Bode plot

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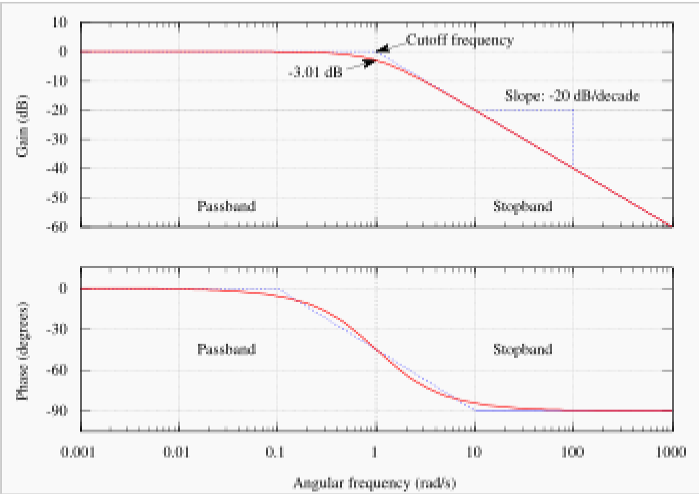

The Bode plot for a first-order (one-pole) lowpass filter

A **Bode plot**, named after Hendrik Wade Bode, is usually a combination of a Bode magnitude plot and Bode phase plot:

A **Bode magnitude plot** is a graph of log magnitude versus frequency, plotted with a log-frequency axis, to show the transfer function or frequency response of a linear, timeinvariant system.

The magnitude axis of the Bode plot is usually expressed as decibels, that is, 20 times the common logarithm of the amplitude gain. With the magnitude gain being logarithmic, bode plots make multiplication of magnitudes a simple matter of adding distances on the graph (in decibels), since

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A **Bode phase plot** is a graph of phase versus frequency, also plotted on a log-frequency axis, usually used in conjunction with the magnitude plot, to evaluate how much a frequency will be phase-shifted. For example a signal described by: $A\sin(\omega t)$ may be attenuated but also phase-shifted. If the system attenuates it by a factor x and phase shifts it by $-\Phi$ the signal out of the system will be $(A/x)\sin(\omega t - \Phi)$. The phase shift Φ is generally a function of frequency.

Phase can also be added directly from the graphical values, a fact that is mathematically clear when phase is seen as the imaginary part of the complex logarithm of a complex gain.

The magnitude and phase Bode plots can seldom be changed independently of each other — changing the amplitude response of the system will most likely change the phase characteristics and vice versa. For minimum-phase systems the phase and amplitude characteristics can be obtained from each other with the use of the Hilbert transform.

If the transfer function is a rational function, then the Bode plot can be approximated with straight lines. These asymptotic approximations are called **straight line Bode plots** or **uncorrected Bode plots** and are useful because they can be drawn by hand following a few simple rules. Simple plots can even be predicted without drawing them.

The approximation can be taken further by *correcting* the value at each cutoff frequency. The plot is then called a **corrected Bode plot**.

Contents

Rules for hand-made Bode plot

The main idea about Bode plots is that one can think of the log of a function in the form:

$$f(x) = A\prod(x+c_n)^{a_n}$$

as a sum of the logs of its poles and zeros:

$$\log(f(x)) = \log(A) + \sum a_n \log(x+c_n)$$

This idea is used explicitly in the method for drawing phase diagrams. The method for drawing amplitude plots implicitly uses this idea, but since the log of the amplitude of each pole or zero always starts at zero and only has one asymptote change (the straight lines), the method can be simplified.

Straight-line amplitude plot

Amplitude decibels is usually done using the $20\log_{10}(X)$

version. Given a transfer function in the form

$$H(s) = A \prod \frac{(s + x_n)^{a_n}}{(s + y_n)^{b_n}}$$

where x_n and y_n are constants, $s = j\omega$, a_n, $b_n > 0$, and H is the transfer function:

- at every value of s where $\omega = x_n$ (a zero), **increase** the slope of the line by $20 \cdot a_n dB$ per decade.
- at every value of s where $\omega = y_n$ (a pole), **decrease** the slope of the line by $20 \cdot b_n dB$per decade.
- The initial value of the graph depends on the boundaries. The initial point is found by putting the initial angular frequency ω into the function and finding $|H(j\omega)|$.
- The initial slope of the function at the initial value depends on the number and order of zeros and poles that are at values below the initial value, and are found using the first two rules.

To handle irreducible 2nd order polynomials, $ax^2 + bx + c$ can, in many cases, be approximated as $(\sqrt{a}x + \sqrt{c})^2$.

Note that zeros and poles happen when ω is *equal to* a certain x_n or y_n. This is because the function in question is the magnitude of H(jω), and since it is a complex function, . Thus at any place where there is a zero or pole involving the term $(s + x_n)$, the magnitude of that term is $\sqrt{(x_n + j\omega)\cdot(x_n - j\omega)} = \sqrt{x_n^2 + \omega^2}$.

Corrected amplitude plot

To correct a straight-line amplitude plot:

- at every zero, put a point $3 \cdot a_n \ \mathrm{dB}$ **above** the line,
- at every pole, put a point $3 \cdot b_n \ \mathrm{dB}$ **below** the line,
- draw a smooth line through those points using the straight lines as asymptotes (lines which the curve approaches).

Note that this correction method does not incorporate how to handle complex values of x_n or y_n. In the case of an irreducible polynomial, the best way to correct the plot is to actually calculate the magnitude of the transfer function at the pole or zero corresponding to the irreducible polynomial, and put that dot over or under the line at that pole or zero.

