# Quantum mechanics of matter

$$Y_{lm}(\theta,\phi) = \Theta_{lm}(\theta)\,\mathrm{e}^{\mathrm{i}m\phi} \qquad \widehat{\mathrm{L}}^2 Y_{lm} = l(l+1)\hbar^2\,Y_{lm} \qquad l = 0,1,2,\ldots, \qquad m = 0,\pm1,\ldots,\pm l$$

$$\widehat{\mathrm{L}}_z Y_{lm} = m\hbar\,Y_{lm} \qquad \widehat{\mathrm{L}}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$\text{parity of } Y_{lm} = (-1)^l \qquad \int_0^{2\pi}\int_0^{\pi} Y^*_{l_1,m_1}(\theta,\phi)\,Y_{l_2,m_2}(\theta,\phi)\,\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi = \delta_{l_1,l_2}\,\delta_{m_1,m_2}$$

$$\widehat{\mathrm{H}}\psi_n = E_n\psi_n \qquad n = 1,2,\ldots \qquad \widehat{\mathrm{H}} = -\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\widehat{\mathrm{L}}^2}{2\mu\,r^2} - \frac{e^2}{4\pi\varepsilon_0\,r}$$

$$\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)\,Y_{lm}(\theta,\phi) \qquad R_{nl}(r) = \left(\frac{r}{a_0}\right)^l\left(\text{polynomial in }\frac{r}{a_0}\right)\mathrm{e}^{-r/na_0}$$

$$\langle r^k\rangle = \int_0^\infty r^{k+2}R_{nl}^2(r)\,\mathrm{d}r \qquad \int_0^\infty R^*_{n_1,l}(r)\,R_{n_2,l}(r)\,r^2\,\mathrm{d}r = \delta_{n_1,n_2}$$

$$E_n = -\frac{E_\mathrm{R}}{n^2} \qquad E_\mathrm{R} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{\mu}{2\hbar^2} = \frac{\hbar^2}{2\mu a_0^2} = \frac{1}{2}\,\frac{e^2}{4\pi\varepsilon_0 a_0} \qquad \mu = \frac{m_1m_2}{m_1+m_2}$$

$$a_0 = \frac{4\pi\varepsilon_0}{e^2}\frac{\hbar^2}{\mu} \qquad E_{nj} = -\frac{E_\mathrm{R}}{n^2}\left[1 + \frac{\alpha^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right] \qquad \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

$$E_\mathrm{R}^{\text{scaled}} = Z^2\frac{\mu}{\mu_\mathrm{H}}E_\mathrm{R} \qquad a_0^{\text{scaled}} = \frac{1}{Z}\frac{\mu_\mathrm{H}}{\mu}a_0 \qquad j = l \pm \tfrac{1}{2}$$

$$L = |l_1-l_2|,\,|l_1-l_2|+1,\,\ldots,\,l_1+l_2-1,\,l_1+l_2 \qquad S = |s_1-s_2|,\,|s_1-s_2|+1,\,\ldots,\,s_1+s_2-1,\,s_1+s_2$$

$$J = |L-S|,\,|L-S|+1,\,\ldots,\,L+S-1, L+S \qquad \mathbf{J} = \mathbf{L}+\mathbf{S} \qquad E_\mathrm{gs} \le \min\frac{\langle\phi_\mathrm{t}|\widehat{\mathrm{H}}|\phi_\mathrm{t}\rangle}{\langle\,\phi_\mathrm{t}|\phi_\mathrm{t}\,\rangle}$$

$$\widehat{\mathrm{H}} = \widehat{\mathrm{H}}^{(0)} + \delta\widehat{\mathrm{H}} \qquad \widehat{\mathrm{H}}^{(0)}\psi_n^{(0)} = E_n^{(0)}\psi_n^{(0)} \qquad E_n \simeq E_n^{(0)} + \langle\psi_n^{(0)}|\delta\widehat{\mathrm{H}}|\psi_n^{(0)}\rangle$$

$$\Psi(x,t) = \sum_k a_k(t)\,\psi_k(x)\,\mathrm{e}^{-\mathrm{i}E_kt/\hbar} \qquad a_k(t) \simeq \delta_{ki} + \frac{1}{\mathrm{i}\hbar}\int_0^t \mathrm{e}^{\mathrm{i}\omega_{ki}t'}V_{ki}(t')\,\mathrm{d}t' \qquad l_f = l_i \pm 1;\; m_f = m_i \text{ or } m_i \pm 1$$