Unit 1:
Pre-Requisites

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1 1.1 Equations and Graphs

Learning Objectives

A student will be able to:

- Find solutions of graphs of equations.
- Find key properties of graphs of equations including intercepts and symmetry.
- Find points of intersections of two equations.
- Interpret graphs as models.

Introduction

In this lesson we will review what you have learned in previous classes about mathematical equations of relationships and corresponding graphical representations and how these enable us to address a range of mathematical applications. We will review key properties of mathematical relationships that will allow us to solve a variety of problems. We will examine examples of how equations and graphs can be used to model real-life situations.

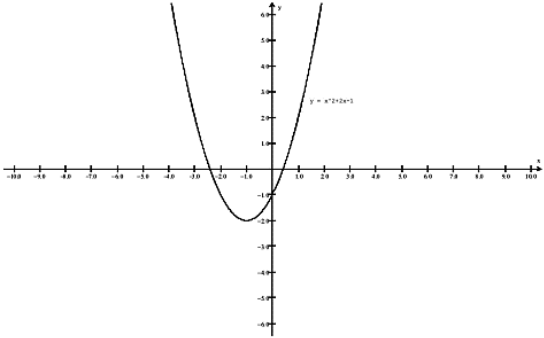
Let's begin our discussion with some examples of algebraic equations:

Example 1: $y = x^2$ The equation has ordered pairs of numbers (x, y) as solutions. Recall that a particular pair of numbers is a solution if direct substitution of the x and y values into the original equation yields a true equation statement. In this example, several solutions can be seen in the following table:

x	$y = x^2 + 2x - 1$

-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14

We can graphically represent the relationships in a rectangular coordinate system, taking the x as the horizontal axis and the y as the vertical axis. Once we plot the individual solutions, we can draw the curve through the points to get a sketch of the graph of the relationship:



We call this shape a

parabola and every quadratic function, $f(x) = ax^2 + bx + c, a \neq 0$ has a parabola-

shaped graph. Let's recall how we analytically find the key points on the parabola. The vertex will lowest point, $(-1, -2)$. In general, the vertex is located at the point $(b/2a, f(b/2a))$. We then can be the identify

points crossing the x and y axes. These are called the intercepts of by setting $x = 0$ in the equation, and then solving for y as follows:

the equation. The $y-$intercept is found $y = 0^2 + 2(0) - 1 =$

-1. The $y-$intercept is located at $(0, -1)$.

The $x-$intercept is found by setting $y =$

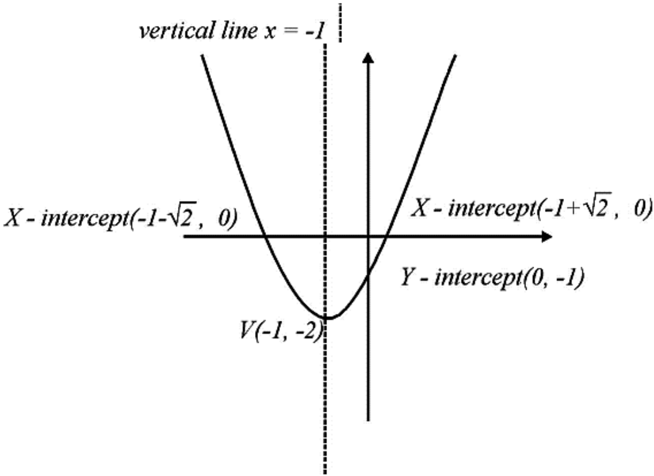
0 in the equation, and solving for x as follows: $0 = x^2 + 2x -$

1 Using the quadratic formula, we find that $x = -1 \pm \sqrt{2}$. $(-1 + \sqrt{2}, 0)$.

The $x-$intercepts are located at $(-1 - \sqrt{2}, 0)$ and Finally,

recall that we defined the symmetry of a graph. We noted

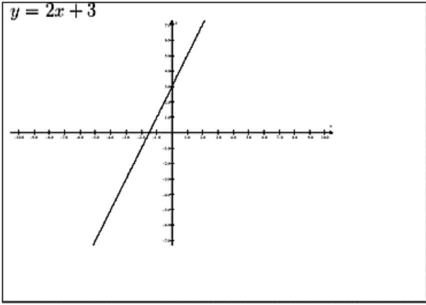
examples of vertical and horizontal line symmetry as well as symmetry about particular points. For the current example, we note that the graph has symmetry in the vertical line $x = -1$.The graph with all of its key characteristics is summarized below:



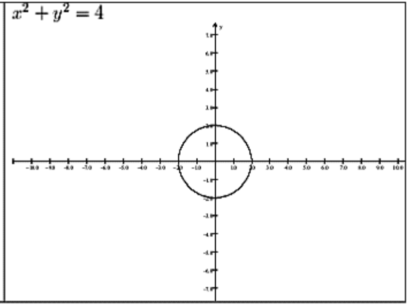
Let’s look at a couple of more examples.