Straight-line phase plot

Given a transfer function in the same form as above:

$$H(s) = A\prod\frac{(s+x_n)^{a_n}}{(s+y_n)^{b_n}}$$

the idea is to draw separate plots for each pole and zero, then add them up. The actual phase curve is given by − arctan(Im[$H(s)$] / Re[$H(s)$]).

To draw the phase plot, for **each** pole and zero:

- if A is positive, start line (with zero slope) at 0 degrees,
- if A is negative, start line (with zero slope) at 180 degrees,
- at every $\omega = x_n$ (for "stable" zeros – $Re(z) < 0$), slope the line **up** at $45 \cdot a_n$ degrees per decade, beginning one decade before $\omega = x_n$ (that is, start at $\frac{x_n}{10}$),
- at every $\omega = y_n$ (for stable poles – $Re(p) < 0$) slope the line **down** at $45 \cdot b_n$ degrees per decade, beginning one decade before $\omega = y_n$ (that is, start at $\frac{y_n}{10}$),
- "unstable" (right half plane) poles and zeros ($Re(s)$ >

0) have opposite behavior

- flatten the slope again when the phase has changed by degrees (for a zero) or $90 \cdot b_n$ degrees (for a pole),
- After plotting one line for each pole or zero, add the lines together to obtain the final phase plot; that is, the final phase plot is the superposition of each earlier phase plot.

Example

A passive (unity pass band gain) lowpass RC filter, for instance has the following transfer function expressed in the frequency domain:

$$H(jf) = \frac{1}{1 + j2\pi fRC}$$

From the transfer function it can be determined that the cutoff frequency point f_c (in hertz) is at the frequency

$$f_c = \frac{1}{2\pi RC}$$

or (equivalently) at

$\omega_c = \frac{1}{RC}$ where $\omega_c = 2\pi f_c$ is the angular cutoff frequency in radians per second.

The transfer function in terms of the angular frequencies becomes:

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}$$

The above equation is the normalized form of the transfer function.

Magnitude plot

The magnitude (in decibels) of the transfer function above, (normalized and converted to angular frequency form), given by the decibel gain expression A_{vdB}:

$$A_{\mathrm{vdB}} = 20 \log |H(j\omega)| = 20 \log \frac{1}{|1 + j\frac{\omega}{\omega_c}|} =$$

when plotted versus input frequency ω on a logarithmic scale, can be approximated by two lines and it forms the asymptotic (approximate) magnitude Bode plot of the transfer function:

- for angular frequencies below ω_c it is a horizontal line at 0 dB since at low frequencies the $\frac{\omega}{\omega_c}$ term is small and can be neglected, making the decibel gain equation above equal to zero,
- for angular frequencies above ω_c it is a line with a slope of −20 dB per decade since at high frequencies the $\frac{\omega}{\omega_c}$ term dominates and the decibel gain expression above simplifies to $-20 \log \frac{\omega}{\omega_c}$ which is a straight line with a slope of -20 dB per decade.

These two lines meet at the corner frequency. From the plot, it can be seen that for frequencies well below the corner frequency, the circuit has an attenuation of 0 dB, corresponding to a unity pass band gain, i.e. the amplitude of the filter output equals the amplitude of the input. Frequencies above the corner frequency are attenuated – the higher the frequency, the higher the attenuation.

Phase plot

The phase Bode plot is obtained by plotting the phase angle of the transfer function given by:

$$\phi = -\arctan \frac{\omega}{\omega_c}$$

versus ω, where ω and ω_c are the input and cutoff angular frequencies respectively. For input frequencies much lower than corner, the ratio $\frac{\omega}{\omega_c}$ is small and therefore the phase angle is close to zero. As the ratio increases the absolute value of the phase increases and becomes –45 degrees when $\omega = \omega_c$. As the ratio increases for input frequencies much greater than the corner frequency, the phase angle asymptotically approaches -90 degrees. The frequency scale for the phase plot is logarithmic.

Normalized plot

The horizontal frequency axis, in both the magnitude and phase plots, can be replaced by the normalized (nondimensional) frequency ratio $\frac{\omega}{\omega_c}$. In such a case the plot is said to be normalized and units of the frequencies are no longer used since all input frequencies are now expressed as multiples of the cutoff frequency ω_c.

Bode plotter

The Bode plotter is an electronic instrument resembling an oscilloscope, which produces a Bode diagram, or a graph, of a circuit's voltage gain or phase shift plotted against frequency in a feedback control system or a filter. It is extremely useful for analyzing and testing filters and the stability of feedback control systems, through the measurement of corner (cutoff) frequencies and gain and phase margins.

This is identical to the function performed by a vector network analyzer, but the network analyzer is typically used at much higher frequencies.

See also

- Nyquist plot
- Analog signal processing

External links

- Explanation of Bode plots with movies and examples (http://www.facstaff.bucknell.edu/mastascu/eControlHT
- How to draw piecewise asymptotic Bode plots (http://www.swarthmore.edu/NatSci/echeeve1/Ref/LPS
- Summarized drawing rules

(http://lims.mech.northwestern.edu/~lynch/courses/ME
(PDF)

- Bode plot applet (http://www.uwm.edu/People/msw/BodePlot/) - Accepts transfer function coefficients as input, and calculates magnitude and phase response
- Bode Plotting on the HP49 (http://mickpc.pbwiki.com/index.php?wiki=BodePlot)
- Circuit analysis in electrochemistry (http://www.abc.chemistry.bsu.by/vi/fit.htm)
- DIN-A4 printing template (pdf)

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Category: Signal processing

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