Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$
 (2.13)

where N is the number of branches connected to the node and i_n is the *n*th current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

To prove KCL, assume a set of currents $i_k(t)$, k = 1, 2, ..., flow into a node. The algebraic sum of currents at the node is

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \cdots$$
 (2.14)

Integrating both sides of Eq. (2.14) gives

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \cdots$$
 (2.15)

where $q_k(t) = \int i_k(t) dt$ and $q_T(t) = \int i_T(t) dt$. But the law of conservation of electric charge requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus $q_T(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.

Consider the node in Fig. 2.16. Applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 (2.16)$$

since currents i_1 , i_3 , and i_4 are entering the node, while currents i_2 and i_5 are leaving it. By rearranging the terms, we get

$$i_1 + i_3 + i_4 = i_2 + i_5 \tag{2.17}$$

Equation (2.17) is an alternative form of KCL:

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Note that KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point. In two dimensions, a closed boundary is the same as a closed path. As typically illustrated in the circuit of Fig. 2.17, the total current entering the closed surface is equal to the total current leaving the surface.

A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in Fig. 2.18(a) can be combined as in Fig. 2.18(b). The combined or equivalent current source can be found by applying KCL to node a.

$$I_T + I_2 = I_1 + I_3$$



Figure 2.16 Currents at a node illustrating KCL.



Figure 2.17 Applying KCL to a closed boundary.

Two sources (or circuits in general) are said to be equivalent if they have the same *i*-v relationship at a pair of terminals.

(2.18)

or

$$I_T = I_1 - I_2 + I_3$$

A circuit cannot contain two different currents, I_1 and I_2 , in series, unless $I_1 = I_2$; otherwise KCL will be violated.

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0$$
 (2.19)

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the *m*th voltage.

To illustrate KVL, consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 (2.20)$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \tag{2.21}$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises
$$(2.22)$$

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been $+v_1$, $-v_5$, $+v_4$, $-v_3$, and $-v_2$, which is the same as before except that the signs are reversed. Hence, Eqs. (2.20) and (2.21) remain the same.

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 2.20(a), the combined or equivalent voltage source in Fig. 2.20(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$



Figure 2.18 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

KVL can be applied in two ways: by taking either a clockwise or a counterclockwise trip around the loop. Either way, the algebraic sum of voltages around the loop is zero.



Figure 2.19 A single-loop circuit illustrating KVL.

or

$$V_{ab} = V_1 + V_2 - V_3 \tag{2.23}$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.



For the circuit in Fig. 2.21(a), find voltages v_1 and v_2 .



Figure 2.21 For Example 2.5.

Solution:

To find v_1 and v_2 , we apply Ohm's law and Kirchhoff's voltage law. Assume that current *i* flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \quad v_2 = -3i$$
 (2.5.1)

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0$$
 or $5i = 20 \implies i = 4$ A

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \quad v_2 = -12 \text{ V}$$

XAM

2.5