Example 2:

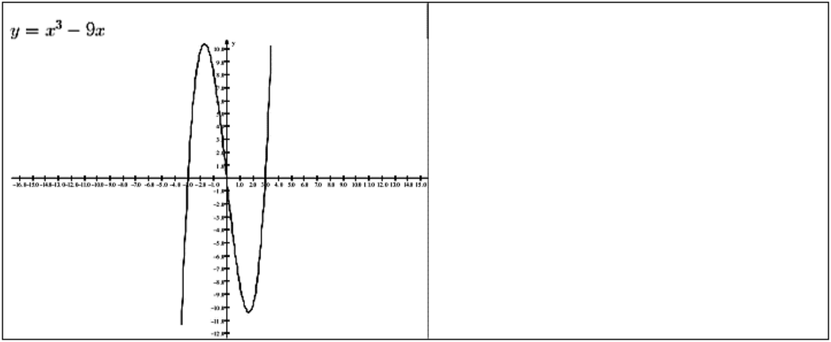
Here are some other examples of equations with their corresponding graphs:



$y = 2x + 3$



$x^2 + y^2 = 4$



Example 3:

We recall the first equation as linear so that its graph is a straight line. Can you determine the intercepts? **Solution:** $x-$intercept at $(-3/2, 0)$and $y-$intercept at $(0, 3)$.

Example 4:

We recall from pre-calculus that the second equation is that of a circle with center $(0, 0)$ and radius $r = 2$. Can you show analytically that the radius is 2?

Solution:

Find the four intercepts, by setting $x = 0$ and solving for y, and then setting $y = 0$ and solving for x. **Example 5:**

The third equation is an example of a polynomial relationship. Can you find the intercepts analytically?

Solution:

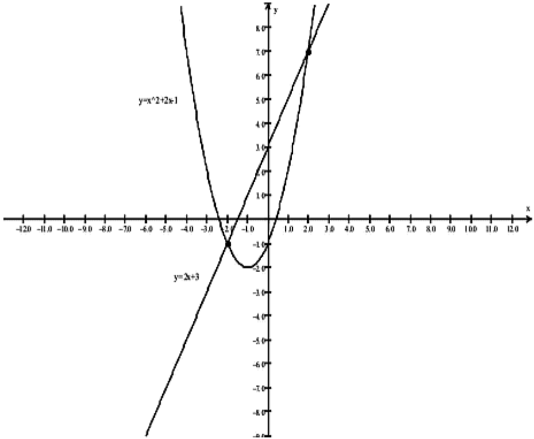
We can find the $x-$intercepts analytically by setting $y = 0$ and solving for x. So, we have

$$x^3 - 9x = 0$$
$$x(x^2 - 9) = 0$$
$$x(x-3)(x+3) = 0$$
$$x = 0, x = -3, x = 3.$$

4 So the $x-$intercepts are located at $(-3, 0)$, $(0, 0)$, and $(3, 0)$. Note that $(0, 0)$ is also the $y-$intercept. The y intercepts can be found by setting $x = 0$. So, we have

$$\begin{aligned} x^3 - 9x &= y \\ (0)^3 - 9(0) &= y \\ y &= 0. \end{aligned}$$

Sometimes we wish to look at pairs of equations and examine where they have common solutions. Consider the linear and quadratic graphs of the previous examples. We can sketch them on the same axes:



We can see that the graphs intersect at two points. It turns out that we can solve the problem of finding the points of intersections analytically and also by using our graphing calculator. Let's review each method. **Analytical Solution**

Since the points of intersection are on each graph, we can use substitution, setting the general $y-$ coordinates equal to each other, and solving for x.

$$2x + 3 = x^2 + 2x - 1$$
$$0 = x^2 - 4$$
$$x = 2, x = -2.$$

We substitute each value of x into one of the original $(-2, -1)$ and $(2, 7)$.

equations and find the points of intersections at **Graphing Calculator Solution**

Once we have entered the relationships on the **Y=** menu, we press **2nd [CALC]** and choose #5 **Intersection** from the menu. We then are prompted with a cursor by the calculator to indicate which two graphs we want to work with. Respond to the next prompt by pressing the left or right arrows to move the cursor near one of the points of intersection and press **[ENTER]**. Repeat these steps to find the location of the second point.

