Since both of our lines go through the origin (0,0), it’s pretty easy to find the slope of our line ***l***. As we said before, slope is rise/run.
      * The two points for line ***l*** are (0,0) and (5,4) so plug that into the slope formula. Slope is:
      * y2 – y1 = slope of ***l***

x2 – x1

* + - * 4-0 = slope of ***l***
      * 5-0
      * ***4/5*** is the slope of ***l***

B is the correct answer.

1. Friends.
   1. Let’s write our given: ***n*** = 3***p***
   2. What are we looking for? The ***value*** of ***p*** for which ***n*** = ***p***.
   3. This problem is not too bad. Start plugging in answers.
      1. ***p*** = ***0***
         * ***n*** = 3***p***
         * ***n*** = 3(***0***)
         * ***n*** = ***0***
         * ***n*** = ***p*** = ***0*** That works. When p = 0, n = p.
      2. ***p*** = ***1/3*** 
         * ***n*** = 3***p***
         * ***n*** = 3(***1/3***)
         * ***n*** = ***1***
         * ***n*** ≠ ***p*** (B) is out.
   4. (C) and (D) are both out because they will produce similar numerical answers like (B), and (E) is out because (A) works.

A is the correct answer.

1. Triangles and lines. MARK UP THE PICTURE!
   1. Let’s write our given: ***z*** = 30
   2. What are we looking for? The ***value*** of ***x*** + ***y***.
   3. To the picture. We know that the angles of a triangle add up to 180°. We also know that straight lines add up to 180°. Mark up the picture and write some formulas that will help us find ***x*** + ***y***. Look at line ***l***. One angle of the triangle is 180° – ***y***° because that angle plus ***y***° equals 180° (of line ***l***). The other angle of the triangle is 180°- ***x***° for the same reason.

***m***

***n***

***y°***

***x°***

***z°***

***180°-y°***

***180°-x°***

***l***

* 1. Now write the formula for the triangle with our angles.
     + - 180° – ***y***° + 180° – ***x***° + ***z***° = 180° Plug in for ***z*** and simplify.
       - 180° – ***y***° + 180° – ***x***° + ***30***° = 180°
       - 390° – ***y***° – ***x***° = 180° Subtract 390° from both sides.

-390° -390°

* + - * – ***y***° – ***x***° = -210°
      * + ***y***° + ***x***° = +210°
      * ***x***° + ***y***° = ***210***°

D is the correct answer.

1. Parabola problem.
   1. Let’s write our given: f(x) = x2 + bx + c
   2. What are we looking for? The ***possible graph*** of f(x).
   3. With parabolas, the sign of the x2 term (the unwritten *a* here) tells us the parabola opens up or down. If the x2 term is positive, it opens up, if it’s negative, it opens down. Our x2 is positive, so it opens up. Answers (A) and (B) are out.
   4. The next thing to look at is the “c.” Think of this like a y-intercept, or ***b*** in a line f(x) = mx + ***b***. When ***b*** is positive, the line crosses the y-axis in positive territory, when ***b*** is negative, the line crosses the y-axis in negative territory. In our equation, c is positive, so we know this parabola crosses the y-axis in positive territory. (C) is out because that graph crosses at y = 0, and (D) is out because that graph crosses the y-axis in negative territory. We’re left with (***E***), and that’s correct because it opens up and crosses the y-axis in positive territory.

E is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + The cube has edges of length 2.
        + ***A*** and ***B*** are midpoints of two edges.
   2. What are we looking for? The ***length*** of ***AB***.
   3. To the picture! Start by labeling the lengths (and half-lengths) of the relevant edges of the cube and the two corners on the bottom right of the cube. Call those points C and D. We know we can’t get straight to ***AB***, so let’s use some triangles to help us out. That’s where C and D come in. ***AB*** is the ***green dashed*** line cutting through the interior of the cube in the picture below.

***A***

***B***

***C***

***2***

***1***

***1***

***D***

* 1. We almost have a triangle with AB as one of the sides. Draw a line between points B and D to create ∆***AB***D. Now let’s first figure out the length of ***BD***.
  2. Look at ∆BCD. We’ve pulled it out of the cube so it’s easier to work with. This is a cube, so ∠C has to be a right angle. That means we can use the Pythagorean Theorem to solve for BD.

***D***

***1***

***2***

***C***

***B***

***√5***

* + - * a2 + b2 = c2
      * ***B***C2 + CD2 = ***B***D2
      * 12 + 22 = ***B***D2
      * 1 + 4 = ***B***D2
      * 5 = ***B***D2
      * √5 = ***B***D
  1. Now if we pull out ∆***AB***D, we should be able to find ***AB***. We have 2 sides, ***A***D (1), and ***B***D (√5). We can use the Pythagorean Theorem again to solve for ***AB***. In this case, ***AB*** is our longest side.
     + - a2 + b2 = c2

***D***

***A***

***B***

***1***

***√5***

***?***

* + - * ***A***D2 + ***B***D2 = ***AB***2
      * 12 + (√5)2 = ***AB***2
      * 1 + 5 = ***AB***2
      * 6 = ***AB***2
      * √***6*** = ***AB***

D is the correct answer.

1. Friends and algebra.
   1. Let’s write our givens:
      1. x is defined as x = x2 – x
      2. ***a*** = ***a*** - 2
   2. What are we looking for? The ***value*** of ***a***.
   3. This looks more tricky than it is. Instead of a number as the input, like 2, we have to input ***a*** and ***a*** – 2. We also know that those two inputs produce equal outputs.
   4. Start with ***a***. Plug in ***a*** wherever there is an x.
      * + x = x2 – x
        + ***a*** = ***a***2 – ***a*** That’s one part of the equation.
   5. Now ***a*** - 2. Plug in ***a*** – 2 wherever there is an x.
      * + x = x2 - x
        + ***a*** - 2 = (***a*** - 2)2 - (***a*** - 2) That’s a little ugly, but we have the second part.
   6. Set the two expressions equal to each other and solve for ***a***.
      * + ***a*** = ***a*** - 2
        + ***a***2 – ***a*** = (***a*** - 2)2 - (***a*** - 2) Simplify the right side first. FOIL and distribute the negative.
        + ***a***2 – ***a*** = ***a***2 - 4***a*** + 4 - ***a*** + 2
        + ***a***2 – ***a*** = ***a***2 - 5***a*** + 6The ***a***2’s go away.
        + – ***a*** = - 5***a*** + 6Add 5***a*** to both sides.
        + 4***a*** =  6
        + ***a*** =  6/4 =  ***3/2***

C is the correct answer.

# Test 8 Section 3

1. Fractions and decimals.
   1. What are we looking for? The ***decimal*** that’s between 1/5 and ¼.
   2. Convert the fractions to decimals.
      * + 1/5 = 0.20
        + ¼ = 0.25
   3. Our range is 0.20 and 0.25. Let’s look at the answers and see which works.

(A) 0.14 Nope. Too small.

(B) 0.15 Nope. Too small.

(C) 0.19 Nope. Too small.

(D) 0.21 Yes.

(E) 0.26 Nope. Too big.

D is the correct answer.

1. Coordinate plane basics. DRAW THE PICTURE!
   1. Let’s write our given: All the points (the answers) are in the xy coordinate plane.
   2. What are we looking for? The ***point*** that is ***closest*** to the ***origin***.
   3. Draw the coordinate plane and mark the coordinates.
   4. To the picture. Remember the origin is the center of the coordinate plane and has coordinates (0,0). We’ve marked that as ***O*** below.
   5. Look at the answers and see which one is closer.
      1. It is pretty clear that (A) and (E) are the farthest away, so we can eliminate those. We are down to (B), (C), and (D).
      2. (B) looks the closest, and it is. Even if we can’t see that, we know that (C) and (D) are the same distance away from ***O***, so neither can be the answer.

***-1***

***1***

***-1***

***1***

***D***

***B***

***A***

***C***

***E***

***O***

* + 1. We could draw triangles and find exact distances, but this is only the 2nd problem in the section, so we know it’s most likely an easy one. We can be comfortable with our hunch here.

B is the correct answer.

1. Geometry and a system of equations. MARK UP THE PICTURE!
   1. Let’s write our given: AB is a line.
   2. What are we looking for? The ***value*** of ***y***.
   3. To the picture. All of those angles below AB add up to ***180°***, because added up, the angles make up a straight line. The angles above line AB, y°, x° and x°, also sum to ***180°***. These are two equations. Convert them to “math speak.”

***y°***

***x°***

***x°***

***x°***

***x°***

***x°***

***x°***

***x°***

***A***

***B***

***180°***

***180°***

* + - * 5***x***° = ***180°***
      * ***y***° + 2***x***° = ***180°***
  1. Solve the first equation for ***x***.
     + - 5***x***° = ***180°*** Divide both sides by 5.
       - ***x*** = ***36***
  2. Get ***y*** by itself in the second equation, and substitute in the value of ***x***.
     + - ***y***° + 2***x***° = ***180°*** Subtract ***2x*** from both sides.
       - ***y***° = ***180°*** - 2***x***°
       - ***y***° = ***180°*** - 2(***36***)°
       - ***y***° = ***180°*** - 72°
       - ***y***° = ***108°***

A is the correct answer.

1. Algebra and a lonely friend.
   1. Let’s write our given: 6,565 = 65(***x***+1)
   2. What are we looking for? The ***value*** of ***x***.
   3. Solve for ***x***.
      * + 6,565 = 65(***x***+1) Distribute the 65 on the right side of the equation.
        + 6,565 = 65***x***+65 Subtract 65 from both sides.
        + 6,500 = 65***x*** Divide both sides by 65.
        + ***100*** = ***x***

C is the correct answer.

1. Exponents and friends.
   1. Let’s write our givens:
      * + m***x***×m7 = m28
        + (m5)***y*** = m15
   2. What are we looking for? The ***value*** of ***x*** + ***y***.
   3. We have to know our exponents rules. When multiplying like-bases, we ***add*** the exponents. This is the situation in the equation with ***x***.
      * + m***x***×m7 = m28
        + m(***x***+7) = m28 Bases are the same, so exponents have to be the same.
        + ***x*** + 7 = 28 Subtract 7 from the left side.
        + ***x*** = ***21***
   4. Now our 2nd equation with ***y***. When raising an exponent to another exponent, we ***multiply*** the exponents.
      * + (m5)***y*** = m15
        + m5***y*** = m15 Bases are the same, so exponents have to be the same.
        + 5***y*** = 15 Divide both sides by 5.
        + ***y*** = ***3***
   5. We have our two pieces; solve for ***x*** + ***y***.
      * + ***x*** + ***y***
        + ***21*** + ***3***
        + ***24***

D is the correct answer.

1. Graph Analysis Problem
   1. What are we looking for? The ***value*** that is closest to the ***decrease per year*** from 1987 to 1990.
   2. First, make sure we are only looking from 1987 to 1990. We don’t care about the time from 1985 to 1987. From 1987 to 1990 is a straight line, and we can approximate the decrease per year from looking at the line. Each block is in 10,000 increments, so keep that in mind as we go through the answers.
      1. 7,000. This doesn’t look right, because that would mean that from one year to the next, the decrease would be less than a full block. From 1987 to 1988, the drop is more than one full block. (A) is out.
      2. 11,500. This could be close, but 11,500 is just over the size of one 10,000 block. If, in looking at 1988 to 1989, the 1989 figure was 130,000, this might be a good answer, but 1989 drops below the 130,000 line, so this decrease is a good bit over 10,000. (B) is out.
      3. ***14,000***. This is the best looking one so far. Look at 1987 to 1988. 1987 is well above the half-way mark between 150,000 and 160,000, and in 1988 the sales drop almost all the way to 140,000, but not quite. That means we have to subtract the little bit between 140,000 up to the 1988 mark to get the drop, so we’re a little below 15,000, or 1 ½ blocks. Check (D) and (E).
      4. 17,500. This would look great if 1988 dropped all the way to 140,000, but it doesn’t. 17,500 is 1 ¾ blocks, and none of our years show that big of a drop.
      5. 42,000. That’s over 4 blocks. That may be the total decrease from 1987 to 1990, but it is not the decrease *per year*.

C is the correct answer.

1. Triangle problem with vertical angles. MARK UP THE PICTURES!
   1. Let’s write our givens:
      * + AE and CD are ⊥ to CE.
        + ***x*** = ***y***
        + AB = 4
        + BD = 8
   2. What are we looking for? The ***length*** of ***CE***.
   3. To find ***CE***, we need the lengths of ***CB*** and ***BE***. To the picture. ∠ABE and ∠CBD are vertical angles. We know vertical angles are equal, so mark that on the graph. Also, ∠E and ∠C are both 90°, because they are right angles. Since ∠E = 90° and ***x*** = ***y***, then ***x***° = ***y***° = 45°, because:
      * + ∠E + ∠A + ∠ABE = 180°
        + 90° + ***x***° + ***y***° = 180°
        + ***x***° + ***y***° = 90°, and since ***x*** = ***y***,
        + ***x***° = ***y***° = 45°

***4***

***8***

***D***

***C***

***B***

***A***

***E***

***y°***

***x°***

***A***

***8***

***D***

***C***

***B***

***E***

***45°***

***45°***

***4***

D

* 1. Look at ∆ABE. It’s a 45°-45°-90°. That means sides AE and ***BE*** = ***s***, and AB = ***s***√2.

***A***

***8***

***D***

***C***

***B***

***E***

***45°***

***45°***

***s***

***s***

We have AB’s length; it’s 4, so by solving for ***s*** we will have the length of ***BE***.

* + - * AB = ***s***√2
      * 4 = ***s***√2
      * 4 = ***s***√2 Divide by √2.
      * 4 = ***s*** Simplify by multiplying the fraction by √2/√2.

√2

* + - * 4√2 = ***s***

√2 × √2

* + - * 4√2 = ***s*** Reduce the fraction.

2

* + - * 2√2 = ***s***
      * ***BE*** = ***s***
      * ***BE*** = ***2√2***
  1. Now look at ∆CBD. We need CB. Remember our vertical angles, ∠ABE and ∠CBD? They are equal. So, ∠CBD = 45°, and for the same reason as with our ∆ABE, ∠D is also 45°. Another 45°-45°-90°, but this time our ***s***√2 = 8 (BD). CB is our ***s***, so let’s find that.

***4***

***A***

***8***

***D***

***C***

***B***

***E***

***45°***

***45°***

***2√2***

***4√2***

* + - * BD = ***s***√2
      * 8 = ***s***√2 Divide both sides by √2. √2/√2 = 1, so on the

right side we’re left with just ***s***.

* + - * 8 = ***s*** Multiply the left side by √2/√2, which is the

√2 same as multiplying it by 1.

* + - * 8 × √2 = ***s*** Multiply across.

√2 √2

* + - * 8√2 = ***s*** Reduce the fraction.

2

* + - * 4√2 = ***s***
      * ***CB*** = ***s***
      * ***CB*** = ***4√2***
  1. Add up ***BE*** and ***CB*** to get ***CE***.
     + - ***BE*** + ***CB*** = ***CE***
       - ***2√2*** + ***4√2*** = ***CE***
       - ***6√2*** = ***CE***

B is the correct answer.

1. Tough fraction problem. This problem is all about keeping units in the right place.
   1. Let’s write our givens:
      * + Price is ***d*** dollars for 8 ounces of ground coffee.
        + 1 ounce makes ***c*** cups of coffee.
   2. What are we looking for? The ***dollar cost*** for ***one*** cup of coffee in terms of ***c*** and ***d***.
   3. Start with the first given. Another way to read that is ***d*** dollars ***per*** 8 ounces of coffee. ***Per*** sounds like a fraction, so write that out.
      * + ***d*** dollars

8 ounces

* 1. Next given. 1 ounce makes ***c*** cups of coffee. That’s the same thing as 1 ounce ***per*** ***c*** cups of coffee. Same thing here. Write that fraction.
     + - 1 ounce

***c*** cups

* 1. We now need to multiply these fractions together to get the total dollar cost of a cup of coffee.
     + - ***d*** dollars × 1 ounce

8 ounces  ***c*** cups

* + - * Look at the units. The ounces in the denominator of the first fraction and the ounce in the numerator of the second fraction cancel each other out. We are left with dollars/cups in terms of units. That’s what we want. Multiply it out.
      * ***d*** dollars

8 × ***c***

* + - * ***d***.

8***c***

A is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. Proportions.
   1. Let’s write our given:
      * + 10 = ***b***

***a*** 12

* 1. What are we looking for? The ***value*** of ***ab***.
  2. Cross-multiply and we’re done.
     + - 10 = ***b***

***a*** 12

* + - * ***ab*** = 10 × 12
      * ***ab*** = ***120***

***120*** is the correct answer.

1. Pattern Problem
   1. Let’s write our givens:
      * + The sequence starts with 150, 30, 6.
        + Each number after the 1st term is 1/5 of the preceding term.
   2. What are we looking for? The ***5th*** term in the sequence.
   3. Convert the “math speak” for the sequence.
      * + Each term is 1/5 the preceding term. Let’s use 150 and 30.
        + 30 = 1/5 × 150
        + So all we do is multiply each term by 1/5 and we have our next term.
   4. 6 is our 3rd term, so to get our 4th, we need to multiply it by 1/5.
      * + 6 × 1/5 = 6/5
   5. 6/5 is our 4th term. Multiply it by 1/5 to get our 5th term.
      * + 6/5 × 1/5 = ***6/25***

***6/25*** is the correct answer. (or ***.24***)

1. Line problem. DRAW THE PICTURE!
   1. Let’s write our givens:
      * + A, B, C, D and E are on a line, not necessarily in that order.
        + AB has length 24.
        + C is midpoint of AB.
        + D is midpoint of AC.
        + DE is 5.
   2. What are we looking for? ***One possible distance*** of AE.

***12***

***D***

***C***

***B***

***A***

***24***

***6***

***E***

***12***

***D***

***C***

***B***

***A***

***24***

***6***

* 1. Draw a line and start with our given information.
     + - AB has a length of 24.
       - C is midpoint of AB.
       - D is midpoint of AC.
       - From this, we can figure out that AC = 12 and AD = 6.
  2. We know that DE is 5, so we know that E is either going to be 1 to the right of A or 1 to the left of C. Let’s make it easy on ourselves and have the E between A and D.
     + - Since DE = 5 and AD= 6, we know ***AE*** can be ***1***.

