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Vector Spaces

WEB

INTRODUCTORY EXAMPLE

Space Flight and Control Systems

Twelve stories high and weighing 75 tons, *Columbia* rose majestically off the launching pad on a cool Palm Sunday morning in April 1981. A product of ten years' intensive research and development, the first U.S. space shuttle was a triumph of control systems engineering design, involving many branches of engineering—aeronautical, chemical, electrical, hydraulic, and mechanical.

The space shuttle's control systems are absolutely critical for flight. Because the shuttle is an unstable airframe, it requires constant computer monitoring during atmospheric flight. The flight control system sends a stream of commands to aerodynamic control surfaces and 44 small thruster jets. Figure 1 shows a typical closed-loop feedback system that controls the pitch of the shuttle during flight. (The pitch is the elevation angle of the nose cone.) The junction symbols (⊗) show where signals from various sensors are added to the computer signals flowing along the top of the figure.

Mathematically, the input and output signals to an engineering system are functions. It is important in applications that these functions can be added, as in Fig. 1, and multiplied by scalars. These two operations on functions have algebraic properties that are completely analogous to the operations of adding vectors in $\mathbb{R}^n$ and multiplying a vector by a scalar, as we shall see in Sections 4.1 and 4.8. For this reason, the set of all possible inputs (functions) is called a *vector space*. The mathematical foundation for systems engineering rests on vector spaces of functions, and Chapter 4 extends the theory of vectors in $\mathbb{R}^n$ to include such functions. Later on, you will see how other vector spaces arise in engineering, physics, and statistics.