We can use equations and graphs to model real-life

situations. Consider the following problem. **Example 6:**

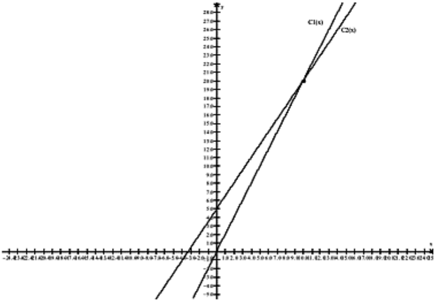
Linear Modeling

5 The cost to ride the commuter train in Chicago is $\$2$. Commuters have the option of buying a monthly coupon book costing $\$5$that allows them to ride the train for $\$1.5$ on each trip. Is this a good deal for someone who commutes every day to and from work on the train?

Solution:

We can represent the cost of the two situations, using the

linear equations and the graphs as follows: $C_1(x) = 2x$ $C_2(x) = 1.5x + 5$



As before, we can find the point of intersection of the lines, or in this case, the break-even value in terms of days, by solving the equation:

$$
\begin{aligned}
C_1(x) &= C_2(x) \\
2x &= 1.5x + 5 \\
x &= 10.
\end{aligned}
$$

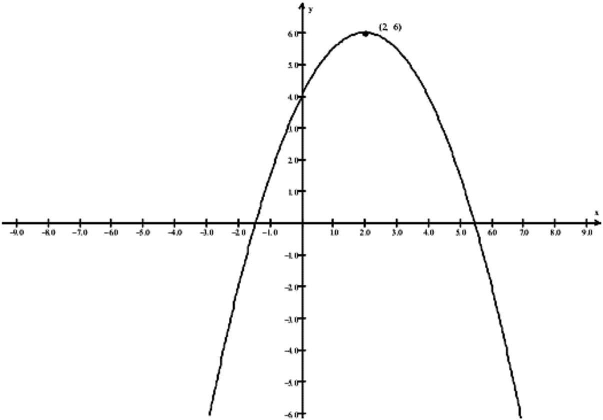
So, even though it costs more to begin with, after 10 days the cost of the coupon book pays off and from that point on, the cost is less than for those riders who did not purchase the coupon book. **Example 7:** Non-Linear Modeling

The cost of disability benefits in the Social Security program for the years 2000 - 2005 can be modeled as a quadratic function. The formula

$$Y = -0.5x^2 + 2x + 4$$

indicates the number of people Y, in millions, receiving Disability Benefits x years after 2000. In what year did the greatest number of people receive benefits? How many people received benefits in that year? **Solution:**

We can represent the graph of the relationship using our graphing calculator.



The vertex is the maximum point on the graph and is located at $(2, 6)$. Hence in year 2002 a total of **6 million** people received benefits.

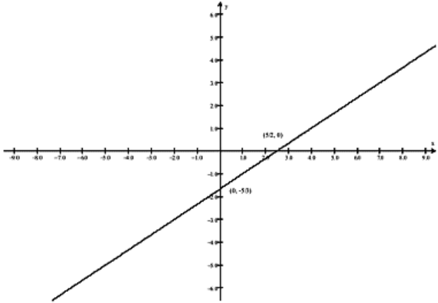
Lesson Summary

1. Reviewed graphs of equations
2. Reviewed how to find the intercepts of a graph of an equation and to find symmetry in the graph
3. Reviewed how relationships can be used as models of real-life phenomena
4. Reviewed how to solve problems that involve graphs and relationships

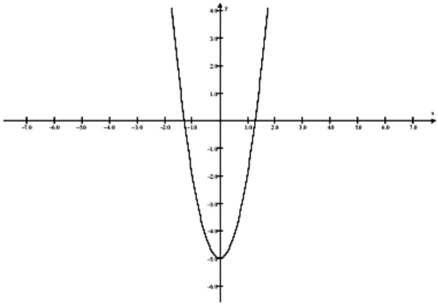
Review Questions

In each of problems 1 - 4, find a pair of solutions of the equation, the intercepts of the graph, and determine if the graph has symmetry.