***1*** is a correct answer, as is ***11***.

1. Integer Problem
   1. Let’s write our given: 5 consecutive integers sum to 185.
   2. What are we looking for? The ***greatest*** of these ***integers***.
   3. Consecutive means in a row, so we have 5 numbers in a row. Where do we start? Let’s divide 185 by 5 and use that as a starting point.
      * + 185/5 = 37
   4. Use that as the lowest number of our 5-number sequence and see if it sums to 185.
      * + 37 + 38 + 39 + 40 + 41 = 195. We’re a little high. Try 35 as the starting number.
        + 35 + 36 + 37 + 38 + ***39*** = 185. That works. And it makes sense, because 37 is the average of 5 numbers that sum to 185, so 37 needs to be the middle number.
   5. What are we looking for again? The ***greatest*** ***integer*** in the sequence. It’s ***39***.
   6. Another way to solve this is to set the greatest number in the sequence as x and define the other numbers in terms of x:
      * + x
        + x - 1
        + x - 2
        + x - 3
        + x - 4
        + Add those x-terms up to 185 and find x.
        + x + x – 1 + x – 2 + x – 3 + x – 4 = 185
        + 5x-10 = 185
        + 5x = 195
        + x = ***39***

***39*** is the correct answer.

1. Word problem.
   1. Let’s write our givens:
      * + Pay is $1200 plus 20% of ***gross sales***.
        + Monthly pay was $2500.
   2. What are we looking for? The dollar amount of ***gross sales***.
   3. Convert the” math speak” in the first given.
      * + Pay is $1200 plus 20% of ***gross sales***
        + Pay = $1200 + 20% × ***gross sales***
        + $2500 = $1200 + 20% × ***gross sales*** Subtract $1200 from both sides.
        + $1300 = 20% × ***gross sales*** Divide both sides by 20%.
        + ***$6500*** =  ***gross sales***

***6500*** is the correct answer.

1. Geometry. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + Each wedge makes a 40° angle with the center.
        + Each disc (uncut) weighs 2.5 grams
   2. What are we looking for? The ***weight*** of each ***wedge***.
   3. To the drawing. How many degrees are there in a circle? 360°. To figure out how many wedges we have, we need to divide 360° by 40°.
      * + 360°/40° = 9, so we have 9 wedges.
   4. If we have 9 wedges in one disc, and each disc weighs 2.5 grams, we can divide 2.5 by 9 to get the grams of 1 ***wedge***.
      * + 2.5 grams/ 9 wedges
        + ***.277*** grams in a ***wedge***

***.277*** is a correct answer.

1. Sneaky difference of squares problem.
   1. Let’s write our givens:
      * + ***x***2 – ***y***2 = 10
        + ***x*** + ***y*** = 5
   2. What are we looking for? The ***value*** of ***x*** – ***y***.
   3. “Difference of perfect squares” bells should be going off in our heads. ***x***2 – ***y***2 is the general form of the difference of perfect squares.
      * + ***x***2 – ***y***2 = (***x*** + ***y***)(***x*** - ***y***)
   4. Now all we have to do is plug in, because the form is already set up.
      * + ***x***2 – ***y***2 = (***x*** + ***y***)(***x*** - ***y***)
        + 10 = (5)(***x*** - ***y***)
        + Since we’re looking for ***x*** – ***y***, divide both sides by 5 and we are done.
        + 10/5 = (5)(***x*** - ***y***)/5
        + ***2*** = ***x*** - ***y***

***2*** is the correct answer.

1. Squares and triangles. MARK UP THE PICTURE!
   1. Let’s write our given: Large square has sides of 3.
   2. What are we looking for? The ***area*** of the tilted interior square.
   3. To the picture! Look at all the triangles. They are all similar because they are locked in with two squares. This means they all have a short side of 1 and a long side of 2. Mark the 1 on the bottom left triangle.
   4. We have two sides of a right triangle, so use the Pythagorean Theorem to find the hypotenuse.
      * + a2 + b2 = ***c***2

***1***

***1***

***2***

***c***

* + - * 12 + 22 = ***c***2
      * 1 + 4 = ***c***2
      * 5  = ***c***2
      * ***√5***  = ***c***
  1. Our ***c*** just happens to be the side of our interior square. The area of a square is side2, and in this case ***c***2.
     + - ***c***2 = area of interior square
       - (***√5***)2 = area of interior square
       - ***5*** = area of interior square

***5*** is the correct answer.

1. Remainder problem.
   1. Let’s write our givens:
      * + ***j*** ***R*** ***k*** is the whole number remainder when ***j*** is divided by ***k***.
        + ***13*** ***R*** ***k*** = 2
   2. What are we looking for? The ***value*** of ***k***.
   3. Remainder refresher: It’s the amount left over in division. Here’s an example:

What’s the remainder when we divide 5 by 2?

2 r ***1*** ***1*** is the remainder in this example

2√5

* 1. Back to our problem. Write the skeleton of the problem, in this case just the division sign.

√

* 1. Now let’s fill it in with the given information.

r 2 From the statement: “when ***j*** is divided by ***k***”

***k*** √***j***

* 1. What numbers work for ***k***? Let’s try 7. Input 7 for ***k***.

1 r 6

***7*** √***13***

Remainder is ***6***. ***k*** can’t be 7. It has to be 2. Before we go too much further, think about this problem. There’s only one number we can divide into 13 and get a remainder of 2, and that’s 11. Try it.

1 r 2

***11***√***13***

***11*** is the correct answer.

1. Average problem with friends to end.
   1. Let’s write our givens:
      * + Average scores of a class of ***p*** students is 70.
        + Average scores of a class of ***n*** students is 92.
        + When the scores are added together, the average is 86.
   2. What are we looking for? The ***value*** of ***p***/***n***.
   3. Think of the average equation for the combined group.
      * + Sum of all student scores = 86

Number of all students

* + - * We know the denominator; it’s ***p*** + ***n*** because those two friends equal the total students.
      * Sum of all student scores = 86

***p*** + ***n***

* + - * The top is a little more difficult. If we can find an expression for the sum of the ***p*** scores and the sum of the ***n*** scores, we can add them to find the sum of ***all*** student scores.
  1. Let’s look at the average in terms of an equation, starting with one of the classes. We’ll use ***p***.
     + - Sum of all scores in ***p*** class = 70

Number of ***p*** students

* + - * Sum of all scores in ***p*** class = 70 Multiply both sides by ***p***.

***p***

* + - * Sum of all scores in ***p*** class × ***p*** = 70 × ***p***

***p***

* + - * Sum of all scores in ***p*** class = 70***p***
      * This will also work for ***n*** so: Sum of all scores in ***n*** class = 92***n***
  1. Back to our overall average. Plug in the sum of scores information.
     + - Sum of all student scores = 86

***p*** + ***n***

* + - * 70***p*** + 92***n*** = 86 Cross-multiply and simplify.

***p*** + ***n***

* + - * 86(***p*** + ***n***) = 70***p*** + 92***n***
      * 86***p*** + 86***n*** = 70***p*** + 92***n*** Subtract 70***p*** and 86***n*** from both sides.
      * 16***p*** = 6***n*** Divide both sides by 16***n***.
      * 16***p*** = 6***n*** The 16’s on the left side cancel out, as do the ***n***’s on the right side.

16***n*** 16***n***

* + - * ***p*** = ***3***

***n*** ***8***

***3/8*** is the correct answer.

# Test 8 Section 7

1. Points for letters.
   1. Let’s write our givens:
      * + Each ***q***, ***x*** and ***z*** is worth 5 points.
        + All other letters are worth 1 point.
   2. What are we looking for? The ***points*** for the word ***exquisite***.
   3. Count the letters. There are a total of 9 letters.
      * + There is one ***q*** and one ***x***, so there are (2 × 5 = 10) points for those.
        + The other 7 letters each are worth one point, so (7 × 1 = 7) points for those.
   4. Combine the two numbers.
      * + 10 + 7 = ***17*** total points

B is the correct answer.

1. Straight-forward friends problem.
   1. Let’s write our given: 2x – 10 = 20
   2. What are we looking for? The ***value*** of x – 5.
   3. The long way is to solve for x like we normally do.
      * + 2x – 10 = 20 Add 10 to both sides.
        + 2x = 30 Divide both sides by 2.
        + x = 15
        + If x = 15, then x – 5 = ***10***.
   4. Now an easier way. Just divide the whole equation by 2 and we’ll have x – 5.
      * + C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmf2x – 10 = 20

2

* + - * x – 5 = ***10***

B is the correct answer.

1. Integer Problem
   1. Let’s write our given: ***t*** represents an ***odd*** integer.
   2. What are we looking for? The ***expression*** that represents an ***even*** integer .
   3. Pick an odd integer. Keep it simple; let ***t*** = ***1***. Try the answer choices.
      1. ***t*** + 2
         * ***1*** + 2 = 3
         * Odd. (A) is out.
      2. 2***t*** - 1
         * 2(***1***) - 1 = 1
         * Odd. (B) is out.
      3. 3***t*** - 2
         * 3(***1***) - 2 = 1
         * Odd. (C) is out.
      4. 3***t*** + 2
         * 3(***1***) + 2 = 5
         * Odd. (D) is out.
      5. 5***t*** + 1
         * 5(***1***) + 1 = 6
         * ***Even***

E is the correct answer.

1. Triangle Problem
   1. Let’s write our givens:
      * + The perimeter of ∆ABC is the same as the perimeter of ∆DEF.
        + ∆ABC is equilateral.
   2. What are we looking for? The ***length*** of ***AB***.
   3. Add up the sides of ∆DEF to get the perimeter.
      * + 4 + 8 + 9 = 21
   4. With this result and our given, we know that the perimeter of ∆ABC is also 21. And we know ∆ABC is equilateral. Equilateral means that all 3 sides are the same, so divide the perimeter by 3, and we’ll have the ***length*** of ***AB***.
      * + 21 / 3 = ***AB***
        + ***7*** = ***AB***

C is the correct answer.

1. Data Analysis Problem
   1. Let’s write our given: The store sold 900 pairs of “Other Brands” jeans.
   2. What are we looking for? The ***total*** pairs of jeans sold at the store.
   3. Find “Other Brands” on the pie chart. It says 20%. So we know 900 pairs of jeans represent 20% of the jeans sold. Set up the proportion to find out the total pairs of jeans sold by the entire store (i.e., 100% of jeans sold).
      * + 20% = 100% Cross multiply and solve for ***total***.

900 ***total***

* + - * 20%(***total***) = 100%(900) Divide both sides by 20%.
      * ***total***  = ***4,500***

E is the correct answer.

1. Geometry. DRAW THE PICTURE!
   1. Let’s write our given: Rectangular floor is 12 feet by 18 feet (1 yard = 3 feet).
   2. What are we looking for? The ***square yards*** of carpeting needed to cover it.
   3. Draw the picture. Label the sides in yards (divide the feet by 3 to get yards)

***12 feet = 4 yards***

***18 feet = 6 yards***

* + - * Area = length × width
      * Area = 6 yards × 4 yards
      * Area = ***24*** square yards

C is the correct answer.

1. System of equations problem.
   1. Let’s write our givens:
      * + Kitten + Bunny = 7
        + Kitten + ***Puppy*** = 8
        + Bunny+ ***Puppy*** = 9
   2. What are we looking for? The weight of ***Puppy***.
   3. Take the first equation and get Kitten by itself.
      * + Kitten + Bunny = 7 Subtract Bunny.
        + Kitten = 7 - Bunny
   4. Plug that into the second equation.
      * + Kitten + ***Puppy*** = 8
        + 7 - Bunny + ***Puppy*** = 8 Simplify.
        + ***Puppy*** - Bunny = 8 - 7
        + ***Puppy*** - Bunny = 1 Get Bunny by itself so we can solve for ***Puppy*** in the 3rd equation.
        + Bunny = ***Puppy*** – 1
   5. Plug that into the 3rd given equation.
      * + Bunny + ***Puppy*** = 9
        + ***Puppy*** – 1 + ***Puppy*** = 9
        + 2***Puppy*** – 1 = 9
        + 2***Puppy*** = 10
        + ***Puppy*** = ***5*** pounds

D is the correct answer.

1. Proportion Problem
   1. Write the givens:
      * + ¼ inch = 16 feet
        + Driveway is 40 feet long.
   2. What are we looking for? The ***length in inches*** that the driveway is on the map.
   3. Set up the proportion. Inches on top and feet on the bottom.
      * + ¼ = ***x*** Cross multiply and solve for ***x***, which is our driveway length in inches.

16 40

* + - * ¼(40) = 16***x*** Divide both sides by 16.
      * ***5*** = ***x***

***8***

B is the correct answer.

1. Friends and the coordinate plane.
   1. Let’s write our givens:
      * + (***p***,0) is a point of intersection of y = -x2 + 9 and y = x2 – 9.
        + ***p*** is positive.
   2. What are we looking for? The ***value*** of ***p***.
   3. In our point (p,0), x = p and y = 0. Plug that into one of our equations. The second one looks a little friendlier, so start there. Since this point is on ***both*** graphs, whatever value we find for p must be on both graphs at a value of (p,0).
      * + (***p***,0)
        + (x,y)
        + 0 = ***p***2 – 9 Difference of perfect squares.
        + 0 =(***p*** + 3)(***p*** - 3)
        + ***p*** = 3 or -3 ***p*** has to be positive (given).
        + ***p*** = ***3***

A is the correct answer.

1. Rate Problem
   1. Let’s write our givens:
      * + Old machine makes 300 bolts per hour.
        + New machine makes 450 per hour.
        + Both machines start at same time.
   2. What are we looking for? The number of ***minutes*** it takes two machines to make 900 bolts.
   3. First let’s look at one hour. In one hour, both machines together make 750 bolts (300 from old machine + 450 from new machine). We need to get to 900 bolts. This can be looked at as a proportion:
      * + 900 bolts = 1.2 This is still in hours, so we need to multiply by 60 to get to minutes.

750 bolts

* + - * ***72*** minutes

B is the correct answer.

1. Function Problem
   1. What are we looking for? The ***function*** that defines ***g***
   2. If there’s a zero, start there, those are always easiest. Start going through the answer choices and solve for ***t*** = ***0*** to see if it gives us a ***g***(***0***) = 2 (given in table).
      1. ***g***(***t***) = ½(***t***) + 1
         * ***g***(***0***) = ½(***0***) + 1
         * ***g***(***0***) = 1 That doesn’t work; the ***g***(***0***) is supposed to be 2. (A) is out.
      2. ***g***(***t***) = -½(***t***) + 1
         * ***g***(***0***) = -½(***0***) + 1
         * ***g***(***0***) = 1 Nope. (B) is out.
      3. ***g***(***t***) = -***t*** + 1
         * ***g***(***0***) = -(***0***) + 1
         * ***g***(***0***) = 1 Nope. (C) is out.
      4. ***g***(***t***) = -***t*** + 2
         * ***g***(***0***) = - (***0***) + 2
         * ***g***(***0***) = 2 (D) could work. Try (E).
      5. ***g***(***t***) = -2***t*** + 2
         * ***g***(***0***) = -2(***0***) + 2
         * ***g***(***0***) = 2 (E) could also work.
   3. We’ve got it narrowed down to two, (D) and (E). Let’s use ***t*** = ***1*** and see if we get ***g***(***1***) = 0 (given in table).
      1. ***g***(***t***) = -***t*** + 2
         * ***g***(***1***) = -(***1***) + 2
         * ***g***(***1***) = 1 (D) doesn’t work; ***g***(***1***) is supposed to be 0. Try (E).
      2. ***g***(***t***) = -2***t*** + 2
         * ***g***(***1***) = -2(***1***) + 2
         * ***g***(***1***) = 0 (***E***) works.

E is the correct answer.

1. Data analysis.
   1. Let’s write our given: The scatterplot includes data for 16 total students.
   2. What are we looking for? The answer choice that’s a ***true*** statement.
   3. Each dot is one student. Grade is on the y-axis and distance is on the x-axis. Go through each answer choice.
      1. Only one student that travels 2 miles. False, because there are two dots on the “2” mile line. (A) is out.
      2. Half of the students travel less than 4 miles. That means that we need 8 students that travel 1, 2 or 3 miles. Count the 1, 2 and 3 mile dots. There are only 5 dots on miles 1, 2, and 3. (B) is out.
      3. More 12th than 11th graders travel 6 ***or more*** miles to school. There are 3 12th graders that travel 6+ miles. There are 2 11th graders that travel 6+ miles. (***C***) is ***true***. Check the others.
      4. All students that travel less than 3 miles are 12th graders. No, there are actually no 12th graders that travel ***less than*** 3 miles. (D) is out.
      5. Half the students that travel 7 miles or more are 9th graders. Nope, there are only 2 9th graders in this group, with 5 students total. That’s 40%, not 50% (half). (E) is out.

C is the correct answer.

1. Number Placement Problem
   1. Let’s write our givens:
      * + 3-digit positive numbers
        + Hundreds digit = ***3***
        + Units digit = ***4***
   2. What are we looking for? ***The number of positive 3-digit integers*** that follow the rules in the given info.
   3. We need a skeleton for this problem. Draw 3 boxes. Put in the 3 and the 4.

***3***

***4***

* 1. We know that the hundreds digit is ***3***, and the units digit is ***4***, so what can change? Only the ***tens*** digit. We have to have a positive number here, so the only numbers that can go there are 0-9, inclusive. That’s 10 possibilities (not 9) that can be in the tens digit. We have ***10*** possible 3-digit numbers.

A is the correct answer.

1. Function problem on a coordinate plane.
   1. Let’s write our givens:
      * + Equation of the line is y = ***m***x + ***b***.
        + m and b are constants.
   2. What are we looking for? The ***graph*** that has the equation y = -3***m***x + ***b***.
   3. This is just the general form of a function. ***m*** is the slope; ***b*** is the y-intercept. First, we need to find the slope and y-intercept of the original graph, y = ***m***x + ***b***.
      * + ***b*** = ***-1*** because the line crosses the y-intercept at -1.
        + ***m***, the slope, is ***-1/3*** because the line goes ***down*** 1 and to the ***right*** 3. ***m*** = ***-1/3***
        + Answers (B), (C) and (E) are all out, because those lines don’t cross the y-axis at -1. We’re left with (A) and (D).
   4. Look at the equation of the function we are trying to find. The new equation is:
      * + y = -3***m***x + ***b*** We found ***b***; it is ***-1***, so:
        + y = -3***m***x + ***-1*** We also know ***m*** = ***-1/3***, so plug that in.
        + y = -3(***-1/3***)x + ***-1*** Simplify. The negative signs and the 3’s go away.
        + y = (1)x + ***-1*** The slope of our new line is positive 1.
   5. Let’s now go back and look at (A) and (D).
      * + Answer (A) has a slope of 1/3 because the line goes *up* 1 and to the *right* 3. (A) is out.
        + Answer (***D***) has a slope of 1 (*up* 1, *right* 1).

D is the correct answer.

1. Cube fun.
   1. Let’s write our given: Cube has a volume of 8.
   2. What are we looking for? The ***shortest distance*** from the center of the cube to the base of the cube
   3. Since the cube has a volume of 8, we know that each edge has a length of 2.
   4. Find the height of the line that goes vertically from the center down to the base (the vertical ***green*** dashed line). This length must be ***1***, because the cube has edges of length 2, and the center must be in the middle.
   5. Look at the other dashed lines from the center to the base. Is there any way that any of those ***blue*** dashed lines can be shorter than the vertical ***green*** dashed line? Not in the real world. ***1*** is the shortest distance.

A is the correct answer.

***1***

***1***

***2***

***2***

***2***

1. A fraction, friends, and an exponent.
   1. Let’s write our given: ***y*** = 5***x***3/***z***
   2. What are we looking for? What happens to ***y*** when ***x*** and ***z*** are both doubled.
   3. Let’s get those friends into number form. We need to pick values for ***x*** and ***z***. How about ***1*** and ***1***?
      * + ***x*** = ***1*** , ***z*** = ***1***
          - ***y*** = 5***x***3/***z***
          - ***y*** = 5(***1***)3/(***1***)
          - ***y*** = 5(***1***) / ***1***
          - ***y*** = ***5***
   4. Now ***double*** ***x*** and ***z*** and see what our new ***y*** is. ***x*** = ***2*** , ***z*** = ***2***
      * + ***y*** = 5***x***3/***z***
          - ***y*** = 5(***2***)3/(***2***)
          - ***y*** = 5(***8***) / ***2***
          - ***y*** = 40 / ***2***
          - ***y*** = ***20***
   5. What happened to ***y***? ***20*** is 4 times ***5***, so when ***x*** and ***z*** are doubled, ***y is multiplied by 4***.

E is the correct answer.

1. Algebra and an exponent.
   1. Let’s write our givens:
      * + Car purchased a car for $5,000.
        + Car value decreases by 20% each year.
        + ***n*** = years
        + Value is V(***n***), where V(***n***) = 5,000(4/5)***n***.
   2. What are we looking for? ***The number of*** ***years*** after purchase it will take for the car’s value to drop to $3,200.
   3. We are looking for ***n*** where V(***n***) = $3,200. Plug it in and solve.
      * + V(***n***) = 5,000(4/5)***n***
        + 3,200 = 5,000(4/5)***n*** Divide both sides by 5,000.
        + 3,200 = (4/5)***n*** Reduce the fraction on the left. Get rid of the 2 zero’s.

5,000

* + - * 32 = (4/5)***n*** Divide 32 and 50 both by 2 to reduce the fraction.

50

* + - * 16 = (4/5)***n*** Distribute the ***n*** into the parenthesis.