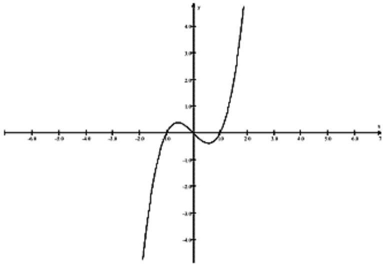
1. $2x - 3y = 5$



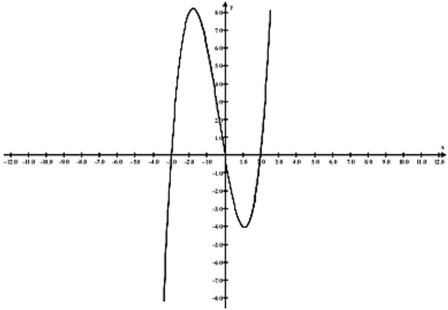
2. $3x^2 - y = 5$



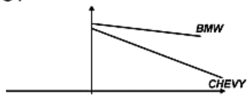
3. $y = x^3 - x$



4. $y = x^3 + x^2 - 6x$



5. Once a car is driven off of the dealership lot, it loses a significant amount of its resale value. The graph below shows the depreciated value of a BMW versus that of a Chevy after t years. Which of



the following

statements is the best conclusion about the data?

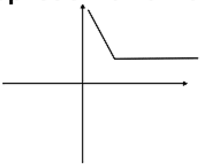
a. You should buy a BMW because they are better cars.

b. BMWs appear to retain their value better than Chevys.

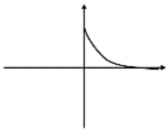
c. The value of each car will eventually be $0.

6. Which of the following graphs is a more realistic representation of the depreciation of cars.

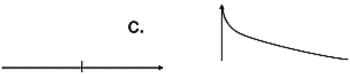
a.



b.



c.



7. A rectangular swimming pool has length that is 25

yards greater than its width.

a. Give the area enclosed by the pool as a function of its width.

b. Find the dimensions of the pool if it encloses an area of 264 square yards.

8. Suppose you purchased a car in 2004 for $18,000. You have just found out that the current year 2008 value of your car is $8,500. Assuming that the rate of depreciation of the car is constant, find a formula that shows changing value of the car from 2004 to 2008.

9. For problem #8, in what year will the value of the vehicle be less than $1,400?

10. For problem #8, explain why using a constant rate of change for depreciation may not be the best way to model depreciation.

Review Answers

1. $(1, -1)$ and $(4, 1)$ are two solutions. The

intercepts are located at $(0, 5/3)$ and $(5/2, 0)$. We have a linear relationship between x and y, so its graph can be sketched as the line passing through any two solutions.

2. By solving for y, we have $y = 3x^2 - 5$, so two solutions are $(-1, -2)$ and $(1, -2)$.

The x-intercepts are located at $\left(\pm\sqrt{\frac{5}{3}}, 0\right)$ and the y-intercept is located at $(0, -5)$.

The graph is symmetric in the y-axis.

3. Using your graphing calculator, enter the relationship on the **Y=** menu. Viewing a table of points, we see many solutions, say $(2, 6)$ and $(-2, -6)$, and the intercepts at $(0, 0)$, $(-1, 0)$ and $(1, 0)$. By inspection we see that the graph is symmetric about the origin.

4.

Using your graphing calculator, enter the relationship on the **Y=** menu. Viewing a table of points, we see many solutions, say $(2, 0)$, and $(-1, 6)$, and the intercepts located at $(0, 0)$, $(-3, 0)$, and $(2, 0)$. By inspection we see that the graph does not have any symmetry.

5. b.

6. c. because you would expect (1) a decline as soon as you bought the car, and (2) the value to be declining more gradually after the initial drop.

7.

a. $A(w) = w^2 + 25w$.

b. The pool has area 264 when width = 8 and length = 33.

8. The rate of change will be $(-9500/4) = -2375.$ The formula will be $y = -2375x + 18000.$

9. At the time x=7, or equivalently in the year 2011, the car will be valued at $1375.

10. A linear model may not be the best function to model depreciation because the graph of the function decreases as time increases; hence at some point the value will take on negative real number values, an impossible situation for the value of real goods and products.

1.2 Relations and Functions

Learning Objectives

A student will be able to:

- Identify functions from various relationships.
- Review function notation.
- Determine domains and ranges of particular functions.
- Identify key properties of some basic functions.
- Sketch graphs of basic functions.
- Sketch variations of basic functions using transformations.
- Compose functions.

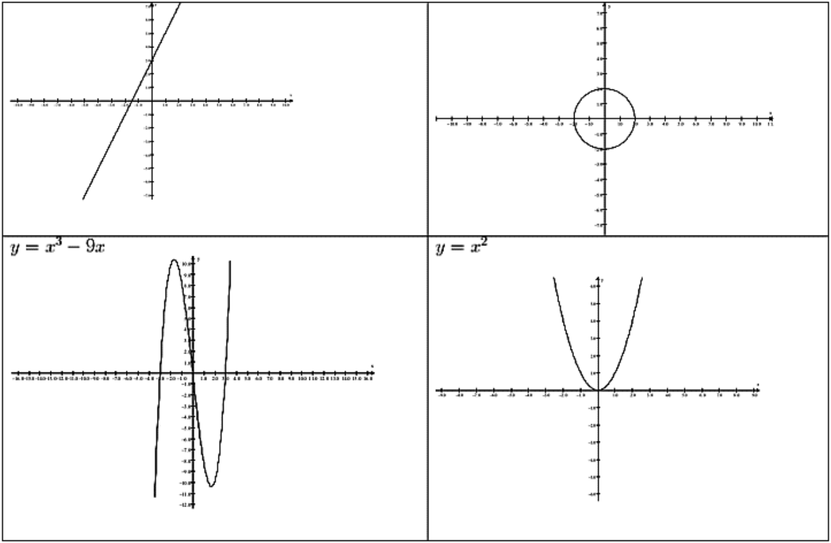
Introduction

In our last lesson we examined a variety of mathematical equations that expressed mathematical relationships. In this lesson we will focus on a particular class of relationships called functions, and examine their key properties. We will then review how to sketch graphs of some basic functions that we will revisit later in this class. Finally, we will examine a way to combine functions that will be important as we develop the key concepts of calculus.

Let’s begin our discussion by reviewing four types of equations we examined in our last lesson.

Example 1:

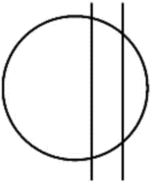
$y = 2x + 3$	$x^2 + y^2 = 4$

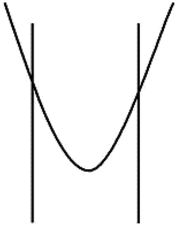


Of these, the circle has a quality that the other graphs do not share. Do you know what it is? **Solution:**

The circle’s graph includes points where a particular $x-$value has example, the points $(1, \sqrt{3})$and $(1, -\sqrt{3})$are both solutions to the

two points associated with it; for equation $x^2 + y^2 = 4$.For each of the other relationships, a particular $x-$value has exactly one $y-$value associated with it.) The relationships that satisfy the condition that for each $x-$value there is a unique $y-$value are called **functions**. Note that we could have determined whether the relationship satisfied this condition by a graphical test, the vertical line test. Recall the relationships of the circle, which is not a function. Let’s compare it with the parabola, which is a function.





If we draw vertical lines through the graphs as indicated, we see that the condition of a particular $x-$value having exactly one $y-$value associated with it is equivalent to having at most one point of intersection with any vertical line. The lines on the circle intersect the graph in more than one point, while the lines drawn on the parabola intersect the graph in exactly one point. So this vertical line test is a quick and easy way to check whether or not a graph describes a function.

10 We want to examine properties of functions such as

function notation, their domain and range (the sets of x and y values that define the function), graph sketching techniques, how we can combine functions to get new functions, and also survey some of the basic functions that we will deal with throughout the rest of this book.

Let’s start with the notation we use to describe functions. Consider the example of the linear function $y = 2x + 3$. We could also describe the function using the symbol $f(x)$ and read as "f *of* x" to indicate the $y-$value of the function for a particular $x-$value. In particular, for this function we would write $f(x) = 2x + 3$ and indicate the value of the function at a particular value, say $x = 4$ as $f(4)$ and find its value as follows: $f(4) = 2(4) + 3 = 11$. This statement corresponds to the solution $(4, 11)$ as a point on the graph of the function. It is read, "f *of* x *is* 11."