25

* + - * 16 = 4***n***

25 5***n***

* 1. Look at those last two fractions. They are equal, so the numerators must be equal:
     + - 16 = 4***n***
       - What’s the square root of 16? It’s 4. 16 is a perfect ***square***. So ***n*** = ***2***. It takes two years for the car’s value to drop to $3,200.
  2. That also checks out with the denominators:
     + - 25 = 5***n***
       - 25 = 5***2***
       - 25 = 25

B is the correct answer.

1. Pattern Problem
   1. Let’s write our givens:
      * + Three wires, ***A***, ***B*** and ***C*** are braided.
        + Pattern (each step) goes left into middle, then right into middle.
   2. What are we looking for? ***The numbered step*** where the pattern starts to repeat the original order.
   3. Start with the given steps and continue the pattern until we get back to the ***A***, ***B***, ***C*** pattern.

|  |  |  |  |
| --- | --- | --- | --- |
| Start | ***A*** | ***B*** | ***C*** |
| Step 1 | ***B*** | ***A*** | ***C*** |
| Step 2 | ***B*** | ***C*** | ***A*** |
| Step 3 | ***C*** | ***B*** | ***A*** |
| Step 4 | ***C*** | ***A*** | ***B*** |
| Step 5 | ***A*** | ***C*** | ***B*** |
| ***Step 6*** | ***A*** | ***B*** | ***C*** |

D is the correct answer.

1. Median Theory Problem
   1. Let’s write our given: Set of eleven ***different*** numbers.
   2. What are we looking for? The answer choice statement that ***will not affect*** the ***median*** of the set.
   3. The ***median*** is the middle number when a set is ordered from least to greatest. Pick a set of numbers. Let’s start with 1, and add 2 so our set is the odd numbers from 1 to 21.

1 3 5 7 9 ***11*** 13 15 17 19 21

The ***median*** of this set is ***11***. Now let’s go through the answer choices and see what happens.

* + 1. Doubling each number
       - Old: 1 3 5 7 9 ***11*** 13 15 17 19 21
       - New: 2 6 10 14 18 ***22*** 26 30 34 38 42
       - The median changed. (A) is out.
    2. Increasing each number by 10
       - Old: 1 3 5 7 9 ***11*** 13 15 17 19 21
       - New: 11 13 15 17 19 ***21*** 23 25 27 29 31
       - The median changed. (B) is out.
    3. Increasing the ***smallest*** number only. It doesn’t say by how much. What if we increased it by 100?
       - Old: ***1*** 3 5 7 9 ***11*** 13 15 17 19 21
       - New: ***100*** 3 5 7 9 ***11*** 13 15 17 19 21
       - New: 3 5 7 9 11 ***13*** 15 17 19 21 ***101***
       - Our median changed. (C) is out.
    4. Decreasing the ***largest*** number only. It doesn’t say by how much. What if we decreased it by 100?
       - Old: 1 3 5 7 9 ***11*** 13 15 17 19 ***21***
       - New: 1 3 5 7 9 ***11*** 13 15 17 19 ***-79***
       - New: ***-79*** 1 3 5 7 ***9*** 11 13 15 17 19
       - Our median changed. (D) is out.
    5. Increasing the ***largest*** number only. Besides being the last answer left, why does this work? Because we’re moving the farthest right number ***even further*** to the right. No effect on median.
       - Old: 1 3 5 7 9 ***11*** 13 15 17 19 ***21***
       - New: 1 3 5 7 9 ***11*** 13 15 17 19 ***25***
       - New: 1 3 5 7 9 ***11*** 13 15 17 19 ***100***
       - It doesn’t matter by how much we increase the largest number. ***11*** is still going to be right in the middle of the set of numbers. ***11*** is still the ***median*** in (***E***).

E is the correct answer.

1. Tough perimeter problem. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + SBT is one-quarter of a circle; center is R.
        + Radius of 6
        + ABCR is a rectangle.
        + Length + width of rectangle is 8.
   2. What are we looking for? The ***perimeter*** of the ***shaded region***.
   3. To the picture. Perimeter is the distance around the shaded region. That is AS + AC + CT + SBT (arc) . Let’s start with the arc SBT.
   4. We know SBT is on the circle. Circumference of a circle is 2πr. Since SBT is ¼ of a circle, we know that the length of the arc SBT = ¼(2πr).
      * + SBT = ¼(2πr)
        + SBT = ¼(2π6)
        + SBT = ¼(12π)
        + SBT = 3π
   5. Now we need to find the other parts of the perimeter. Those are AS, AC and CT.
      * + AC is probably the most straight-forward. We’ll find it through BR. Diagonals of a rectangle are equal. The other diagonal of this rectangle, BR, is a radius of the circle at center R with a length of 6. So, this means AC = 6.

***R***

***x***

***S***

***A***

***B***

***T***

***C***

***6***

***R***

***S***

***A***

***B***

***T***

***C***

***x***

***6 - x***

***8 - x***

***x - 2***

* 1. AS and CT are a little more involved. Remember we are dealing with *perimeter* here, not area. We need a variable. Since we’ve got information regarding the length plus the width of the rectangle, let’s let AR = ***x***.
     + - Let’s get AS first. We know SR is a radius, so SR = 6. That means AS = 6 - ***x***.
       - CT is the tough one, and takes two steps. We need to use our other given, that the length (AR) + the width (RC) = 8. Write an expression for RC first.

RC + AR = 8

RC + ***x*** = 8

RC = 8 - ***x***

* + - * We know that RT is a radius, so it must equal 6. Write an expression for CT.

RC + CT = 6

8 - ***x*** + CT = 6 Subtract 8 and add ***x***  to both sides.

CT = -2 + ***x***

CT = ***x*** - 2

* 1. Now put all of the sides of the shaded region together to make an equation.
     + - arc SBT + AS + AC + CT = ***perimeter***
       - 3π + 6 - ***x*** + 6 + ***x*** - 2 = ***perimeter***
       - Look what happens to those ***x***’s. One positive, one negative, they cancel each other out. Nice.
       - 3π + 6 + 6 - 2 = ***perimeter***
       - ***3π + 10*** = ***perimeter***

B is the correct answer.

# Test 8 Section 9

1. Plug and chug to get us going.
   1. Let’s write our given: Expression is 3***m*** – 1.
   2. What are we looking for? The ***value*** of ***m*** that will make the expression be greater than 10.
   3. Try the answer choices.
      1. ***m*** = ***4***
         * 3(***4***) - 1
         * 12 - 1 = 11. 11 > 10. That works. ***m*** = ***4***
   4. Check the others if you like, but 4 is is the answer.

A is the correct answer.

1. Multiplication Identity Problem
   1. Let’s write our given: ***a*** x ***k*** = ***a*** for all values of ***a***.
   2. What are we looking for? The ***value*** of ***k***.
   3. We can try all the answers for k, but we will quickly see that 1 is the answer. The only number we can multiply by ***a*** and get a result of ***a*** is ***1***.
      * + ***a*** x ***k*** = ***a***
        + ***a*** x ***1*** = ***a***
        + ***k*** = ***1***

D is the correct answer.

1. Parallel lines and vertical angles. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + l ∥ m
        + ***x*** = 80
        + ***y*** = 70
   2. What are we looking for? The ***value*** of ***z***.
   3. To the picture! Mark the parallel lines and the values of ***x*** and ***y***.

***80***°

***70***°

***z***°

* + - * ***80***° and ***70***° are both vertical angles with the 2 angles that form the triangle that has ***z*** as the third angle. That means those 3 angles must add up to 180°.
      * ***80***° + ***70***° + ***z***° = 180°
      * 150° + ***z***° = 180° Subtract 150° from both sides.
      * ***z***° = ***30***°

A is the correct answer.

1. System of equations word problem.
   1. Let’s write our givens:
      * + ***Scenic*** route is 5 km more than ***direct*** route.
        + Going by the ***scenic*** route and returning by ***direct*** route, the total trip is 35 km.
   2. What are we looking for? The number of ***km*** in the ***direct*** route.
   3. Name some friends.
      * + ***d*** = ***direct*** route
        + ***s*** = ***scenic*** route
   4. Write some equations from the given information. Convert the “math speak.”
      * + ***Scenic*** route is 5 km more than ***direct*** route
        + ***s***  = 5 + ***d***
        + ***s*** = 5 + ***d***
        + Going by the ***scenic*** route and returning by ***direct*** route, the total trip is 35 km
        + ***s*** + ***d***  = 35
        + ***s*** + ***d*** = 35
   5. We’ve got two equations. Plug in the value of ***s*** from the 1st equation into the 2nd equation.
      * + ***s*** + ***d*** = 35
        + (5 + ***d***)+ ***d*** = 35
        + 5 + 2***d***  = 35 Subtract 5 from both sides.
        + 2***d***  = 30 Divide both sides by 2.
        + ***d***  = ***15***

C is the correct answer.

1. Probability problem.
   1. Let’s write our givens:
      * + Total cycle is 80 seconds.
        + Green for 40 seconds
        + Amber for 10 seconds
        + Red for 30 seconds
   2. What are we looking for? The ***probability*** that the light ***will*** ***not be*** red at a randomly chosen time.
   3. Sketch out a table for this problem.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Color | Red | Amber | Green | Total |
| Seconds | 30 | 10 | 40 | 80 |

* 1. Probability is part/whole, or in this case ***not red***/total. Add up amber and green to see how many seconds the light is ***not red***.
     + - Green + Amber = ***not red***
       - 40 + 10 = ***not red***
       - 50 = ***not red***
  2. Our total seconds are 80, so we have the two pieces of our ***probability***.
     + - ***probability*** that the light is ***not red*** = 50/80, which reduces to ***5/8***.

B is the correct answer.

1. Proportion problem.
   1. Let’s write our givens:
      * + Increase in heating expenses is directly proportional to increase in water temperature.
        + Heating expense goes up $24 when water temp goes up 20 degrees.
   2. What are we looking for? ***The increase in*** heating expenses when water temp increases by 15 degrees.
   3. Straight proportion problem. On one side is our given info; on the other side is the answer we seek.
      * + $24 = $***x***  Cross multiply and solve.

20 15

* + - * 20***x*** = 24 × 15
      * 20***x*** = 360
      * ***x*** = ***18***

B is the correct answer.

1. Triangles and averages.
   1. What are we looking for? The ***average*** of ***u***, ***v***, ***w***, ***x*** and ***y*** in the triangles.
   2. An average is the sum of the terms divided by the number of the terms. That is:
      * + ***u*** + ***v*** + ***w*** + ***x*** + ***y*** = ***average***

5

* 1. We don’t know the values of the terms, but we can find the sum of them. Look at the first triangle. The angles must sum to 180°. The angles are ***x***, ***y*** and a right angle. A right angle has 90°, so ***x*** + ***y*** has to be ***90***°. Now our average is:
     + - ***u*** + ***v*** + ***w*** + ***90***° = ***average***

5

* 1. What about ***u***, ***v***, ***w***? Those are three angles of the 2nd triangle, so they sum to 180°. So, ***u*** + ***v*** + ***w*** = ***180***°. Now our average looks like:
     + - ***180***° + ***90***° = ***average***

5

* + - * 270° = ***average***

5

* + - * ***54***° = ***average***

E is the correct answer.

1. Number Line Problem
   1. Let’s write our given: x3 < x2 < x

***x***

***x3***

***x2***

* 1. What are we looking for? A possible value of ***x***.
  2. Let’s go through the answer choices.
     1. In (A) and (B), the negative numbers are tempting, but what happens when we square a negative number? It turns positive, so that would mean ***x***2 would be to the right of ***x***. That’s not the case on our number line. (A) and (B) are both out.
     + x = ¾ When ¾ is squared, x2 = 9/16, which is smaller than ¾. x3 = 27/64 which is smaller than 9/16. (***3/4***) works. We’re done.

C is the correct answer.

1. Line Problem
   1. Let’s write our given: Line l passes through the origin.
   2. What are we looking for? The ***value*** of ***k/h***.
   3. We know the slope of this line because it goes through the origin (0,0) and point (1,3). Slope is rise over run, or the difference in y’s over the difference in x’s.
      * + rise = (y2 – y1) = 3 – 0 = 3

run (x2 – x1) 1 - 0

* 1. Now look at h and k. We know they are going to be in the ratio of 3/1, because they are on the line with the slope of 3. Let’s say ***h*** = ***-2*** and ***k*** = ***-6***. Use the origin and plug into the slope formula. While it may be a little confusing because the point is in negative territory, the negatives in the slope formula cancel out, and we’re left with ***k***/***h*** = ***3***.
     + - ***k*** - 0

***h*** - 0

* + - * ***-6*** - 0

***-2*** - 0

* + - * ***-6*** = ***3***

***-2***

A is the correct answer.

1. Absolute value problem.
   1. Let’s write our givens:
      * + |***m*** – 3| = 5
        + |***k*** + 7| = 15
        + ***m*** < 0
        + ***k*** < 0
   2. What are we looking for? The ***value*** of ***m*** – ***k***.
   3. Solve each absolute value equation.
      * + |***m*** – 3| = 5
          1. ***m*** – 3 = 5 ***m*** – 3 = -5
          2. ***m*** = 8 ***m*** = -2
          3. If ***m*** < 0, ***m*** = ***-2***.
        + |***k*** + 7| = 15
          1. ***k*** + 7 = 15 ***k*** + 7 = -15
          2. ***k*** = 8 ***k*** = -22
          3. If ***k*** < 0, ***k*** = ***-22***.
   4. Solve for the ***value*** of ***m*** - ***k***.
      * + ***m*** – ***k***
        + ***-2*** – (***-22)***
        + ***20***

E is the correct answer.

1. Data Analysis Problem
   1. What are we looking for? ***How many times*** as fast does ***5W*** oil flow versus ***20W*** oil.
   2. Looking at the chart, as we go down from 20W to 10W, each decrease in oil rating increases the speed by 2 times. Since 20W is the slowest, let’s give it a flow rate of 1. Now scale it up according to the table to determine the value for 5W and our answer.
      * + ***20W*** = 1
        + 15W = 2 (2 times as fast as 20W)
        + 10W = 4 (2 times as fast as 15W)
        + ***5W*** = ***8*** (2 times as fast as 10W)
        + So, ***5W*** flows ***8*** times as fast as ***20W***.

C is the correct answer.

1. Similar triangles.
   1. Let’s write our givens:
      * + P, A and B are equally spaced on line l.
        + P, Q and R are equally spaced on line m.
        + PB = 4
        + PR = 6
        + AQ = 4
   2. What are we looking for? The ***perimeter*** of quadrilateral QABR.
   3. Perimeter is the length around the edge. Draw the picture. The first and second givens are long ways of saying that A is right in the middle of P and B, and Q is right in the middle of P and R. That means PA = AB and PQ = QR. Mark that and also the distances.

***A***

***l***

***P***

***m***

***2***

***2***

***3***

***3***

***4***

***6***

***4***

***B***

***Q***

***R***

* 1. We need to find BR so we can find the perimeter of QABR. We can find this because ∆PAQ is similar to ∆PBR. In similar triangles, corresponding sides have the same ratio.
     + - PA/ PB = 2/4
       - PQ/PR = 3/6
       - Those both reduce to ½.
  2. Now look at BR. That ratio would be:
     + - AQ/BR = ½ Plug in AQ.
       - 4/BR = ½ Cross multiply.
       - BR = 8
  3. Add up all the sides of QABR.
     + - AQ + QR + BR + AB = ***perimeter*** QABR
       - 4 + 3 + 8 + 2 = ***perimeter*** QABR
       - ***17*** = ***perimeter*** QABR

E is the correct answer.

1. Functions and friends.
   1. Let’s write our givens:
      * + ***g***(n) = n2 + n
        + ***h***(n) = n2 - n
   2. What are we looking for? The ***value*** of ***g***(5) - ***h***(4).
   3. Inputs and outputs. For ***g***, put the 5 in for the n, and for ***h***, put the 4 in for the n. Then subtract.
      * + ***g***(n) = n2 + n
          - ***g***(5) = 52 + 5
          - ***g***(5) = 25 + 5
          - ***g***(5) = 30
        + ***h***(n) = n2 - n
          - ***h***(4) = 42 - 4
          - ***h***(4) = 16 - 4
          - ***h***(4) = 12
        + ***g***(5) - ***h***(5) = ***?***
        + 30 - 12 = ***18***

D is the correct answer.

1. Functions and friends continued.
   1. Let’s write our givens:
      * + ***g***(n) = n2 + n
        + ***h***(n) = n2 - n
   2. What are we looking for? The ***expression equivalent*** to ***h***(m + 1).
   3. Instead of a number this time, we are plugging in the expression (m + 1) for n. No problem.
      * + ***h***(n) = n2 - n
        + ***h***(m+1) = (m+1)2 - (m+1) Now simplify. FOIL the (m + 1)2. Distribute the negative in (m+1).
        + ***h***(m+1) = m2 + 2m + 1 – m – 1 1’s cancel each other out, and we’re left with one m.
        + ***h***(m+1) = m2 + m
   4. Now look at that equation we found. Doesn’t it look just like ***g***(n) = n2 + n ? Yes, it does. So answer (A), ***g***(m) looks like our answer. Check it.
      * + ***g***(n) = n2 + n Plug in m for n.
        + ***g***(m) = m2 + m
        + That’s the same as ***h***(m+1).
        + ***h***(m+1) = ***g***(m)

A is the correct answer.

1. Word problem with percentages.
   1. Let’s write our givens:
      * + Store sells a sweater for $28.
        + The sales price is 40% more than what the ***store pays for it***.
        + At the end of the season, employees can buy the shirt at a 30% discount of the ***store’s cost***.
   2. What are we looking for? The ***price*** ***the employee would pay*** for a sweater ***bought at the end of the season.***
   3. First thing is to find the store’s cost, and then we will take the steps to get to the employee cost. Let ***store*** ***cost*** = ***c***.
      * + 28 = ***c*** × 1.40 (1.40 = 1 + 40%)
        + 28 = ***c*** × 1.40 Divide by 1.40.
        + ***20*** = ***c***
   4. Now let’s find the employee’s price. The employee gets a 30% discount off the ***store’s cost***. That’s the same thing as paying 70% of the ***store’s cost***.
      * + ***Employee price*** = ***c***  × .70
        + ***Employee price*** = ***20*** × .70
        + ***Employee price*** = ***14***

B is the correct answer.

1. Rectangle to end the test. DRAW A PICTURE!
   1. Let’s write our givens:
      * + Rectangle ABCD
        + E is midpoint of BC.
        + Area of ABED is 2/3.
   2. What are we looking for? The ***area*** of rectangle ***ABCD***.
   3. Start with the picture. Mark the midpoint E (BE = EC) and draw in DE so we can see ABED.

***A***

***B***

***D***

***C***

***E***

* + - * This doesn’t really give us too much. But what if we draw a line from E to the middle of AD? Look at the picture below. We now have two rectangles that are the same size. Also, if we look at DE, it is a diagonal of the bottom rectangle, and a diagonal of a rectangle cuts the rectangle into two triangles with equal areas.

***A***

***B***

***D***

***C***

***E***

* + - * The top rectangle, too, can be divided into 2 equal triangles. That means we have ***4 equal triangles*** that make up the entire ***area*** of rectangle ***ABCD***.

***A***

***B***

***D***

***C***

***E***

* 1. Back to the given information. We’re given that the area of ABED is 2/3. How many triangles is that? It’s 2 from the top rectangle and 1 from the bottom rectangle, so 3 total triangles make up the area ABED. 4 triangles make up the area of ABCD, so set up a ratio. Let ***x*** be the area of ***ABCD***.
     + - ABED = 3 triangles = 3 Use that ¾ ratio and plug in the area of ABED.

***ABCD*** 4 triangles 4

* + - * ABED = 3 = 2/3

***ABCD*** 4 ***x***

* + - * 3 = 2/3 Cross multiply and solve for ***x***.

4 ***x***

* + - * 3 ***x*** = 2/3 × 4
      * 3 ***x*** = 8/3
      * 3 ***x*** = 8/3 Divide both sides by 3.
      * ***x*** = 8/3 × 1/3
      * ***x*** = ***8/9***

C is the correct answer.