We can now begin to discuss the properties of functions, starting with the ***domain*** and the ***range*** of a function. The ***domain*** refers to the set of $x-$values that are inputs in the function, while the ***range*** refers to the set of $y-$values that the function takes on. Recall our examples of functions:

Linear Function $g(x) = 2x + 3$

Quadratic Function $f(x) = x^2$

Polynomial Function $p(x) = x^3 - 9x$

We first note that we could insert any real number for an $x-$value and a well-defined $y-$value would come out. Hence each function has the set of all real numbers as a domain and we indicate this in interval form as $D : (-\infty, \infty)$.

Likewise we see that our graphs could extend up in a positive direction and down in a negative direction without

the set of all real numbers

end in either direction. Hence we see that the set of $R : (-\infty, \infty)$.

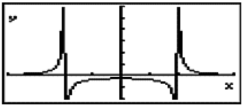
$y-$values, or the range, is **Example 2:**

Determine the domain and range of the function. $f(x) = 1/(x^2 - 4)$.

Solution:

We note that the condition for each $y-$value is a fraction that includes an x term in the denominator. In deciding what set of $x-$values we can use, we need to exclude those values that make the denominator equal to 0. Why? **(Answer: division by 0 is not defined for real numbers.)** Hence the set of all permissible $x-$values, is all real numbers except for the numbers $(2, -2)$, which yield division by zero.

So on our graph we will not see any points that correspond to these $x-$values. It is more difficult to find the range, so let's find it by using the graphing calculator to produce the graph.



11 From the graph, we see that every $y \neq 0$ value in $(-\infty,$

$\infty)$(or "All real numbers") is represented; hence the range of the function is $\{-\infty, 0\} \cup \{0, \infty\}$. This is because a fraction with a non-zero numerator never equals zero.

Eight Basic Functions

We now present some basic functions that we will work with throughout the course. We will provide a list of eight basic functions with their graphs and domains and ranges. We will then show some techniques that you can use to graph variations of these functions.

Linear

Square (Quadratic)

$f(x) = x$ $\qquad$ $f(x)$

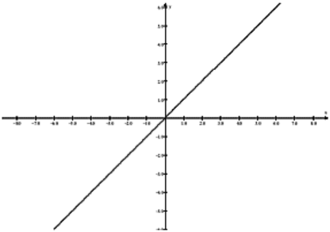
$= x^2$

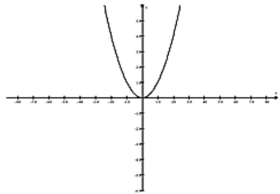
Domain =All reals

Domain =All reals

Range =All reals

Range $= \{y \geq 0\}$





Cube (Polynomial)

Square Root

$f(x) = x^3$

$f(x)$

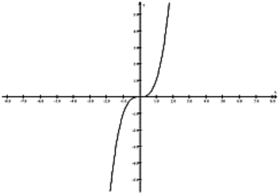
$= \sqrt{x}$

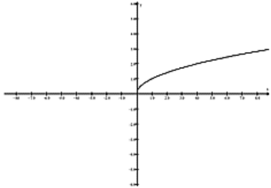
Domain =All reals

Domain $= \{x \geq 0\}$

Range =All reals

Range $= \{y \geq 0\}$





12 ***Absolute Value***

Rational

$f(x) = |x|$

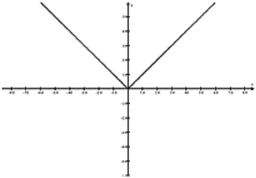
$f(x)$

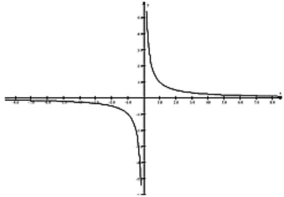
$= 1/x$

Domain =***All reals***

Domain $= \{x \neq 0\}$
Range $= \{y \geq 0\}$

Range $= \{y \neq 0\}$





Sine

$f(x) = \sin x$

Domain = All reals

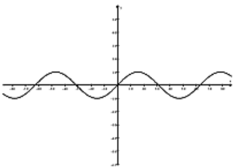
Range $= \{-1 \leq y \leq 1\}$

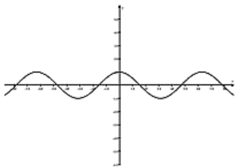
Cosine

$f(x) = \cos x$

Domain = **All reals**

Range $= \{-1 \leq y \leq 1\}$



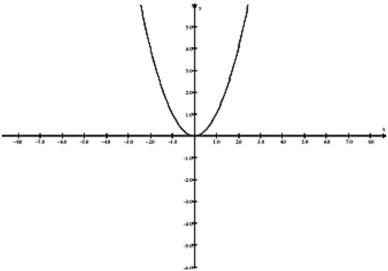


Graphing by Transformations

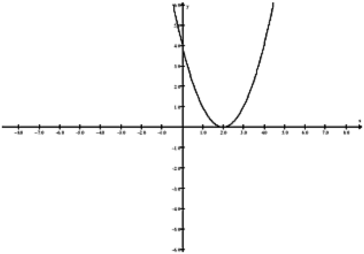
Once we have the basic functions and each graph in our memory, we can easily sketch variations of these. In general, if we have $f(x)$ and c is some constant value, then the graph of $f(x-c)$ is just the graph of $f(x)$ shifted c units to the right. Similarly, the graph of $f(x+c)$ is just the graph of $f(x)$ shifted c units to the left.

Example 3:

$f(x) = x^2$



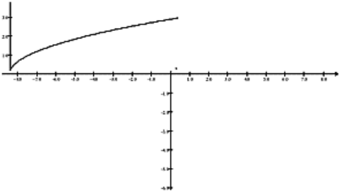
$f(x) = (x-2)^2$



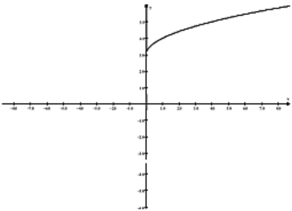
In addition, we can shift graphs up and down. In general, if we have $f(x)$, and c is some constant value, then the graph of $f(x)+c$ is just the graph of $f(x)$ shifted c units up on the $y-$axis. Similarly, the graph of $f(x)-c$ is just the graph of $f(x)$ shifted c units down on the $y-$axis.

Example 4:

$$f(x)=\sqrt{x}$$



$$f(x) = \sqrt{x} + 3$$

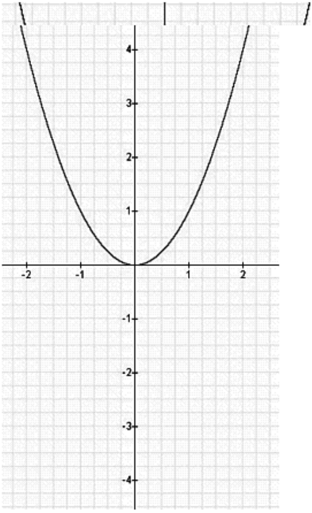


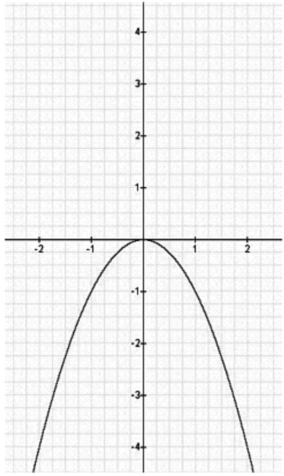
We can also flip graphs in the $x-$axis by multiplying by a negative coefficient.

$f(x) = x^2$

$f(x) =$

$-x^2$

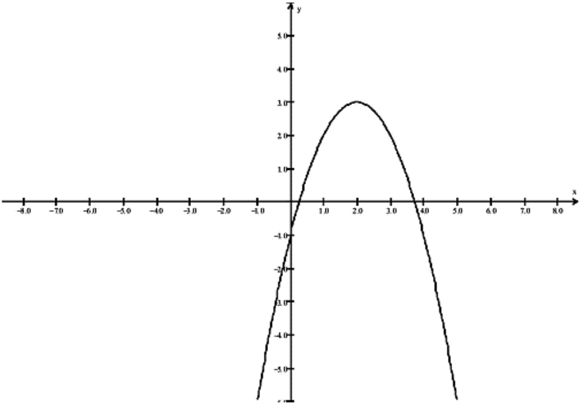




Finally, we can combine these transformations into a single example as follows.

Example 5:

$f(x) = -(x-2)^2 + 3$. The graph will be generated by taking $f(x) = x^2$, flipping in the $y-$axis, and moving it two units to the right and up three units.