# Test 9 Section 2

1. Sets
   1. What are we looking for? ***How many*** numbers in set X are in set Y.
   2. See which numbers are in both sets and count them 32 and 33 are in both sets So, ***two*** numbers.

A is the correct answer.

1. Rate Problem
   1. Let’s write our givens:
      * + Peg traveled 10 miles in 2 hours.
        + Linda traveled twice as far in half the time
   2. What are we looking for? ***Linda’s average speed*** in miles per hour.
   3. Figure out Linda’s distance It’s twice as far as Peg.
      * + 10 miles x 2 = 20 miles for Linda
   4. Now Linda’s time It’s half as long as Peg’s.
      * + 2 hours / 2 = 1 hour for Linda
   5. Combine them for miles per hour.
      * + Linda miles/Linda hour = Linda miles/hour = 20/1 = ***20*** ***mph***

C is the correct answer.

1. Friends.
   1. Let’s write our given: x = k(k – 2)
   2. What are we looking for? The ***expression*** for x + 1.
   3. Add 1 to x and then simplify the expression.
      * + x = k(k – 2)
        + x ***+ 1*** = k(k – 2) ***+ 1***
        + x + 1 = k(k – 2) + 1 Distribute the k into the parenthesis.
        + x + 1 = ***k2 – 2k + 1***

C is the correct answer.

1. Functions and the coordinate plane.
   1. Let’s write our givens:
      * + Given line is y = ***a***x + ***b***.
        + ***a*** and ***b*** are constants.
   2. What are we looking for? The ***graph*** that best represents y = 2***a***x + ***b***.
   3. y = ***a***x + ***b*** looks pretty familiar That’s just the general form a function, but instead of slope being m, it’s a b is the y-intercept Look at the two lines.
      * + y = ***a***x + ***b***
        + y = 2***a***x + ***b***
        + Both have the same y-intercept, so we know that our answer has to cross the y-axis at the same point The given graph crosses the y-axis at 1, so answers (A), (D) and (E) are .
   4. We’re left with (B) and (C) Look at those graphs. (B) looks steeper than the original, while (C) looks more flat than the original. What determines the steepness of a line? The slope. The original slope is ***a***, and we know the new slope is twice the old one. We don’t know ***a***, but if we just said it was 1, then the original slope is 1. The new line’s slope would be 2 because it’s 2***a***. Slope is rise/run, so a slope of 2 versus a slope of 1 means that it goes up 2 for every 1 move to the right instead of just going up 1. So, the new line is steeper than the original, like answer (***B***).

B is the correct answer.

1. Sneaky special triangle Mark up the picture.
   1. Let’s write our given: The perimeter of the triangle is 4 + 2√2.
   2. What are we looking for? The ***value*** of ***x***.
   3. To the picture This is a right triangle with 2 legs of the same size That’s a 45° – 45° – 90° The sides of a 45° – 45° – 90° triangle are s, s and s√2. Mark that, but use ***x*** instead of s.

***x***

***x***

***x√2***

* + - * We know that the perimeter is 4 + 2√2, and we have all our sides, so write the equation.
      * ***x*** + ***x*** + ***x***√2 = 4 + 2√2 Combine those ***x*** terms.
      * 2***x*** + ***x***√2 = 4 + 2√2 We have a regular term, 4 and, on each side, a √2 term.
  1. If 2***x*** = 4 and ***x***√2 = 2√2, we’ll be good. Solve.
     + - 2***x*** = 4 Divide both sides by 2. ***x***√2 = 2√2 Divide both sides by √2.
       - ***x*** = ***2 x*** = ***2***
       - So, if ***x = 2***, the perimeter is 4 + 2√2.

A is the correct answer.

1. Data analysis and a median.
   1. Let’s write our given:
      * + Test scores of 16 students before Sam takes the test
        + ***Sam*** gets a ***95*** on the test the following week.
   2. What are we looking for? The ***median*** score for the test after ***Sam*** takes it.
   3. When Sam takes the test, there will be 17 scores The ***median*** score is the one right in the middle when the scores are lined up from least to greatest, or the  number List the scores with Sam’s ***95*** included.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  | 10th | 11th | 12th | 13th | 14th | 15th | 16th | 17th |
| 60 | 70 | 70 | 75 | 75 | 80 | 80 | 80 | ***85*** | 90 | 90 | 90 | 90 | 95 | 95 | ***95*** | 100 |

* + - * Our ***median*** test score, the 9th number, is ***85***.

C is the correct answer.

1. Friends and containers.
   1. Let’s write our givens:
      * + 16 containers of one size have total capacity of ***x*** gallons.
        + 8 containers of another size have total capacity of ***x*** gallons.
        + ***x*** > 0
   2. What are we looking for? The ***capacity*** of one of the ***larger*** containers.
   3. First we need to figure out which container is larger If it takes 16 containers of one size to make ***x*** gallons, and only takes 8 containers of another size to make x gallons, the 8 containers are larger. That’s a start.
   4. What if we give ***x*** a value? How about ***x*** = 32 gallons Now we can write a couple equations.
      * + Larger containers
        + 8(# of gallons per container) = ***x*** gallons Divide both sides by 8.
        + 8(# of gallons per container) = ***32*** gallons

8 8

* + - * # of gallons per container = 4 gallons
      * We need our answer in terms of ***x*** We divided our ***x*** (***32***) by 8 to give us our gallons per container.
      * ***x***/8 = ***capacity*** of larger container

D is the correct answer.

1. Tricky coordinate plane problem Draw a picture.
   1. Let’s write our given: Rectangle ABCD’s sides are not parallel to the axes.
   2. What are we looking for? The ***product*** of the slopes of all four sides of ABCD.
   3. Draw the picture. Keep it simple. Let’s use a rectangle with points at (0,1), (1,0), (0,-1) and (-1,0). All squares are rectangles, so this satisfies the requirement. And it’s tilted, so the sides are not parallel to the axes.

***A***

***B***

***C***

***D***

***(0,1)***

***(1,0)***

***(0,-1)***

***(-1,0)***

* 1. We’ve got our four points, so let’s get the slopes. The slopes are (y2 – y1)/(x2 – x1). Let’s work our way around the figure.
  2. Slope of AB
     + - (***0***,***1***) and (***1***,***0***)
       - (y2 – y1)/(x2 – x1)
       - (***0*** – ***1***)/(***1*** – ***0***)
       - -1/1 = -1 = Slope of AB
  3. Slope of BC
     + - (***0***,-***1***) and (***1***,***0***)
       - (y2 – y1)/(x2 – x1)
       - (***0*** – ***-1***)/(***1*** – ***0***)
       - 1/1 = 1 = Slope of BC
  4. Because ABCD is a rectangle, AB and CD are parallel, and and BC are also parallel Parallel means same slope so:
     + - Slopes of AB and CD = -1
       - Slopes of BC and = 1
  5. Now we can find the products.
     + - Slope AB × Slope BC × Slope CD × Slope
       - -1 × 1 × -1 × 1
       - 2 negative signs cancel out, and we’re left with a ***product*** of ***1***.

D is the correct answer.

*The following question starts the series of student-response questions If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. Fractions and TV.
   1. Let’s write our givens:
      * + hourlong TV show
        + 20 minutes of commercials
   2. What are we looking for? The ***fraction*** of the TV show that was ***not*** commercials.
   3. First, we need to subtract to find out how many minutes are not commercials.
      * + Total - Commercials = Non-Commercials
        + 60 min - 20 min = Non-Commercials
        + 40 min = Non-Commercials
        + A fraction is just a part/whole.
        + Non-Commercials = 40 Reduce the fraction.

Total 60

* + - * 40/60 = ***2/3***

***2/3*** is the correct answer.

1. “ Math Speak” Problem
   1. Let’s write our given:
      * + Product of 0.3 and a ***number*** is equal to 1.
   2. What are we looking for? That ***number***.
   3. Translate the “math speak.”
      * + Product of 0.3 and a ***number*** is equal to 1
        + 0.3 × ***number*** = 1
   4. Solve the equation.
      * + 0.3 × ***number*** = 1 Divide both sides by 0.3.
        + ***number*** = ***3.33***

***3.33*** is the correct answer. (or ***10/3*** if we kept it in fractions and didn’t convert to decimals.)

1. Weird function problem (For this problem we are using the symbol ***xyz*** to replace the triangle in the book.
   1. Let’s write our given: ***xyz*** = ***xy*** - ***zy***
   2. What are we looking for? The ***value*** of ***xyz*** when ***x*** = ***10***, ***y*** = ***3*** and ***z*** = ***5***
   3. Plug and chug.
      * + ***xyz*** = ***xy*** - ***zy***
        + ***xyz*** = ***103*** - ***53***
        + ***xyz*** = 1000 - 125
        + ***xyz*** = ***875***

***875*** is a correct answer.

1. A rectangle and a square MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + PQST is a rectangle.
        + URST is a square.
        + PU = 5
        + UT is a positive *integer*.
        + Area of PQST is greater than 10 and less than 30.
   2. What are we looking for? A possible ***value*** of ***UT***.
   3. To the picture. We’re looking for ***UT***, so let that be ***x***. Since URST is a square, all the sides are ***x***, and since PQST is a rectangle, QP = ST = ***x***. Mark all of that and also that PU = 5. This means QR is also 5.

***P***

***T***

***Q***

***S***

***U***

***R***

***x***

***x***

***x***

***x***

***x***

***5***

***5***

* 1. Now let’s work with the area of the rectangle. Area is length × width. In this case, PT × QP. The result of that has to be greater than 10 and less than 30. First write the equation:
     + - Area = PT × QP
       - Area = (5 + ***x***) × ***x*** It’s probably easier to just pick a value for ***x*** and see if it works Let’s try ***2***.
       - Area = (5 + ***2***) × ***2***
       - Area = 7 × ***2***
       - Area = 14 That’s greater than 10 and less than 30 ***x*** can be ***2***.

***2*** is a correct answer.

1. Proportions and balloons It’s a party.
   1. Let’s write our givens:
      * + Balloons are ***red***, ***green*** and ***blue***.
        + 1/3 of balloons are ***red***.
        + Half as many ***green*** balloons as ***red*** balloons
        + 18 balloons are ***blue***.
   2. What are we looking for? The ***total number*** of balloons in the box.
   3. Convert the “math speak” from our givens.
      * + 1/3 of balloons are ***red***
        + 1/3(***total***) = ***red***
        + Half as many ***green*** balloons as ***red*** balloons
        + ***green*** =1/2 × ***red***
        + ***blue*** = ***18***
        + ***total*** = ***red*** + ***green*** + ***blue***
   4. Plug in our expressions into the ***total*** equation.
      * + ***total*** = ***red*** + ***green*** + ***blue***
        + ***total*** = 1/3(***total***) + ½(***red***) + ***18*** Substitute 1/3(***total***) for ***red***.
        + ***total*** = 1/3(***total***) + ½( 1/3(***total***)) + ***18*** Solve for ***total***.
        + ***total*** = 1/3(***total***) + 1/6(***total***) + 18 Convert 1/3 to 2/6 so we can add those fractions.
        + ***total*** = 2/6(***total***) + 1/6(***total***) + 18
        + ***total***  = 3/6(***total***) + 18
        + ***total***  = 1/2(***total***) + 18 Subtract ½(***total***) from both sides.
        + ½(***total***) = 18
        + ½(***total***)×2 = 18 ×2
        + ***total***  = ***36***

***36*** is the correct answer.

1. Combination problem in the form of parallel lines.
   1. Let’s write our givens:
      * + P, Q and R lie on line ***l***.
        + S, T, U and V are on another line that is parallel to ***l***.
   2. What are we looking for? The ***number*** of ***lines*** that can be drawn so that each line contains only 2 of the 7 points.

***V***

***U***

***T***

***S***

***R***

***Q***

***P***

* 1. To the picture.
  2. We can draw all the lines, but let’s just look at the top line We have three points We can draw 4 lines from point P, so that also means we can draw 4 lines from points Q and R 4 lines from each of P, Q & R means:
     + - 4 × 3 = ***12 lines***.

***12*** is a correct answer.

1. Exponents Problem
   1. Let’s write our given: 2***x*** + 2***x*** + 2***x*** + 2***x*** = 27
   2. What are we looking for? The ***value*** of ***x***.
   3. Let’s simplify this a bit Combine the like terms and calculate 27.
      * + 2***x*** + 2***x*** + 2***x*** + 2***x*** = 27 There are 4 2***x***’s 27 = 128.
        + 4(2***x***) = 128 Much better Now simplify Divide both sides by 4.
        + 2***x*** = ***32***
        + We can try to do this with logs, but since it’s a simple base of 2, let’s just multiply 2’s until we get to ***32***. However many 2’s it takes to get to 32 is our answer.
        + 22 = 2 × 2 = 4
        + 23 = 2 × 2 × 2 = 8
        + 24 = 2 × 2 × 2 × 2 = 16
        + 2***5*** = 2 × 2 × 2 × 2 × 2 = ***32***
        + ***x*** = ***5***

***5*** is the correct answer.

1. Averages.
   1. Let’s write our givens:
      * + 5 people with notecards each write a ***positive*** ***integer***.
        + Average of 5 numbers is 15.
   2. What are we looking for? The ***greatest possible number*** that could be on one of the cards.
   3. We are dealing with an average here, so the formula is:
      * + Average = sum of terms

# of terms

* + - * 15 = sum of terms

5

* 1. If we are looking for the greatest number, it makes sense that we’d want the 4 other numbers to be as small as possible. The problem does not say that the numbers must be different, but they must be ***positive integers***. The smallest possible positive integer is 1. Let our ***greatest possible integer*** be ***x***.
     + - 15 = 1 + 1 + 1 + 1 + ***x*** Simplify.

5

* + - * 15 = 4+ ***x*** Multiply both sides by 5.

5

* + - * 75 = 4+ ***x*** Subtract 4.
      * ***71*** = ***x***

***71*** is the correct answer.

1. Distance proportion problem Draw a picture here.
   1. Let’s write our givens:
      * + ***Alice*** and ***Corinne*** walk 10 steps in opposite directions and stop.
        + ***Alice*** then turns around and walks ***17*** steps back to ***Corinne***.
   2. What are we looking for? The length of one of ***Alice’s*** steps compared to that of ***Corinne’s***.
   3. To the picture. ***Alice*** walks out ***10*** steps and then turns around to walk back. It takes her ***10*** steps to get back to the starting point, and only ***7*** more steps to cover the same distance as ***Corinne’s 10*** steps. Right away you can tell that Alice’s steps are longer (she needs fewer steps to cover the same distance).

***17***

***10***

***10***

***10***

***10***

***17***

***10***

***7***

***Alice – BLUE***

***Corrine - RED***

* 1. Now we can write an equation We have ***7*** steps for ***Alice*** to ***10*** steps for ***Corinne***.
     + - ***7*** × (length of ***Alice’s*** step) = ***10*** × (length of ***Corinne’s*** step)
       - Divide by ***7*** and we have our proportion.
       - length of ***Alice’s*** step = ***10*** × (length of ***Corinne’s*** step)

***7***

***10/7*** is the correct answer.

* + - * This one is confusing, and to validate that we’ve divided correctly, remember that Alice takes fewer steps once she gets back to the starting point (7) than Corinne (10) so Alice’s steps must be longer. 10/7 × the length of Corrine’s steps will give a longer length (the length of Alice’s steps), and that makes sense.

1. Friendly function to end.
   1. Let’s write our givens:
      * + f(x) = x2 + 18
        + ***m*** is a ***positive*** number.
        + f(2***m***) = 2f(***m***)
   2. What are we looking for? The ***value*** of ***m***.
   3. Remember that with functions, whatever is in the inside of the parenthesis is the input When we have a number attached to the ***outside*** of thefunction, it’s just like multiplying 2f(x) is treated the same way as 2 times any other variable Instead of a simple variable (like x or y), it is just f(x).
   4. Find the expressions for f(2***m***) and 2f(***m***) and set them equal.
   5. f(2***m***) 2***m*** is our “input” so wherever there’s an x, put a 2***m***.
      * + f(x) = x2 + 18
        + f(2***m***) = (2***m***)2 + 18
        + f(2***m***) = 4***m***2 + 18
   6. 2f(***m***) ***m*** is our “input” here.
      * + f(x) = x2 + 18
        + 2f(***m***) = 2(***m***2 + 18) We have to multiply both sides by 2, because it’s 2f(***m***) here.
        + 2f(***m***) = 2***m***2 + 36
   7. Set the two expressions equal (given) and solve for ***m***.
      * + f(2***m***) = 2f(***m***)
        + 4***m***2 + 18 = 2***m***2 + 36
        + -2***m***2 -18 -2***m***2 -18
        + 2***m***2 = 18 Divide both sides by 2.
        + ***m***2 = 9 Take the square root.
        + ***m*** = 3 or -3 We’re given that it’s a ***positive*** number.
        + ***m*** =  ***3***

***3*** is the correct answer.

# Test 9 Section 5

1. Sequence Problem
   1. Let’s write our givens:
      * + Sequence is 2, 6, 14, 30, ...
        + Each number after 1st is obtained by adding 1 to the previous number and doubling it.
   2. What are we looking for? The ***6th number*** in the sequence.
   3. We already have 4 numbers, so start with 30 and continue the pattern twice to get the ***6th number***.
      * + (30 + 1) × 2 = 62 That’s the 5th term.
        + (62 + 1) × 2 = ***126*** That’s the ***6th term***.

E is the correct answer.

1. Friends.
   1. Let’s write our givens:
      * + a(x + y) = 45
        + ***ax*** = ***15***
   2. What are we looking for? The ***value*** of ***ay***.
   3. Start by distributing the ***a*** in the first given.
      * + a(x + y) = 45
        + ***ax*** + ***ay*** = 45 We know ***ax*** = 15, so plug that in.
        + ***15*** + ***ay*** = 45
        + ***ay*** = 30 Subtract 15 from both sides and we’re done.

E is the correct answer.

1. Data problem.
   1. What are we looking for? The ***speed*** in miles per hour shown on the speedometer.
   2. Look at the miles per hour line. There are four equal spaces in between 30 and 60 and our needle is on the mark after the first space. If we figure out miles per hour of one space, we can add that to 30 and be done.
      * + 60 – 30 = 4 × space
        + 30 = 4 × space Divide by 4.
        + 7.5 = 1 space Now add 7.5 mph to 30 mph and we’ve got our speed.
        + 30 mph + 7.5 mph = ***37.5*** mph.

B is the correct answer.

1. Digit problem.
   1. Let’s write our givens:
      * + 3-digit integers
        + 4, 5 and 6 must all be used in the integer.
   2. What are we looking for? The number of ***different integers*** that can be formed using 4, 5 and 6.
   3. Start with 4 and make the list.
      * + 456 Flip the 5 and 6.
        + 465 Now put 5 first.
        + 546 Flip the 4 and 6.
        + 564 Now 6 first.
        + 645 Flip the 4 and 5.
        + 654
        + That’s ***6*** different 3-digit numbers.

C is the correct answer.

1. Prism Problem
   1. Let’s write our givens:
      * + Each rectangular face has an area of ***r***.
        + Each triangle face has an area of ***t***.
   2. What are we looking for? The ***total surface area*** of the figure in terms of ***r*** and ***t***.
   3. For surface area, we just need to add up all the faces. We already have the area of the faces in terms of ***r*** and ***t*** , we just need to count them up.
      * + There are 3 rectangles, so 3***r*** takes care of that surface area.
        + There are 2 triangles, so 2***t*** takes care of that surface area. Add them up.
        + ***total surface area*** = 3***r*** + 2***t***

B is the correct answer.

1. Friends, fractions and exponents.
   1. Let’s write our givens:
      * + ***n*** is a positive integer.
        + (***n*** + 1)/2***n*** = ½
   2. What are we looking for? The ***value*** of ***n***.
   3. We could cross-multiply and then solve, but that won’t get us too far. Let’s just start going down our answer choices and plugging in for ***n***.
      1. n = 1
         * (***n*** + 1)/2***n***  = ½
         * (***1*** + 1)/2***1*** = ½
         * 2/2 = ½
         * 1 = ½ That’s false. (A) is out.
      2. n = 2
         * (***n*** + 1)/2***n*** = ½
         * (***2*** + 1)/2***2***  = ½
         * 3/4 = ½ That’s false. (B) is out.
      3. n = 3
         * (***n*** + 1)/2***n*** = ½
         * (***3*** + 1)/2***3*** = ½
         * 4/8 = ½
         * 1/2 = ½ That’s true. (***C***) looks good. Check the others to make sure.
      4. n = 4
         * (***n*** + 1)/2***n*** = ½
         * (***4*** + 1)/2***4*** = ½
         * 5/16 = ½ False. (D) is out.
      5. n = 5
         * (***n*** + 1)/2***n*** = ½
         * (***5*** + 1)/2***5*** = ½
         * 6/32 = ½
         * 3/16 = ½ False. (E) is out.

C is the correct answer.

1. Average problem with friends.
   1. Let’s write our given: Average of the weights of ***14*** books is ***p*** pounds.
   2. What are we looking for? The ***total weight*** of the books in terms of ***p***.
   3. The formula for average is:
      * + Average = sum of terms In our case, it is:

# of terms

* + - * ***p*** = ***total weight*** Sum of weights is the same as ***total weight***, so simplify and

***14*** multiply both sides by 14.

* + - * ***14p*** = ***total weight***

E is the correct answer.

1. Lines and distance on a coordinate plane.
   1. Write the given: B is the midpoint of AC.
   2. What are we looking for? The ***value*** of ***t***.
   3. To the picture. If B is the midpoint of AC, that means that the distance between A and B is the same as the distance between B and C. Look at the x-coordinates of A and C. They are both 2. That means that this is a vertical line, so all of the distance for AC comes from the y-coordinates. We are looking for the y-coordinate of point B, which is ***t***.
   4. How do we find the middle of 2 y-coordinates? Add them up and divide by 2.
      * + 5 + -1 = 4
        + 4/2 = 2
        + ***t*** = ***2***.

C is the correct answer.

1. Friends and algebra.
   1. Let’s write our givens:
      * + ***k***(2x + 3)(x – 1) = 0
        + x > 1
   2. What are we looking for? The ***value*** of ***k***.
   3. We know this whole equation has to equal zero. We could try a couple x-values and see what happens but since x must be positive, there is no way that either of the two expressions in the parenthesis can be 0. The only option left to make the whole equation equal to zero is if ***k*** = ***0***. Try x = 2 to check it. Remember, though, we are looking for values of ***k*** ***NOT*** values of x.
      * + x = 2
        + ***k***(2x + 3)(x – 1) = 0
        + ***k*** ×(7) × (1) = 0
        + 7***k*** = 0
        + ***k*** = 0
        + There is no other answer that will work for all x > 1.

B is the correct answer.

1. Logic Problem
   1. Let’s write our given: All the men in the Williams family are over 6 feet tall.
   2. What are we looking for? The ***statement*** in the answer choices that must be ***true***.
      1. (***A***) sounds good. This is the exact same thing as all men in the family are over 6 feet tall. Check the others.
      2. Nope. This would mean that every single man on earth > 6 feet tall is a part of the Williams family.
      3. Nope. There are 6 feet tall men in other families.
      4. Nope. What about women? They could be over 6 feet tall, too.
      5. Nope. There are ***no*** men under 6 feet tall in the Williams family.

A is the correct answer.

1. Circle Problem
   1. Let’s write our given: The circle has a circumference of π.
   2. What are we looking for? The ***radius*** of the circle.
   3. Start with the formula for the circumference of a circle.
      * + Circumference = 2πr We are given the circumference is π, so plug that in.
        + π = 2πr Simplify. Divide both sides by π.
        + 1 = 2r Divide both sides by 2.
        + ***½*** = ***r***

B is the correct answer.

1. Direct Proportionality Problem
   1. Let’s write our givens:
      * + ***y*** is directly proportional to ***x***2.
        + ***y*** = ***1/8*** when ***x*** = ***½***
        + What are we looking for? The ***positive value*** of ***x*** when ***y*** = ***9/2***.
   2. Directly proportional means that the relationship between ***x*** and ***y*** is defined by the relationship below, when ***k*** is a constant.
      * + ***kx***2 = ***y*** Plug in our givens to find ***k***.
        + ***k***(***½***)2 = ***1/8***
        + ***¼k*** = ***1/8*** Multiply both sides by 4.
        + ***k*** = (***1/8***)(4)
        + ***k*** = ***½***
   3. Solve the equation for ***x*** when ***y*** = ***9/2***. Plug in the value of ***k***.
      * + ***kx***2 = ***y*** Get ***x*** by itself first. Divide both sides by ***k***.
        + ***x***2 = ***y***/***k*** Plug in values for ***y*** and ***k***.
        + ***x***2 = (***9/2***)/( ***1/2***)Dividing by a fraction is like multiplying by the reciprocal.
        + ***x***2 = (***9/2***)×(***2***)
        + ***x***2 = 9
        + ***x***  = 3 or -3. We’re looking for the positive value.
        + ***x***  = ***3***

D is the correct answer.

1. Sneaky fraction problem.
   1. Let’s write our given: 4***x*** = 6***u*** = 5***v*** = 7***w*** > 0
   2. What are we looking for? The answer choice that is a ***true*** statement.
   3. First let’s get these in order from lowest number in front of the variable to highest.
      * + 4***x*** = 6***u*** = 5***v*** = 7***w***
        + 4***x*** = 5***v*** = 6***u*** = 7***w***
   4. Think about this for a second. If 4***x*** = 5***v***, that means that ***x*** has to be bigger than ***v***, because there are more ***v***’s (5) than ***x***’s (4) and we get the same number. So that relationship is:
      * + ***x*** > ***v*** That can be continued straight up the line.
        + ***x*** > ***v*** > ***u*** > ***w*** Flip theorder as all the answers have < signs.
        + ***w*** < ***u*** < ***v*** < ***x*** is our ***true*** statement.

D is the correct answer.

1. Function Problem
   1. Let’s write our given: h(***t***) = 2(***t***3 – 3)
   2. What are we looking for? The ***value*** of 2 – 3***t*** when h(***t***) = -60.
   3. First let’s find ***t*** when h(***t***) = -60.
      * + h(***t***) = 2(***t***3 – 3)
        + -60 = 2(***t***3 – 3) Divide both sides by 2.
        + -30 = ***t***3 – 3 Add 3 to both sides.
        + -27 = ***t***3 Take cube root of both sides.
        + ***-3*** = ***t*** Take cube root of both sides.
   4. Now that we have the value of ***t***, find the value of 2 – 3***t***.
      * + 2 – 3***t***
        + 2 – 3(-***3***)
        + 2 + 9
        + ***11***

B is the correct answer.

1. Factors problem with friends.
   1. Let’s write our givens:
      * + ***x*** is divisible by 3.
        + ***y*** is divisible by 5.
   2. What are we looking for? The ***choices*** that must be divisible by 15.
      * + Pick some values for x and y. Let ***x*** = ***6*** and ***y*** = ***10***, and try the choices.

***xy***

***6***×***10***

60 60’s divisible by 15. Try ***x*** = ***3*** and ***y*** = ***15.*** They multiply to 45 (also divisible by 15). ***I*** looks good. Try II.

3***x*** + 5***y***

3(***6***)+ 5(***10***)

18 + 50

68 That’s not divisible by 15. II is out.

5***x*** + 3***y***

5(***6***)+ 3(***10***)

30 + 30

60 That’s divisible by 15. Again, try ***3*** and ***15.***

5(***3***) + 3(***15***)

15 + 45

60 Divisible by 15. ***III*** is also good.

D is the correct answer.

1. Triangle problem. MARK UP THE DRAWING!
   1. What are we looking for? The ***value*** of ***y*** + ***z***.
   2. To the drawing. Since a straight line has 180°, and we have one of the values on that line, we can find expressions for all of the angles of the triangle. 180° – 115° = 65° gives us our first angle. Fill in the values

of the other angles, ***y*** and ***z,*** written as expressions.

***y°***

***z°***

***115°***

***180° - z°***

***180° - y°***

***65°***

* + - * We have all the angles of a triangle, so let’s write an equation. The angles of a triangle sum to 180°.
      * 65° + (180° - ***z***°) + (180° – ***y***°) = 180° Simplify that equation.
      * 65° + 180° - ***z***° + 180° - ***y***° = 180°
      * 425° - ***z***° - ***y***° = 180° Subtract 425° from both sides.
      * - ***z***° - ***y***° = -245° Multiply the entire equation by -1.
      * ***z***° + ***y***° = ***245***°

E is the correct answer.

1. Integers and algebra.
   1. Let’s write our givens:
      * + The sum of 3 consecutive odd integers is 111.
        + ***n*** is the least of these integers.
   2. What are we looking for? The ***equation*** in the answer choicesthat represents the given statement.
   3. 3 consecutive odd integers that start at ***n*** all increase by 2, so write down 3 expressions for those integers.
      * + 1st integer: ***n***
        + 2nd integer: ***n*** + 2
        + 3rd integer: (***n*** + 2) + 2
   4. Now that we have expressions for all the integers, add them up and make them equal 111.
      * + ***n*** + ***n*** + 2 + (***n*** + 2) + 2 = 111 Combine like terms. We have 3 ***n***’s and 3 2’s on the left side.
        + ***3n + 6 = 111***

D is the correct answer.

1. Circumference and arcs.
   1. Let’s write our givens:
      * + Picture shown is part of a circle with circumference of 45.
        + Full circle has 18 arcs each of length ***2*** and ***b***.
   2. What are we looking for? The ***degree measure*** of each arc ***b***.
   3. Start by getting an expression for the entire circumference in terms of ***2*** and ***b***. We know that there are 18 of each so:
      * + 18(***2*** + ***b***) = 45 From that we can find the length of ***b***.
        + 36 + 18***b*** = 45
        + 18***b*** = 9
        + ***b*** = ***½***
   4. Now we know the length of ***b***, and we also have the circumference of 45. Those two measurements give a ratio that is equal to the central measure angle of arc ***b*** to the total degrees in a circle. Total degrees in a circle are 360. Set up the proportion and solve for the ***degree measure***.
      * + ***½*** = ***degree measure*** Cross multiply and simplify.

45 360

* + - * 360 × (***½***) = 45 × ***degree measure***
      * 180 = 45 × ***degree measure*** Divide both sides by 45.
      * ***4°*** = ***degree measure***

A is the correct answer.

1. Exponential Function:
   1. Let’s write our givens:
      * + $300 for car maintenance this year
        + Each year, car maintenance goes up 10%.
        + Cost of car maintenance is ***c***.
        + ***n*** is the number of years from now.
        + Function for cost of car maintenance is ***c***(***n***) = 300***xn.***
   2. What are we looking for? The value of ***x***.
   3. The best way to attack this is to look ahead 1 year and solve the function. We know that in 1 year, the cost of maintenance will be 10% higher. Forget about the function for a second, and just calculate the maintenance cost in one year: Cost in 1 year = cost this year + 10%. That’s the same thing as:
      * + Cost in 1 year = cost this year × (1.10%) We know the cost this year ($300).
        + Cost in 1 year = $300 × (1.10%)
        + Cost in 1 year = $***330***
   4. We know our cost in 1 year, so see if we can use the function to get an ***x*** that will give us the same $330 cost.
      * + ***c***(***n***) = 300***xn***
        + ***330*** = 300***x1*** ***x1*** is just ***x***.
        + ***330*** = 300***x*** Divide both sides by 300.
        + 330/300 = ***x***
        + ***1.1*** = ***x***

C is the correct answer.

1. Tricky special triangles to finish. MARK UP THE PICTURE!
   1. Let’s write our given: All the line segments in the figure are congruent (equal length).
   2. What are we looking for? The ***ratio*** of ***AC*** to ***BD***.
   3. To the picture.
      1. If all of the line segments are the same, that means that ∆ABD and ∆BCD are equilateral and congruent. So, all those angles of the triangles are 60°.
      2. Also, the line ***AC*** cuts ∠A and ∠C in half, so each is 30°. That means that the central angles are all 90° because a triangle has 180°.
      3. Mark the angles on the picture.
      4. We’ve also named the center point E to make things easier on us.

***60°***

***A***

***B***

***C***

***D***

***60°***

***60°***

***60°***

***30°***

***30°***

***30°***

***30°***

***90°***

***90°***

***E***

***30°***

***x***

***x√3***

***2x***

***60°***

***90°***

* 1. Triangles with angles of 30°-60°-90° are special triangles, and their sides follow the formulas given in the picture on the right above. We can use those expressions and fill in the lengths of the sides of ∆ABE and ∆BCE. Remember, we’re looking for the ***ratio*** of ***AC*** to ***BD***, not necessarily the exact lengths.

***60°***

***A***

***B***

***C***

***D***

***60°***

***60°***

***60°***

***30°***

***30°***

***30°***

***30°***

***90°***

***90°***

***E***

***x√3***

***x√3***

***x***

***x***

***2x***

* 1. We basically have 4 congruent triangles in here. We marked AB as 2x, but we really don’t need that for our ratio. What we need are the ***x***’s and the ***x√3***’s that we filled in for the pieces of ***AC*** and ***BD***. Now we need to set up the ratio of ***AC*** to ***BD***.
     + - ***AC*** = x√3 + x√3

***BD*** x + x

* + - * ***AC*** = 2x√3 The 2’s and ***x***’s cancel each other out.

***BD*** 2x

* + - * ***AC*** = ***√3***

***BD*** ***1***

B is the correct answer.

# Test 9 Section 8

1. A friend to start with.
   1. Let’s write our given: 6,700 = 100(6***k*** + 7)
   2. What are we looking for? The ***value*** of ***k***.
   3. Simplify and get ***k*** by itself. Divide by 100 first, because that is easier than distributing the 100 first.
      * + 6,700 = 100(6***k*** + 7)

100 100

* + - * 67 = 6***k*** + 7 Subtract 7 from both sides.
      * 60 = 6***k*** Divide both sides by 6.
      * ***10*** = ***k***

C is the correct answer.

1. Negative and positive algebra.
   1. Let’s write our givens:
      * + 3 more than ***n*** is negative.
        + 5 more than ***n*** is positive.
   2. What are we looking for? A ***possible value*** of ***n***.
   3. First, notice that we have both positive and negative answers. If ***n*** were positive, how could we *add* to a positive number and satisfy our given “3 more than ***n*** is negative.” We couldn’t. So, ***n*** has to be negative. Eliminate (D) and (E).
   4. Let’s try the other answer choices.
      1. n = -5
         * -5 + 3 = -2 That’s negative.
         * -5 + 5 = 0 That’s not positive. It’s supposed to be. (A) is out.
      2. n = -4
         * -4 + 3 = -1 That’s negative.
         * -4 + 5 = 1 That’s positive. (***B***) is the winner. Try (C) just to make sure.
      3. n = -3
         * -3 + 3 = 0 That’s not negative. It’s supposed to be. (C) is out.

B is the correct answer.

1. Angles. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + ***x*** = 70
        + ***y*** = 40
        + The dotted lines bisect the ***x***° and ***y***° angles.
   2. What are we looking for? The ***value*** of ***z***.
   3. To the picture! Bisect means cut in half, so mark those angles.

***x°***

***y°***

***z°***

* 1. ***z*** is made up of half the value of ***x*** and half the value of ***y*** because the dotted lines bisect those angles, and ***z*** stretches from one dotted line to the other. Set up the equation and solve:
     + - ***z***° = ½(***70***°) + ½(***40***°)
       - ***z***° = 35° + 20°
       - ***z***° = ***55***°

E is the correct answer.

1. Probability. Kind of.
   1. Let’s write our givens:
      * + Piece of fruit is pulled at random.
        + Probability that it is an apple is 2/5.
   2. What are we looking for? The ***number*** that ***cannot*** be the total pieces of fruit in the basket.
   3. Just look at the answers. They’re all divisible by 5 except for one, letter (***C***). Pretty good bet that (C) is our answer.
   4. Write the equation for a probability.
      * + Probability = Specific Outcome

Total Possible Outcomes

* + - * In this case = Apple = 2

Total Fruits 5

* + - * Our “Total Fruits” must be a multiple of 5, and ***52*** clearly is not.

C is the correct answer.

1. Square and triangle perimeter problem.
   1. Let’s write our givens:
      * + Square and equilateral triangle have equal perimeters.
        + Square has sides of length 3.
   2. What are we looking for? The ***length*** of one side of the triangle.
   3. Calculate the perimeter of the square. Add up the 4 sides of length 3.
      * + 3 + 3 + 3 + 3 = 12
   4. So now we know that the triangle also has a perimeter of 12. And all the sides are equal. Divide 12 by 3 to get the length of one side.
      * + 12/3 =  ***4***

C is the correct answer.

1. Friends.
   1. Let’s write our givens:
      * + ***x*** = ***-1***
        + ***k*** > 0
   2. What are we looking for? The ***answer*** with the ***greatest*** value.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfStart by picking a value for ***k***. Remember, ***k*** > 0 so let’s say ***k*** = ***1***.
   4. Before we start going through the answers, look at ***x***. It’s ***-1***, and in our answers ***x*** has an exponent. Any exponent that is even is going to give us a number with a positive value. If the exponent is odd, the result is going to be negative. So, we can get rid of answers (A), (C) and (E), which will all have negative values. Now go through (B) and (D) and see which one is greater.
      1. 4***kx***2
         * 4(***1***)(***-1***)2 = 4
      2. 8***kx***4
         * 8(***1***)(***-1***)4 = 8 ***8*** is the greatest value.

D is the correct answer.

1. Data Analysis Problem
   1. Let’s write our givens:
      * + Josephine swims, bikes and runs, in that order.
        + She runs faster than she swims.
        + She bikes faster than she runs.
   2. What are we looking for? The ***graph*** that best shows her distance covered during her exercise routine.
   3. This problem is all about slopes and rates of change. Time is on the x-axis of the graph, and distance is on the y-axis of the graph. If we are comparing two activities, the faster one is going to be steeper on the graph. Why? Because a steeper line means that she’s covering more distance in the same amount of time. That being said, if she runs faster than she swims and bikes faster than she runs, then the slopes, in order of steepest to flattest, are going to be: biking > running > swimming
   4. But the order of activity, which the graph represents, is swimming, then biking, then running. Start with swimming.
      * + Swimming: This is the flattest slope (meaning she covers less distance in a certain amount of time than the other activities). That means (B), (C) and (D) are out because in all of those, the first part of the line, the swimming part, is steeper than one or both of the other segments. We’re left with (A) and (E). Move on to Biking.
        + Biking: Both (A) and (E) have the same slopes for the second segment (and the first, too). Move on to Running.
        + Running: (A) has running as the steepest (or fastest) activity. We know that’s not true, because the given says she bikes faster than she runs. (A) is out. (***E***) is the only graph that has the running faster than swimming and the biking faster than running.

E is the correct answer.

1. Coordinate plane problem with intersecting functions. It’s not as bad as it sounds.
   1. Let’s write our givens:
      * + (***√6***,***k***) is a point of intersection of the following functions.
          - ***y*** = ***x***2 - 7
          - ***y*** = -***x***2 + ***j***
        + ***j*** is a constant.
   2. What are we looking for? The ***value*** of ***j***.
   3. Let’s start by finding a value for ***k*** using the first equation. In the point (***√6***,***k***), ***√6*** is the ***x*** -value and ***k*** is the ***y***-value. Plug that into the first equation and we’ll know ***k***.
      * + ***y*** = ***x***2 - 7
        + ***k*** = (***√6***)2 – 7 Square the ***√6***.
        + ***k*** = 6 - 7
        + ***k*** = ***-1***
        + (***√6***, ***-1***) is the point of intersection.
   4. Plug that same point into the second function to find ***j***, because it is the intersection of the two functions, meaning it works in both equations.
      * + ***y*** = -***x***2 + ***j***
        + ***-1*** = -(***√6***)2 + ***j*** Remember PEMDAS; exponents come ***before*** subtraction (the minus).
        + ***-1*** = -6 + ***j***
        + ***-1*** = -6 + ***j***
        + ***5*** = ***j***

A is the correct answer.

1. Absolute value ***and*** an inequality. Wooooo.
   1. Let’s write our given: |2 – ***x***| < 3
   2. What are we looking for? A ***possible value*** of ***x***.
   3. We could write two equations and solve them, but it’s easier to just plug and chug through the answers.
      1. ***x*** = ***4***
         * |2 – ***x***| < 3
         * |2 – ***4***| < 3
         * |-2| < 3
         * 2 < 3 That is true. (A) works, ***x*** can be ***4***.

A is the correct answer.

1. Geometry Problem
   1. Let’s write our given: All the interior angles of the polygon are congruent (the same).
   2. What are we looking for? The ***value*** of ***x***.
   3. First count up the angles. There are 5. When all the interior angles are the same (a regular polygon) the angle formed by continuing one edge of the figure past the point is called an exterior angle. In a regular polygon, the measure of an exterior angle = 360°/(# of interior angles)\*. Plug and chug.
      * + exterior angle = 360°/(# of interior angles)
        + ***x*** = 360/5
        + ***x*** = ***72***

C is the correct answer.

*\*Know this rule! It’s not in the chart at the beginning of the sections.*

1. Word Problem
   1. Let’s write our givens:
      * + The drawing is 3/8 the length of the actual tool.
        + The drawing is 6 inches long.
   2. What are we looking for? The ***actual length*** of the tool.
   3. Let the ***actual length*** be ***x***. Set up the equation and plug and chug.
      * + 3/8(***x***) = drawing length
        + 3/8(***x***) = 6 Multiply both sides by 8/3.
        + ***x*** = 6 × 8/3
        + ***x*** = 48/3
        + ***x*** = ***16***

C is the correct answer.

1. Integer Problem
   1. Let’s write our given:
      * + (***x*** + 3)/2 is an integer.
   2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfWhat are we looking for? The answer choice that represents what ***x*** must be.
   3. Think about this. We’re dividing by 2. What kinds of numbers are always divisible by 2? ***Even*** numbers, so the top of our fraction must be an ***even*** number. We don’t really care if it’s positive or negative, because integers can be both positive and negative. We just need the top to be even. How do we make sure the top of the fraction is always even? ***x*** must be an ***odd number***. Try it.
      * + ***x*** = ***3***
        + (***x*** + 3)/2
        + (***3*** + 3)/2
        + 6/2
        + 3 Integer. Good to go. What about a negative ***x***?
        + ***x*** = ***-5***
        + (***-5*** + 3)/2
        + -2/2
        + -1 Integer. Good to go.

E is the correct answer.

1. Circles on a coordinate plane. MARK UP THE PICTURE!
   1. Let’s write our given: Q and S are the centers of 2 circles that are tangent to the x-axis.
   2. What are we looking for? The ***slope*** of line QS.
   3. We can find the slope of a line if we have 2 points of that line. We have two points, Q and S, so if we find the coordinates of those, we can find the slope:
      * + Slope of a line = (y2 – y1)/(x2 – x1) And in our case:
        + ***slope*** of QS = (yS – yQ)/(xS – xQ)
   4. Look at the diameters of the 2 circles. They form right angles with the x-axis. That means that the diameters are vertical lines, so Q is going to have the same x-coordinate as P (3), and S is going to have the same x-coordinate as R (11).
   5. Now we need the y-values of Q and S. Since the diameter of circle Q is vertical, the length of the diameter comes completely from the y-coordinate of point P (6). The center of a circle is at the exact midpoint of the diameter, so the y-coordinate of Q is going to be 3. By the same logic, Point S is going to have a y-coordinate of 5. We now have two points:
      * + Q: (3,3)
        + S: (11,5)
   6. Use those two points to find the ***slope*** of QS.
      * + ***slope*** = (yS – yQ)/(xS – xQ)
        + ***slope*** = (5 – 3)/(11 – 3)
        + ***slope*** = 2/8
        + ***slope*** = ***1/4***

B is the correct answer.

1. Tough factor problem.
   1. Let’s write our givens:
      * + ***n*** and ***p*** are integers greater than 1.
        + ***p*** is a factor of both ***n*** + 3 and ***n*** + 10.
   2. What are we looking for? The ***value*** of ***p***.
   3. This is a tough one, and one where we just have to start working without knowing exactly where we’re going. Instead of starting with ***p***, we need to start with ***n*** and try some numbers. We know that ***n*** > 1, so let’s try ***n*** = ***2***, and see if we can find a ***p*** that is both an integer greater than 1 and an integer factor of both ***n*** + 3 and ***n*** + 10.
      * + ***n*** = ***2***
        + ***n*** + 3 ***n*** + 10
        + ***2*** + 3 ***2*** + 10
        + 5 12
        + There’s no way ***p*** can be a factor of both 12 and 5 and still be an integer (whole number). Try ***n*** = ***3***.
        + ***n*** = ***3***
        + ***3*** + 3 ***3*** + 10
        + 6 13
        + Same thing here. ***p*** can’t be a factor of both 13 and 6 and be an integer (whole number). Try ***n*** = ***4***.
        + ***n*** = ***4***
        + ***4*** + 3 ***4*** + 10
        + 7 14
        + We have something here. Both 14 and 7 are divisible by ***7*** , which is an integer, and it is greater than 1. So ***p*** = ***7***.

B is the correct answer.

1. A tough angles problem.
   1. Let’s write our givens:
      * + The figure is a cube.
        + Points B, C and E are midpoints of three edges.
   2. What are we looking for? The ***angle*** with the ***least measure***.
   3. To explain this problem, let’s look at two simple right triangles that have one side that is the exact same length (x). The 2 remaining sides in one of the triangles are longer than the 2 remaining sides in the other. We are trying to see the relationship between the angles and the lengths of the sides. We’re not going to give them lengths, we’re just going to look at them.

***x***

***x***

In the triangle on the left, the 2 “other sides” (i.e., not x) are shorter, and the ***red*** angle is bigger. In the triangle on the right, the 2 “other sides” are longer, and the ***blue*** angle is smaller. This same logic can be used in our problem. The angle with the interior point (A, B, C, D or E) furthest from line XY will have the smallest angle, which is the ***angle*** with the ***least measure***. Out of those 5 points, which one is furthest from the line XY? D is all the way across the base of the cube, so XD and YD are the longest “other sides” of our triangles. So, ***∠XDY*** is going to have the ***angle*** with the ***least measure***.

D is the correct answer.

1. Ugly friends to finish.
   1. Let’s write our givens:
      * + ***xy*** = 7
        + ***x*** – ***y*** = 5
   2. What are we looking for? The ***value*** of ***x***2***y*** – ***xy***2.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfWe could simplify the second given and plug it into the first given, but let’s take another look at what we’re trying to evaluate, the ***x***2***y*** – ***xy***2. Let’s try to simplify this. There are some common terms that we may be able to factor out.
      * + ***x***2***y*** – ***xy***2 Divide both terms by ***xy.*** Notice that ***xy*** = 7 (given). We’ll come back to this.
        + ***x*** – ***y***Notice that this equals 5 (given). And how did we get here? We divided by ***xy***. So if we go back and multiply *this* by ***xy***, we’ll have the equivalent of ***x***2***y*** – ***xy***2  in simpler form.
        + ***xy***(***x*** - ***y***) Looks like we’re multiplying our givens.
        + (7)(5)
        + ***35***

D is the correct answer.

# Test 10 Section 2

1. Price changes.
   1. Let’s write our givens:
      * + Total cost of 3 equally priced pencils is $4.50.
        + Cost if each pencil is increased by $0.50.
   2. What are we looking for? How much ***5 pencils*** will ***cost*** at the new price.
   3. First we need to find out the original cost of each pencil.
      * + $4.50 total cost

3 pencils

* + - * $1.50 per pencil = old cost
  1. New cost is higher by $0.50.
     + - $1.50 + $0.50 = $2.00 = new cost
  2. 5 pencils at new cost is:
     + - (# of pencils) × (new cost of pencils)
       - 5 × $2.00
       - ***$10.00***

E is the correct answer.

1. Friends and a relationship.
   1. What are we looking for? The ***linear equation*** that describes the relationship between ***x*** and ***y*** as shown in the table.
   2. Pick a pair, say ***x*** = ***1*** and ***y*** = ***3***, and start going through the answers to see if we can eliminate any of them.
      1. ***y*** = ***x*** + 1
         * ***3*** = ***1*** + 1
         * ***3*** = 2 That’s not a true statement. (A) is out.
      2. ***y*** = ***x*** + 4
         * ***3*** = ***1*** + 4
         * ***3*** = 5 Nope. (B) is out.
      3. ***y*** = 3***x***
         * ***3*** = 3(***1***)
         * ***3*** = 3 That works. (C) could be our answer, but check the others.
      4. ***y*** = 4***x***
         * ***3*** = 4(***1***)
         * ***3*** = 4 Nope. (D) is out.
      5. ***y*** = 4***x*** - 1
         * ***3*** = 4(***1***)- 1
         * ***3*** = 3 That works. (E) could also be the answer.
   3. So we’ve eliminated (A), (B) and (D). We need to pick another pair and test (C) and (E) to see which one works for another pair of ***x*** and ***y***. Let’s try ***2*** and ***7***.
      1. ***y*** = 3***x***
         * ***7*** = 3(***2***)
         * ***3*** = 6 That doesn’t work. (C) is out. Try (E).
      2. ***y*** = 4***x*** - 1
         * ***7*** = 4(***2***)- 1
         * ***7*** = 8 - 1
         * ***7*** = ***7*** That works. (***E***) works for ***1*** and ***3*** as well as ***2*** and ***7***.

E is the correct answer.

1. Circumferences.
   1. Let’s write our givens:
      * + The two circles are tangent at point B.
        + AC = 6
        + Circumference of circle A is twice the circumference of circle C.
   2. What are we looking for? The ***length*** of ***BC***.
   3. ***BC*** is the radius of the smaller circle C. We know that radius and circumference are directly proportional. That means if we take the circumference of a circle and multiply it by 2, then the radius of that circle will also be 2 times greater. The same thing works in reverse. (This only works for circumference, diameter and radius calculations, NOT AREA).
   4. This means we know circle A’s circumference is twice that of circle C’s. That also means we know that:
      * + radius of circle A is twice that of circle C.
        + AB = 2 × ***BC***
   5. Don’t we know what AB + BC is? Yes, it’s 6. And they told us the circles were tangent, so we know AC is a straight line. Set up the equation, substitute in for AB, and solve for ***BC***.
      * + AB + ***BC*** = 6 Substitute in the above expression for AB.
        + 2***BC*** + ***BC*** = 6 Solve for BC.
        + 3***BC*** = 6 Divide both sides by 3.
        + ***BC*** = ***2***

B is the correct answer.

1. Coordinate plane work.
   1. What are we looking for? The ***point*** that has coordinates (***x***, ***y***) that satisfy: |***x***| - |***y***| = 3
   2. First we need to label our points. This is simple counting. Then we can plug the values into our equation.
      * + Point A: (***-3***, ***-6***)
          - |***x***| - |***y***| = 3
          - |***-3***| - |***-6***| =
          - 3 - 6 = -3 That’s not 3. (A) is out.
        + Point B: (***-4***, ***-1***)
          - |***x***| - |***y***| = 3
          - |***-4***| - |***-1***| =
          - 4 - 1 = 3 That’s it. (***B***) works. Check the others.
        + Point C: (***-2***, ***5***)
          - |***x***| - |***y***| = 3
          - |***-2***| - |***5***| =
          - 2 - 5 = -3 That’s not 3. (C) is out.
        + Point D: (***4***, ***3***)
          - |***x***| - |***y***| = 3
          - |***4*** | - |***3***| =
          - 4 - 3 = 1 That’s not 3. (D) is out.
        + Point E: (***3***,- ***5***)
          - |***x***| - |***y***| = 3
          - |***3***| - |-***5***| =
          - 3 - 5 = -2 That’s not 3. (E) is out.

B is the correct answer.

1. Data analysis with a pie chart.
   1. Let’s write our givens:
      * + Survey of 1,000 people asked their ages.
        + Survey results are divided into 4 categories of age groups.
        + The age answer they gave is ***x***.
   2. What are we looking for? The number of people that said their age (***x***) was **less than 40**.
   3. First, we can eliminate 2 categories immediately: ***x*** ≥ 60 and 40 ≤ ***x*** < 60. These are out because people in these groups are clearly 40 or older.
   4. We’re left with ***x*** < 20 (***30%***) and 20 ≤ ***x*** < 40 (***20%***), which make up our less than 40 crowd. First, we’ll

add the 2 percentages of those categories and then we’ll multiply the result by the total people surveyed.

* + - * ***30%*** + ***20%*** = ***50%*** less than 40
      * ***50%*** × (total people surveyed) = ***people*** less than 40
      * ***50%*** × (1,000) = ***people*** less than 40
      * ***500*** = ***people*** less than 40

D is the correct answer.

1. Remainder problem. Times 4.
   1. Let’s write our given: 4 consecutive positive integers are divided by 3.
   2. What are we looking for? A possible list of ***remainders*** for the above given.
   3. What is a remainder? It’s the amount left over in division. Here’s an example:

What’s the remainder when we divide 5 by 3?

1 r ***2*** ***2*** is the remainder in this example.

3|5

* 1. Back to our problem. Write the skeleton of the problem, in this case just the division sign.

|

* 1. Now let’s fill it in with the given information. We just need to pick a starting positive integer. Where to start? Since we’ll be dividing by 3, let’s start with a positive integer that is at least 3 to make it easy for us.
     + - Starting with ***3***.

1 r ***0*** We’re dividing by ***3***. The ***0*** is the remainder.

***3***|***3***

* + - * Now try ***4***.

1 r ***1***

***3***|***4***

* + - * Now try ***5***.

1 r ***2***

***3***|***5***

* + - * Now try ***6***.

2 r ***0***

***3***|***6***

It looks like (D) might be our answer, but let’s do a couple more just to make sure.

* + - * Now try ***7***.

2 r ***1***

***3***|***7***

* + - * Now try ***8***.

2 r ***2***

***3***|***8***

* 1. There’s the pattern. ***0***, ***1***, ***2***, ***0***

D is the correct answer.

1. Inverse proportions.
   1. Let’s write our givens:
      * + ***y*** is inversely proportional to ***x***.
        + ***y*** = ***15*** when ***x*** = ***5***
   2. What are we looking for? The ***value*** of ***y*** when ***x*** = ***25***.
   3. We have to remember our proportion rules. Inversely proportional means that when x goes up, y goes down, and vice versa. The relationship is ***y*** = ***k***/***x*** where ***k*** is a constant. Memorize this!
   4. We first need to find ***k*** by using our given information. Plug it in.
      * + ***y*** = ***k***/***x***
        + ***15*** = ***k***/***5*** Multiply both sides by 5 to get ***k***.
        + ***75*** = ***k***
   5. So our equation is now ***y*** = ***75***/***x***. Plug in ***25*** for ***x*** to see what ***y*** is.
      * + ***y*** = ***75***/***x***
        + ***y*** = ***75***/***25***

***y = 3*** This makes sense because ***x*** went up and ***y*** went down. That’s inversely proportional.

C is the correct answer.

1. System of equations with 3 friends.
   1. Let’s write our givens:
      * + 2***x*** + ***z*** = 2***y***
        + 2***x*** + 2***y*** + ***z*** = 20
        + What are we looking for? The ***value*** of ***y***.
   2. Usually with systems we need as many equations as variables, which we don’t have here. Let’s start by getting the first given’s friends all on one side of the equation so it matches the second given at least in terms of order.
      * + 2***x*** + ***z*** = 2***y*** Subtract 2***y*** from both sides. We’re left with 0 on the right side.
        + 2***x*** + ***z*** - 2***y*** = 0 Now put it in alphabetical order. Switch the ***z*** term and the ***y*** term.
        + 2***x*** - 2***y*** + ***z*** = 0
   3. Now put the re-arranged 1st given and the 2nd given together.
      * + 2***x*** - 2***y*** + ***z*** = 0
        + 2***x*** + 2***y*** + ***z*** = 20
   4. If we multiply the second equation by -1, and add the two equations together, does anything fall out?
      * + 2***x*** - 2***y*** + ***z*** = 0
        + -2***x*** - 2***y*** - ***z*** = -20
        + - 4***y*** = -20
        + The ***x***’s and ***z***’s fell out and we’re left with the ***y***, which is what we’re looking for. Solve for ***y***.
        + -4***y*** = -20 Divide both sides by -4.
        + ***y*** = ***5***

A is the correct answer.

*The following question starts the series of student-response questions. If you don’t know the answer, guess, because you lose no points for an incorrect “write-in” answer.*

1. A simple friend.
   1. Let’s write our given: 2(***x*** – 3) = 7
   2. What are we looking for? The ***value*** of ***x***.
   3. Solve for ***x***.
      * + 2(***x*** – 3) = 7 Distribute that 2.
        + 2***x*** – 6 = 7 Add 6 to both sides.
        + 2***x*** = 13 Divide both sides by 2.
        + ***x*** = ***13/2*** Divide both sides by 2.

***13/2*** is the correct answer. (or ***6.5***)

1. Coordinate Problem
   1. Let’s write our givens:
      * + Point P is on the line ***y*** – 4 = 3(***x*** – 2).
        + The ***x*** -coordinate of P is ***4***.
   2. What are we looking for? The ***y***-value of point P.
   3. Plug in ***4*** for ***x*** in the equation of the line.
      * + ***y*** – 4 = 3(***x*** – 2)
        + ***y*** – 4 = 3(***4*** – 2)
        + ***y*** – 4 = 3(2)
        + ***y*** – 4 = 6 Add 4 to both sides.
        + ***y*** = ***10***

***10*** is the correct answer.

1. Rate Problem.
   1. Let’s write our givens:
      * + Car ***A*** traveled 60 miles.
        + Car ***A*** averaged 20 miles per gallon (MPG).
        + Car ***B*** averaged 15 miles per gallon (MPG).
   2. What are we looking for? ***How far Car B had gone*** when it had used the same amount of gas as A used to go 60 miles. Huh?
   3. First we need to figure out the amount of gas (gallons) that A used to go 60 miles.
      * + Miles/Gallons = MPG This can be re-arranged.
        + Miles/MPG = Gallons
        + ***60*** miles /20 MPG = 3 Gallons for Car ***A***
   4. Now we have to find the distance Car ***B*** traveled when it had used that same amount of gallons (3 gallons). We need to multiply Car ***B***’s Gallons by MPG to get the total miles ***B*** traveled.
      * + 3 gallons x 15 MPG = ***45*** miles traveled by Car ***B***

***45*** is the correct answer.

1. Funny four-sided shape and a line. Mark up the picture.
   1. Let’s write our givens:
      * + A, D and E are on a line.
        + ABCD is a four-sided figure.
   2. What are we looking for? The ***value*** of ***x***.
   3. To the picture. We know that ADE is a straight line, and a straight line has 180°, so ∠ADC = 180° – ***x***°.

***A***

***B***

***C***

***D***

***E***

***100°***

***65°***

***120°***

***180°- x°***

***x°***

* 1. We also know that the interior angles of a 4-sided figure add up to 360°. Always. Now we can write an equation for the angles of ABCD and solve for ***x***.
     + - 360° = 65° + 100° + 120° + 180° -  ***x*** °
       - 360° = 465° -  ***x*** °
       - -105° = - ***x*** °
       - 105° = ***x*** °

***105*** is the correct answer. Don’t write in the ° sign in the answer.

1. Sequence Problem
   1. Let’s write our givens:
      * + 1st term is 20.
        + 2nd term is 8.
        + 3rd and each term thereafter is the ***average*** of the ***previous 2 terms***.
   2. What are we looking for? The ***value*** of the term that you first come across that is ***not an integer***.
   3. Integer means whole number. So we’re looking for the first term that’s not a whole number. Let’s first find the 3rd term. It’s the average of 20 and 8. Average = (sum of the terms)/(number of terms).
      * + 3rd term = (1st + 2nd)/2
        + = (20 + 8)/2
        + = 28/2
        + 3rd term = 14 That’s an integer. Find the 4th term.
        + 4th term = (2nd + 3rd)/2
        + = (8 + 14)/2
        + = 22/2
        + 4th term = 11 That’s an integer. Keep going.
        + 5th term = (3rd + 4th)/2
        + = (14 + 11)/2
        + 5th term = ***25/2*** Not an integer.

***25/2*** is the correct answer. (or ***12.5***)

1. Fractions and friends.
   1. Let’s write our givens:
      * + ***x*** is 1/5 of ***y***.
        + ***y*** is 3/10 of ***z***.
        + ***z*** > 0
   2. What are we looking for? ***x*** is ***what fraction*** of ***z***.
   3. Convert the ‘math speak” in the givens. We ultimately want to get the ***y*** by itself in both problems so that we’re left with only ***x*** and ***z***. Let’s start with ***x***.
      * + ***x*** is 1/5 of ***y***
        + ***x*** = (1/5)***y*** Now multiply both sides by 5.
        + 5***x*** = ***y***
        + Now for the second given.
        + ***y*** is 3/10 of ***z***
        + ***y*** = (3/10)***z***
   4. Now we have two expressions for ***y***. Set them equal to each other.
      * + 5***x*** = ***y***
        + ***y*** = (3/10)***z***
        + 5***x*** = (3/10)***z***
        + Again, what are we looking for? We want to know the fraction of ***z*** that ***x*** is. That means we need to get ***x*** by itself in the equation. Divide both sides by 5.
        + 5***x*** = (3/10)***z***  Dividing by 5 is just like multiplying by 1/5.
        + 5 5
        + ***x*** = 3***z*** × 1

10 5

* + - * ***x*** = (***3***)***z***

***50***

* + - * ***x*** is ***3/50*** of ***z***.

***3/50*** is a correct answer. (or ***.06***)

1. Geometry. A square and a special triangle. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + EBCD is a square.
        + AE = 8
   2. What are we looking for? The ***area*** of EBCD.
   3. To the picture. We know that the triangle has a right angle (90°) and a 60° angle. The other one must be 30°. That’s a 30-60-90 triangle, and it has specific rules regarding the lengths of its sides. Let’s focus on the triangle first. If we find BE, we will know one edge of the square. The short side of a 30-60-90 triangle is ***x***, and in our case ***x*** = ***8***. The other leg in this triangle, BE, is going to be:

***60°***

***A***

***B***

***E***

***30°***

***8***

***8√3***

***C***

***D***

* + - * BE = ***x***√3
      * BE = ***8***√3
  1. Now we have a side of our square. Area of a square is side2, or in this case BE2.
     + - BE = ***8***√3
       - Area of EBCD = BE2
       - Area of EBCD = (***8***√3)2
       - Area of EBCD = 64 × 3
       - Area of EBCD = ***192***

***192*** is the correct answer.

1. System of equations with a proportion wrinkle.
   1. Let’s write our givens:
      * + Mixture of ***cashews*** and ***peanuts***
        + Ratio of weight of ***peanuts*** to ***cashews*** is ***5*** to ***2***.
        + Mixture is 4 pounds total.
   2. What are we looking for? ***The number of pounds*** of ***cashews*** in the mixture.
   3. First let ***cashews*** be ***c*** and ***peanuts*** be ***p***. Now we need to set up some equations.
      * + The total weight, or ***cashews*** and ***peanuts***, is four pounds.
        + ***c*** + ***p*** = 4
        + Ratio of ***peanuts*** to ***cashews*** is ***5*** to ***2***.
        + ***p***/***c*** = ***5***/***2***
   4. Now we’ve got 2 friends and 2 equations. We’re ready to solve. We’ll do this in two steps. Since we’re looking for the pounds of cashews, let’s first isolate ***p*** in our 2nd equation, which will leave us with ***c.*** Then we’ll substitute ***c*** into the 1st equation.
      * + ***p***/***c*** = ***5***/***2*** Multiply both sides by ***c***.
        + ***p*** = 5/2(***c***)Now plug that expression into ***p*** in the 1st equation.
        + ***c*** + ***p*** = 4
        + ***c*** + 5/2(***c***) = 4 Get a common denominator for the ***c*** terms (2).
        + 2/2(***c***) + 5/2(***c***) = 4 Simplify.
        + 7/2(***c***) = 4 Multiply both sides by 2/7.
        + ***c***  = 4 × 2/7
        + ***c***  = ***8/7***

***8/7*** is the correct answer. (or ***1.14***)

1. Slope problem. MARK UP THE PICTURE!
   1. Let’s write our given: Line ***m*** passes through the origin (O) and cuts AB somewhere between A and B.
   2. What are we looking for? A ***possible slope*** of ***m***.
   3. To the picture. First, draw in line ***m***.

***A***

***B***

***l***

***y***

***O***

***m***

***(8,3)***

***x***

* 1. Next, look at OB. That’s a horizontal line. The slope of a horizontal line is what? Zero. Always zero. So the slope of ***m*** has to be greater than zero.
  2. Now look at line l. We can find the slope of that line because we have 2 points: (0,0) and (8,3). Slope is rise/run, or (y2 – y1)/(x2 – x1).
     + - Slope OA = (y2 – y1)/(x2 – x1)
       - Slope OA = (3 – 0)/(8 – 0)
       - Slope OA = 3/8
       - Now we have two boundaries for our slope: 0 on the low side, and 3/8 on the high side. Pick a positive fraction less than 3/8 and we’re good. How about ***1/8***. Perfect.

***1/8*** is a correct answer. ( any answer greater than 0 and less than 3/8 or .375 is correct.)

1. Data Analysis and a mystery median to end the section.
   1. Let’s write our givens:
      * + 5 years of enrollment data
        + ***x*** is 1992 enrollment.
        + The ***median*** over the 5 years is ***1351***.
        + No two years have the same enrollment.
   2. What are we looking for? The ***greatest possible value*** of ***x***.
   3. A median is the number in the middle when all the numbers are placed in order from least to greatest. So let’s put the known enrollment figures in order from least to greatest:
      * + 1238 ***1351*** 1459 1552 (and an ***x*** somewhere in there)
   4. If ***1351*** is the median of all 5 numbers, ***x*** must be on the left-side of the sequence, or less than ***1351***. What’s the closest we can get to ***1351*** without having two years with an enrollment of ***1351***? ***1350***.

***1350*** is the correct answer.

# Test 10 Section 5

1. Sequence problem.
   1. Let’s write our given: ***x*** = 39

***x*** - 2 37

* 1. What are we looking for? The ***value*** of ***x***.
  2. We can cross-multiply this thing and then find x, but when two fractions are equal, the numerators (tops) of both of them have to be equal and so do the denominators (bottoms). Set the tops of the fractions equal.
     + - ***x*** = ***39*** That’s it. No work to do here.

B is the correct answer.

1. Table Analysis.
   1. Let’s write our given: Each letter represents the number of students in the category.
   2. What are we looking for? The ***expression*** that must equal ***z***. ***z*** is the total and includes all categories.
   3. Start going through the answers.
      1. k + s That leaves out junior girls and senior boys. (A) is out.
      2. m + x That leaves out junior girls. (B) is out.
      3. r + s That leaves out all boys. (C) is out.
      4. r + s + t That leaves out all boys and also counts all girls twice. (D) is out.
      5. ***k + n + r + s*** Besides being the only answer left, this equals ***z*** ; it counts all the categories once.

E is the correct answer.

1. Triangle problem. MARK UP THE PICTURE!
   1. What are we looking for? The ***value*** of ***x***.
   2. To the picture. A triangle has 180°. A straight line has 180°. We can figure out ∠ACB, and once we have that, we will have 2 of three angles in the triangle.

***25°***

***120°***

***B***

***A***

***C***

***x°***

* + - * ∠ACB = 180° – 60°
      * ∠ACB = ***120***°
      * 25° + ***120***° + ***x***° = 180°
      * 145° + ***x***° = 180°

***60°***

-145° -145°

* + - * ***x***° = ***35***°

C is the correct answer.

1. “Math Speak” Problem
   1. Let’s write our givens:
      * + $300 to fix old fridge
        + $900 for new fridge
        + $15 savings/month with new fridge
        + In ***x*** months after buying new fridge, the amount saved is equal to the difference between cost of new fridge and cost of fixing the old one.
   2. What are we looking for? The ***value*** of ***x***.
   3. Translate the “math speak.”
      * + They save $15 per month with the new fridge, so in ***x*** months, they will save $15 × ***x***, or 15***x***.
        + amount saved is equal to the difference between cost of new fridge and cost of fixing the old one.
        + 15***x*** = 900 - 300.
   4. Solve for ***x***.
      * + 15***x*** = 900 - 300
        + 15***x*** = 600 Divide both sides by 15.
        + ***x*** = ***40***

D is the correct answer.

1. Equilateral triangles and perimeters. DRAW A PICTURE!
   1. Let’s write our givens:
      * + ∆ABC and ∆DEF are equilateral.
        + Perimeter of ∆ABC is 3 times that of ∆DEF.
        + Perimeter of ∆DEF is 10.
   2. What are we looking for? The ***length*** of ***one side*** of ∆ABC.
   3. Draw a picture. We know ABC has a perimeter of 30 (3 x 10), so draw that. What does equilateral mean? It means all the sides are the same, so each side of ∆ABC is 30/3, or ***10***.

***10***

***B***

***C***

***A***

***E***

***F***

***D***

B is the correct answer.

1. Rate problem.
   1. Let’s write our givens:
      * + Machine mints one coin per second.
        + Machine runs 10 hours per day.
   2. What are we looking for? The number of ***days*** it will take to mint 360,000 coins.
   3. We need an equation.
      * + Total coins = (coins per day) × ***days***
        + We don’t have coins per day, we have coins per second, so we need to scale that up. How many seconds in a day?
        + 60 seconds × 60 minutes × 10 hours = 36,000 seconds/day

1 minute 1 hour 1 day

* With one coin minted per second, that’s 36,000 coins per day.
  1. Back to our equation. We have total coins and coins per day. We can now find ***days***.
     + - Total coins = (coins per day) × ***days*** Get ***days*** by itself. Divide by coins per day.
       - Total coins = ***days***

coins per day

* + - * 360,000 = ***days***

36,000

* + - * ***10*** = ***days***

A is the correct answer.

1. Average problem with a friend.
   1. Let’s write our given: Average of ***x*** and 3***x*** is ***12***.
   2. What are we looking for? The ***value*** of ***x***.
   3. The formula for average is:
      * + Average = sum of terms We have two terms, so the # of terms is 2.

# of terms

* + - * ***12*** = ***x*** + 3***x*** Simplify.

2

* + - * ***12*** = 4***x***

2

* + - * ***12*** = 2***x*** Divide both sides by 2.
      * ***6*** = ***x***

C is the correct answer.

1. Logic Problem
   1. Write the givens:
      * + Some members of chess club are on swim team.
        + No members of swim team are 10th graders.
   2. What are we looking for? The answer choice ***statement*** that must be ***true***.
   3. Start going through the answers to eliminate bad choices.
      1. Not true. Some chess clubbers who aren’t on the swim team might be 10th graders. (A) is out.
      2. Not true. Some chess clubbers who aren’t on the swim team might be 11th graders. (B) is out.
      3. ***True***. If some members of the chess club are on the swim team, then some are not in 10th grade because no swim teamers are in 10th grade. Check the others.
      4. Nope. No swim teamers are in the 10th grade, so (D) is out.
      5. Nope. We can’t conclude that this must be true because we are not given any information about number of members in the chess club. (E) is out.

C is the correct answer.

1. Friends and algebra.
   1. Let’s write our given: 3***x*** + ***n*** = ***x*** + 1
   2. What are we looking for? The ***value*** of ***n*** in terms of ***x***.
   3. Get ***n*** by itself, and we’ll have ***n*** in terms of ***x***.
      * + 3***x*** + ***n*** = ***x*** + 1 Subtract 3***x*** from both sides.
        + ***n*** = -2***x*** + 1
        + ***n*** = 1 - 2***x***

D is the correct answer.

1. Weird function and multiples.
   1. Let’s write our given: ***k*** is the set of all multiples of k.
   2. What are we looking for? The ***set*** that also has all of its numbers in the sets 2, 3 and 5.
   3. We know that the set for 2 is 2, 4, 6, 8, 10, etc. Let’s start going through the answers.
      1. 5 No. Those numbers, 5, 10, 15, etc. are not all in 2. Specifically, 2, 4, 6, 8. (A) is out.
      2. 6 No. Those numbers, 6, 12, 18, etc. are not all in 5. Specifically, 5, 10, 15, 20. (B) is out.
      3. 10 No. Those numbers, 10, 20, 30, etc. are not all in 3. Specifically, 6, 9, 12. (C) is out.
      4. 21 No. Those numbers, 7, 14, 21, etc. are not all in 2. Specifically, 2, 4, 6, 8. (D) is out.
      5. ***60 Yes***. Those numbers, 60, 120, 180, etc. are all in the sets 2, 3, and 5.

E is the correct answer.

1. Geometry problem. MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + AD, BE and CF intersect at O.
        + ∠AOB is 80°.
        + CF bisects ∠BOD.
   2. What are we looking for? The ***measure*** of ∠EOF.
   3. To the picture. If CF bisects ∠BOD, that means that ∠BOC and ∠COD are the same. Mark those.

***80°***

***A***

***B***

***C***

***E***

***F***

***D***

***O***

***?***

* + - * AD is a straight line, so ∠AOB, ∠BOC and ∠COD add up to 180°. Let ∠BOC = ∠COD = ***x***.
      * ∠AOB + ∠BOC + ∠COD = 180°
      * 80° + ***x*** + ***x*** = 180°
      * 80° + 2***x*** = 180° Subtract 80 from both sides.
      * 2***x*** = 100° Divide by 2.
      * ***x*** = ***50***° Divide by 2.
      * That means ∠BOC = ***50***°.
  1. We are looking for the ***measure*** of ∠EOF. ∠EOF and ∠BOC are vertical angles. Vertical angles are equal. So:
     + - ∠EOF = ∠BOC = ***50***°

B is the correct answer.

1. Friend and a root.
   1. Let’s write our givens:
      * + ***k*** is an integer.
        + is our expression.
   2. What are we looking for? The ***least value*** of ***k*** for which our expression is an integer.
   3. We’ve got a square root here with a fraction inside. The fraction has to be a perfect square. For this problem, instead of going through every integer and seeing if it’s in our answers, let’s go through our answers, starting with the lowest value.
      1. = = That’s not an integer. (A) is out. Next.
      2. = = Not even a whole number. (B) is out. Next.
      3. = = = 5 5 is an integer. ***k*** = ***15*** works, and 15 is less than answer choices 25 and 60.

C is the correct answer.

1. Geometry problem. All about visualization. MARK UP THE ANSWERS!
   1. What are we looking for? The ***possible*** shapes when ***only*** the three given figures are combined.

***2***

***1***

***1***

***1***

***1***

* 1. We are given the following building blocks:
  2. Try the different answers.

***1***

***2***

***3***

***3***

* + - * (I) Where’s that green one going to go? (I) does not work.

***4***

***2***

***1***

***1***

***2***

* + - * (II) That’s our shape. (***II***) works.
      * (III) Where’s that green one go? Also, the blue

covers an area that’s not a part of the figure.

(III) is out.

C is the correct answer.

1. Integers, primes, factors, what?
   1. What are we looking for? The number of ***integers*** that are greater than 20 and less than 30, and are the product of ***exactly*** 2 ***different prime*** numbers.
   2. Let’s do some defining.
      * + Product means that our integers are the result of multiplying 2 numbers.
        + Prime means numbers that are only divisible by themselves and 1. Like 2, 3, 5, 7, 11, etc.
   3. Now what integers are between 20 and 30? There are 9: 21, 22, 23, 24, 25, 26, 27, 28, 29
   4. We can eliminate 24, 25, 27 and 28 because they are divisible by more than just two prime numbers.
      * + 24: 1, 2, 3, 5, 6, 8, 12, 24 (6,8,12 are not prime)
        + 25: 1, 5, 25
        + 27: 1, 3, 9, 27 (9 is not prime)
        + 28: 1, 2, 4, 7, 14, 28 (4 and 14 aren’t prime)
   5. We’re left with 21, 22, 23, 26 and 29.
      * + 21 works. 3 and 7 are its factors and they are both prime. That’s 1.
        + 22 works. 2 and 11 are its factors and they are both prime. That’s 2.
        + 23 doesn’t work. It’s prime.
        + 26 works. 2 and 14 are its factors and they are both prime. That’s 3.
        + 29 doesn’t work. It’s prime.
        + That’s a total of ***3 integers*** that work.

D is the correct answer.

1. Pythagorean Theorem with an algebra twist.
   1. Let’s write our given: The figure is a right triangle with sides (7 – ***x***) and (7 + ***x***), and hypotenuse 10.
   2. What are we looking for? The ***value*** of 49 + ***x***2.
   3. Use the Pythagorean Theorem to find ***x***.
      * + a2 + b2 = c2
        + (7 - ***x***)2 + (7 + ***x***)2 = 102 Foil the expressions in the two parentheses. Fun.
        + (7 - ***x***)2 = 49 - 14***x*** + ***x***2
        + (7 + ***x***)2 = 49 + 14***x*** + ***x***2 Put those expressions back into the equation.
        + 49 - 14***x*** + ***x***2 + 49 + 14***x*** + ***x***2 = 102 the +/- 14***x***’s cancel out.
        + 49 + ***x***2 + 49 + ***x***2 = 102 Combine the ***x***2’s and 49’s.
        + 2***x***2 +98 = 100 Subtract 98 from both sides.
        + 2***x***2 = 2 Divide by 2.
        + ***x***2 = ***1***
   4. Plug that into 49 + ***x***2.
      * + 49 + ***x***2
        + 49 + ***1***
        + ***50***

A is the correct answer.

1. Quadratic function and x-intercepts (where the graph crosses the x, or horizontal, axis). MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + h(***2***) is the maximum of this function.
        + h(***a***) = 0
   2. What are we looking for? A possible value of ***a***.
   3. In functions, the number inside the parentheses (like (***2***)) is the input or ***x***-value of a specific point on the function. That is the high point in this case. In h(***a***), ***a*** is the input or x-value, and o is the output or y-value. Looking at the graph, we can see there are two points where the y-value is zero (intersects x-axis).
   4. We don’t have the actual function, so we have to work with what we are given. Mark ***2*** on the x-axis. That’s the ***2***-value of the vertex (given high point). This is important because the two points where the graph crosses the x-axis must be equal in distance from this 2.

***1***

***1***

***2***

* 1. We don’t know exactly where the graph crosses the x-axis, but we can see that one point is negative and one point is greater than 2. Let’s look at the answer choices and see which one could be a possible value for ***a***, the point where the graph crosses the x-axis.
     1. -1 This looks good. We know one of our points is negative. Let’s go through the rest to check.
     2. 0 No. We have already seen that one point is negative and one point is greater than 2. (B) is out.
     3. 2 No. One point clearly crosses the x-axis at a point greater than 2. (C) is out.
     4. 3 No. While it looks like 3 could be a point, if this were true, then the other point where the graph would cross the x-axis would be at 1. Remember, the points have to be equally distant from 2. (D) is out.
     5. 4. No. Same as (D). If 4 were one of the intersecting points, the other point would have to be 0, which we’ve already ruled out.

The only possible answer here is ***-1***.

A is the correct answer.

1. Friends and FOILing.
   1. Let’s write our givens:
      * + ***k*** and ***h*** are constants.
        + x2 + ***k***x + 7 is equivalent to (x + 1)(x + ***h***).
   2. What are we looking for? The ***value*** of ***k***.
   3. This looks ugly because of all the friends. Just start working. Try to factor the first given, the x2 + ***k***x + 7.
      * + x2 + ***k***x + 7 The signs have to be positive, because the entire given is positive.
        + (x + ?)(x +?) We have to use factors of 7, and there are only two: 1 and 7.
        + (x + 1)(x +7) Now compare that to the second given:
        + (x + 1)(x + ***h***)
   4. Those two expressions have to be the same, and they are already almost identical. If ***h*** = ***7***, they are the same, so ***h*** = ***7***. We’re not done yet; we have to find ***k.***
   5. Put the ***h*** = ***7***  into the expression and then FOIL it.
      * + (x + 1)(x + ***h***)
        + (x + 1)(x + ***7***) FOIL it.
        + x2 + 8x + 7 Compare that to the first given:
        + x2 + ***k***x + 7 The two expressions are the same if ***k*** = ***8***.

D is the correct answer.

1. Tough coordinate plane triangle problem.
   1. Let’s write our given: The two legs are parallel to the axes.
   2. What are we looking for? The ***possible lengths*** of the sides of the triangle.
   3. This is a right triangle, with the hypotenuse (AB) that has the slope of a line going through the origin. Combined with the given point (4,10), we can find the slope of AB. Let’s do that. Slope = Rise/Run.
      * + Slope = (y2 – y1)/(x2 – x1)
        + Slope = (10 – 0)/(4 – 0)
        + Slope = 10/4 Reduce that.
        + Slope = ***5***/***2*** So ***5*** could be our vertical side, and ***2*** could be our horizontal side.
   4. Assuming that our answers are in the order: ***horizontal side***, ***vertical side***, hypotenuse, then with a slope of 5/2, every answer’s ***vertical***/***horizontal*** ratio must equal (or reduce to) 5/2. Check them.
      1. 5/2 Yes that works.
      2. 5/2 Indeed.
      3. 3/3 ≠ 5/2 (C) is out.
      4. 4/3 ≠ 5/2 (D) is out.
      5. 5/4 ≠ 5/2 (E) is out.
   5. We’re left with (A) and (B). This is a right triangle, so the sides and hypotenuse must work in the Pythagorean Theorem. Try them.
      1. 2, 5, √29
         * 22 + 52 = (√29)2
         * 4 + 25 = 29 That’s true. ***2, 5, √29*** is a good answer. Check (B) just in case.
      2. 2, 5, 7
         * 22 + 52 = (7)2
         * 4 + 25 = 49 Nope. (B) is out.

A is the correct answer.

1. Function with friends:
   1. Let’s write our givens:
      * + f(x) = 2x – 1
        + ½f(√***t***) = 4
   2. What are we looking for? The ***value*** of ***t***.
   3. First get that second given cleaner by getting the f(√***t***) by itself.
      * + ½f(√***t***) = 4 Divide both sides by ½ (same as multiplying both sides by 2).
        + f(√***t***) = ***8***
   4. Now we’ve got two regular functions with an input. With our first function, to get a value for f(x), we just take a number for x and plug it into the right side of the equation.
      * + For example, let’s say x = 2.
        + f(x) = 2x – 1
        + f(2) = 2(2) – 1 = 3.
        + f(2) = 3
   5. With our second function, f(√***t***), we can follow the same process. The only difference is that we need to put in a √***t*** wherever there is an x. We also already know what the ***output*** of f(√***t***) is. It’s ***8***. Use the first given to set up an equation for f(√***t***).
      * + f(x) = 2x – 1
        + f(√***t***) = 2(√***t***) – 1 We know that f(√***t***) = ***8***. Plug that in.
        + ***8*** = 2(√***t***) – 1 Simplify. Add 1 to both sides.
        + 9 = 2(√***t***) Divide both sides by 2.
        + 9/2 = √***t*** Square both sides.
        + (9/2)2 = ***t*** Distribute the 2.
        + 92/22 = ***t***
        + ***81/4***  = ***t***

E is the correct answer.

1. Tough integer problem to end.
   1. Let’s write our given: ***k*** is a positive integer.
   2. What are we looking for? The ***expression*** that ***must*** represent an even integer that is ***twice*** the value of an ***odd*** integer.
   3. We need to go through the answers choices and identify an answer that can be something *other than* an even integer that is twice the value of an odd integer, so it can be eliminated.
      1. 2***k***. This feels good at first. But say ***k*** is ***2***.
         * 2***k***
         * 2(***2***)
         * 4 That’s an even integer, but it’s twice an ***even*** integer, not an ***odd*** integer. (A) is out.
      2. 2***k*** + 3. Let ***k*** = ***2***.
         * 2***k*** + 3
         * 2(***2***) + 3
         * 7 That’s not an even integer. (B) is out.
      3. 2***k*** + 4. Let ***k*** = ***2***.
         * 2***k*** + 4
         * 2(***2***) + 4
         * 4 + 4
         * 8 That’s an even integer, but it’s twice an ***even*** integer, not an ***odd*** integer. (C) is out.
      4. 4***k*** + 1. Let ***k*** = ***2***.
         * 4(***k***) + 1
         * 4(***2***) + 1
         * 9 That’s not an even integer. And an odd integer can’t be twice another integer. (D) is out.
      5. By elimination, this is our answer, but we’ve come this far, let’s do it. 4***k*** + 2. Let ***k*** = ***2***.
         * 4(***k***) + 2
         * 4(***2***) + 2
         * 10 That’s an even integer and it’s twice an odd integer (5). What if ***k*** is 0dd? Let ***k*** = ***3***.
         * 4(***3***) + 2
         * 12 + 2
         * 14 That’s an even integer and it’s twice an odd integer (7). ***4(k) + 2*** is our answer.

E is the correct answer.

# Test 10 Section 8

1. Combinations.
   1. Let’s write our givens:
      * + 8 dinners
        + 3 desserts
   2. What are we looking for? ***The number of*** dinner/dessert ***combos*** there are.
   3. Not too much to this one, each dinner can go with each dessert. Multiply the numbers.
      * + 8 × 3 = ***24*** ***combos***

A is the correct answer.

1. Pure “math speak” conversion. No solving, just converting.
   1. What are we looking for? The ***equation*** that expresses the given.
   2. Convert the “math speak.”
      * + The sum of 3x and 5 is equal to the product of x and 1/3.
        + 3x + 5 = x × 1/3
        + ***3x + 5 = (1/3)x***

E is the correct answer.

1. Probability Problem
   1. Let’s write our givens:
      * + ***90*** ***total*** trash cans
        + ***15*** ***blue*** trash cans
        + Document is thrown in one trash can; it’s equally likely that it’s any of them.
        + What are we looking for? The ***probability*** that the document is in a ***blue*** trash can.
   2. Figure out the probability. Probability = Part/Total, in this case ***blue***/ ***total***. Plug in and solve.
      * + ***Probability*** = ***blue***/***total***
        + ***Probability*** = ***15***/***90*** Reduce the fraction (divide top and bottom by 15).
        + ***Probability*** = ***1/6***

C is the correct answer.

1. Friends and fractions.
   1. Let’s write our given: (x,y) is an integer pair. (Fancy way of saying x is an integer and y is an integer.)
   2. What are we looking for? The ***number*** of ***integer pairs*** that satisfy x/y = ½
   3. Think about this for a second. There are an infinite number of ways to get to 1/2. 2/4, 3/6, 5/10, 500/1000, and that’s just 4. What about -1/-2, -10/-20? All of those fractions can be reduced to ½.

Way ***more than 4*** is the answer.

E is the correct answer.

1. Data Analysis Problem
   1. What are we looking for? The ***2-month period*** with the least number of books sold.
   2. These two months have to be consecutive. Start going through the answers and ballpark it. (The books sold are in hundreds, but we’ll just refer to them in tens to make things simpler.)
      1. June and July. Looks like a little less than 50. Good starting point to compare. Continue.
      2. July and August. 50. That’s more than (A). (B) is eliminated.
      3. ***August and September***. A little more than 40. That’s less than (A). (A) is eliminated.
      4. September and October. Almost 60. More than (C). (D) is out.
      5. October and November. More than 50. That’s more than (C).

C is the correct answer.

1. Line segments. Yes! MARK UP THE PICTURE!
   1. Let’s write our givens:
      * + AC = 24
        + AB = BC
        + D is between points A and B.
        + AD = DB
   2. What are we looking for? The ***length*** of ***DC***.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfTo the picture. If AB = BC, that means B is right in the middle of AC, so AB = BC = 12. Same logic goes for AD and DB. They are equal, so D is in the middle of AB. That means AD = DB = 6. Mark all that.

***A***

***D***

***B***

***C***

***12***

***6***

***6***

* 1. Now we’ve got our segments filled in. We’re looking for ***DC***. Add DB and BC.
     + - ***DC*** = DB + BC
       - ***DC*** = 6 + 12
       - ***DC*** = ***18***

D is the correct answer.

1. Fractions and negative exponents.
   1. Let’s write our given: ***n*** is a positive integer.
   2. What are we looking for? The ***expression*** that is equivalent to (6 × 10-***n***) + (1 × 10-***n***).
   3. Refresher on negative exponents. Who cares that it’s an ***n***. We know it’s an integer, so treat it like a 1. Or a 2. All a negative exponent does is change the sign on the exponent and shift the base (in this case, 10) to the other side of the fraction.
      * + 10-***n*** = 10-***n***/1 Flip the 10-***n*** into the bottom and make the exponent positive.
        + 10-***n***/1= 1/10***n***
   4. Now look at the first parenthesis and simplify it with our new expression.
      * + (6 × 10-***n***)
        + 6 × (1/10***n***) This is just multiplying a fraction. 6 = 6/1 so the bottom won’t change.
        + (6/1) × (1/10***n***) Multiply across the tops (numerators) and bottoms (denominators).
        + (6 × 1)/(1 × 10***n***)
        + 6/10***n***
   5. The exact same logic can be used on the second parenthesis (1 × 10-***n***).
      * + (1 × 10***-n***) = 1/10***n***
   6. Now put them together to find the ***expression***.
      * + (6 × 10-***n***) + (1 × 10-***n***)
        + 6/10***n*** + 1/10***n*** We can add fractions with similar denominators. (Even if they’re 10***n***.)
        + ***7/10n***

B is the correct answer.

1. “Math speak” problem with degrees.
   1. What are we looking for? ***How many more*** degrees of arc there are in ¼ of a circle than in 1/5 of a circle.
   2. How many degrees in a circle? 360°. Multiply the two fractions to find out how many degrees are in each.
      * + ¼ × 360° = 90°
        + 1/5 × 360° = 72°
   3. Now the “math speak” part. How many ***more*** degrees are in the ¼ circle? That sounds like a difference, so let’s subtract our 1/5 circle degrees from our ¼ circle degrees.
      * + 90° – 72° = ***18°***

B is the correct answer.

1. Reading a function.
   1. What are we looking for? The ***values*** of ***x*** for which f(x) is ***negative***.
   2. f(x) just means the y-values (up and down) for specific x-values (left and right) of the function. This question is just a fancy way of asking what are the x-values of the points where the y-values are negative.
   3. It looks like there’s only one place where the y-values are negative. The origin is one point (***0***,0), and the other point is (***6***,0). But remember, we want the ***x-values*** for those points. Those ***x-values*** and ***all the ones in between*** produce negative y’s, or negative f(x)’s. So, the x-values must be between ***0*** and ***6***.
      * + ***0 < x < 6***

B is the correct answer.

1. Geometry Problem
   1. Let’s write our givens:
      * + Pedestal with 4 layers
        + Each layer has a square base.
        + Each layer is 1 foot high.
   2. What are we looking for? How many ***cubic feet*** of marble make up the pedestal.
   3. We are trying to find the volume of four boxes. Volume is base (length x width) × height. This problem is a bit simpler because all of our heights are 1. When we multiply base × height, we’ll have base × 1, or just base. So, all we need to do is find the area of each of the four bases, and then add them up, which will give us our volume (the answer). Start at the top.
      * + length × width = area
        + 1 × 1 = 1 For the 2nd step we have to add 1 foot to each side.
        + 2 × 2 = 4 And repeat for the 3rd.
        + 3 × 3 = 9 And for the 4th.
        + 4 × 4 = 16 Now add up all the areas.
        + 1 + 4 + 9 + 16 = ***30***

C is the correct answer.

1. Friends and exponents.
   1. Let’s write our givens:
      * + ***x*** and ***y*** are positive integers.
        + 4(2***x***) = 2***y***
   2. What are we looking for? The ***expression*** of ***x*** in terms of ***y***.
   3. The first inclination is to divide by 4 to start peeling away toward the x. But then it gets ugly with logs. There’s an easier way. What is 4 in terms of base 2? 4 = 22. So, rewrite the left side of the equation.
      * + 22×2***x*** = 2***y*** When multiplying like bases, we ***add*** the exponents. Simplify the left side.
        + 2(2+***x***) = 2***y*** Now we have the same bases on both sides. The exponents must be equal.
        + 2+***x***  = ***y*** Get ***x*** by itself. Subtract 2 from both sides.
        + ***x***  = ***y - 2***

A is the correct answer.

1. Triangle Problem
   1. Let’s write our given: The angles of a triangle are in the ratio of 2:3:4.
   2. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfWhat are we looking for? ***How many degrees greater*** isthe largest angle over the smallest angle.
   3. We need a friend, something we can use with that ratio to add up to the 180° of the triangle. How about ***x***? If the ratio is 2:3:4, we can say 2***x*** is our small angle, 3***x*** is the medium angle, and 4***x*** is the largest. Write an equation for the degrees in the triangle.
      * + 2***x*** + 3***x*** + 4***x*** = 180° Combine like terms.
        + 9***x*** = 180° Divide by 9.
        + ***x*** = ***20°***
   4. Now we need to use that to find the measure of our smallest and largest angles. We don’t need the medium.
      * + smallest: 2***x***
        + 2***x*** = 2(***20°***)
        + 2***x*** = 40°
        + largest: 4***x***
        + 4***x*** = 4(***20°***)
        + 4***x*** = 80° Now subtract the smallest from the largest.
        + 4***x*** - 2***x***
        + 80° - 40° = ***40°***

C is the correct answer.

1. “Math Speak” Problem
   1. Let’s write our givens:
      * + First minute of a call is 50 cents.
        + Each minute after the first is 30 cents.
        + phone call is ***n*** minutes, and ***n*** is an integer.
   2. What are we looking for? The ***function*** that describes the cost ***in dollars*** of a call that is ***n*** minutes.
   3. First get our costs in dollars. 50 cents is .. 30 cents is ..
   4. Write out an expression for the cost of the call in words.
      * + f(***n***) = (cost of 1st minute)×(first minute) + (cost of rest of minutes)×(the rest of the minutes)
        + f(***n***) = .50×(first minute) + .30×(the rest of the minutes)
        + The first part can be simplified to .50, because the first minute is just 1.
        + f(***n***) = .50 + .30(the rest of the minutes)
   5. Now we need to know the minutes part of the expression in terms of ***n***. The number of minutes is:
      * + (first minute) + (the rest of the minutes) = ***n***  The 1st minute = 1.
        + 1 + (the rest of the minutes) = ***n*** Subtract 1 from both sides.
        + (the rest of the minutes) = ***n*** - 1
   6. Take the expression for the rest of the minutes, and plug it into our equation above.
      * + f(***n***) = .50 + .30(the rest of the minutes)
        + f(***n***) = .50 + .30(***n*** - 1)

D is the correct answer.

1. Parallel lines and angle rules. MARK UP THE PICTURE!
   1. Let’s write our givens: l ∥ m
   2. What are we looking for? The ***expression*** of ***z***° in terms of ***x***° and ***y***°.
   3. To the picture. ***x***° and ***z***° are part of the triangle, so let’s name that other angle. Let’s call it ***v***°. If we know ***v***°, we can add it to ***x***° and ***z***° and get 180°, because those angles form the triangle.
      * + ***x***° + ***z***° + ***v***° = 180°
   4. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441880[1].wmfNow look at angle ***v***° and angle ***y***°. Light bulb? They are alternate exterior angles. Remember those parallel line rules. Alternate exterior angles have to be equal.

***x°***

***z°***

***v°***

***y°***

***m***

***l***

So ***v***° = ***y***°. Plug in ***y***° for ***v***° in the equation above.

* + - * ***x***° + ***z***° + ***v***° = 180°
      * ***x***° + ***z***° + ***y***° = 180°
      * Subtract ***x***° and ***y***° to get ***z***° by itself.
      * ***z***° = 180° - ***x***° - ***y***°

E is the correct answer.

1. A tough fraction, exponent and friend problem.
   1. Let’s write our givens:
      * + ***n*** and ***k*** are positive integers.
        + ***n*** × 1 × ***n*** = 5

***n*** – 1 ***n*** ***n*** + 1 ***k***

* 1. What are we looking for? The ***value*** of ***k***.
  2. This is ugly, no doubt. But we need to start somewhere. We’re multiplying fractions here, so let’s look for some things to cancel out. We have two “stand alone” ***n***’s on the top and one on the bottom. That means one ***n*** on top cancels out that one ***n*** on the bottom. Progress.
     + - ***n*** \_ × 1 × ***n*** = 5

***n*** – 1 ***n*** ***n*** + 1 ***k***

* + - * 1 \_ × 1 × \_ ***n*** = 5 Simplify that. Get rid of the 1/1 in the middle.

***n*** – 1 1 ***n*** + 1 ***k***

* + - * 1 \_ × ***n*** \_ = 5 Let’s simplify this.

***n*** – 1 ***n*** + 1 ***k***

* 1. Look at the numerators here. With fraction equations, the numerators on the left have to multiply to give us the numerator on the right (same thing for denominators). Set up that equation.
     + - 1 × ***n*** = 5 That’s nice.
       - ***n*** = ***5***
  2. The same rule that we used for the numerators applies to the denominators. Plug in ***5*** for ***n*** in the denominators.
     + - (***n*** – 1) × (***n*** + 1) = ***k***
       - (***5*** – 1) × (***5*** + 1) = ***k***
       - 4 × 6 = ***k***
       - ***24*** = ***k***

C is the correct answer.

1. Ugly friends to finish.
   1. Let’s write our givens:
      * + ***m*** co-workers agree to pay for lunch.
        + ***y*** is the total cost of the lunch.
        + ***p*** co-workers end up not paying.
   2. What are we looking for? The ***additional amount*** that the paying co-workers had to chip in for the lunch.
   3. C:\Users\RyanG\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\NOW7I4J2\MC900441890[1].wmfOne expression we need is for the co-workers who actually paid: ***m*** – ***p***
   4. Let’s find an expression for the amount each co-worker owes for lunch without taking into account who paid and who didn’t.
      * + cost of lunch = ***y*** So ***y***/***m*** is how much each co-worker owes.

all coworkers ***m***

* 1. Now let’s write an equation for the cost of the lunch. We have two buckets of people, those that paid, and those that didn’t.
     + - (What each person owes)×(Non-payers) + (What each person owes)×(Payers) = cost of lunch
       - (***y***/***m***) × ( ***p***) + (***y***/***m***) × (***m*** – ***p***) = ***y***
  2. Remember, we are only concerned with how much ***extra*** the payers had to chip in (i.e., how much they had to chip on account of the non-payers). This amount is just the first portion of the above equation, (***y***/***m***) × (***p***), or ***py***/***m***.
     + - That amount the non-payers owe has to be divided amongst the payers. The payers are ***m*** – ***p***.
       - amount non-payers owe

payers

* + - * ***py***/***m*** Consider ***m*** – ***p*** to be the fraction ***m*** – ***p***/1. Dividing a fraction by a

***m*** – ***p*** fraction is just like multiplying that fraction by 1/fraction.

* + - * ***py*** ×1Multiply across.

***m*** ***m*** - ***p***

* + - * ***py*** = extra amount each payer will have to chip in.

***m***(***m*** - ***p***)

E is the correct answer.