Function Composition

The last topic for this lesson involves a way to combine functions called **function composition**. Composition of functions enables us to consider the effects of one function followed by another. Our last example of graphing by transformations provides a nice illustration. We can think of the final graph as the effect of taking the following steps:

$$x \to -(x-2)^2 \to -(x-2)^2 + 3$$

We can think of it as the application of two functions. First, $g(x)$ takes x to $-(x-2)^2$ and then we apply a second function, $f(x)$ to those $y-$values, with the second function adding $+3$ to each output. We would write the functions as

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$f(g(x)) = -(x-2)^2 + 3$ where $g(x) = -(x-2)^2$ and $f(x) = x + 3$. We call this operation the composing of f with g and use notation $f \circ g$. Note that in this example, $f \circ g \neq g \circ f$. Verify this fact by computing $g \circ f$ right now. (Note: this fact can be verified algebraically, by showing that the expressions $f \circ g$ and $g \circ f$ differ, or by showing that the different function decompositions are not equal for a specific value.)

Lesson Summary

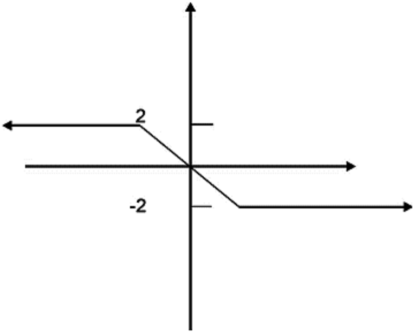
1. Learned to identify functions from various relationships.

2. Reviewed the use of function notation.
3. Determined domains and ranges of particular functions.
4. Identified key properties of basic functions.
5. Sketched graphs of basic functions.
6. Sketched variations of basic functions using transformations.
7. Learned to compose functions.

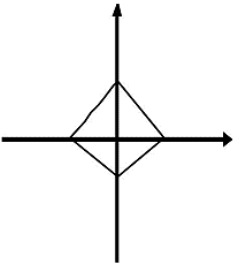
Review Questions

In problems 1 - 2, determine if the relationship is a function. If it is a function, give the domain and range of the function.

1.



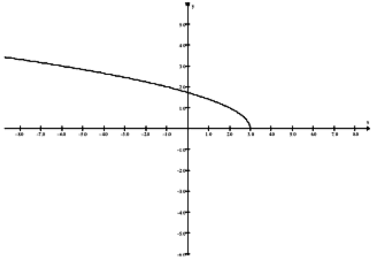
2.



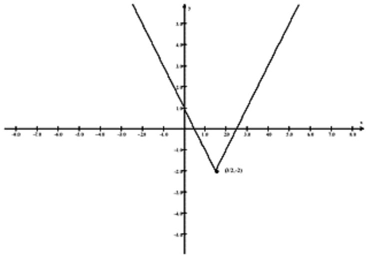
In problems 3 - 5, determine the domain and range of the function and sketch the graph if no graph is provided.

3. $f(x) = \frac{3x^2}{x^2-1}$ (no graph provided)

4. $y = \sqrt{-x+3}$ graph:



5. $f(x) = |2x - 3| - 2$ graph:



In problems 6 - 8, sketch the graph using transformations of the graphs of basic functions. 6. $f(x) = -(x+2)^2 + 5$

7. $f(x) = -\frac{1}{x-2} + 3$

8. $y = -\sqrt{-x-2} + 3$

9. Find the composites $f \circ g$ and $g \circ f$ for the following functions. $f(x) = -3x + 2, g(x) = \sqrt{x}$ 10. Find the composites $f \circ g$ and $g \circ f$ for the following functions. $f(x)$

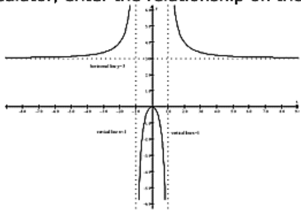
$= x^2, g(x) = \sqrt{x}$ **Review Answers**

1. The relationship is a function. Domain is All Real Numbers and range $= \{-2 \leq y \leq 2\}$. 2. The relationship is not a function.

3. The domain is $\{x \neq -1, 1\}$. Use the graph

and view the table of solutions to determine the range.

Using your graphing calculator, enter the relationship on the **Y=** menu. Graph/table shows that



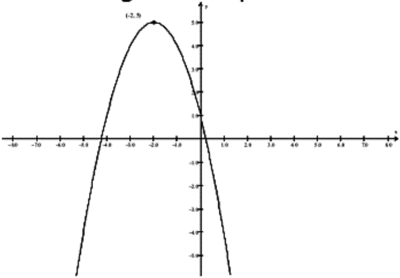
range = $(-\infty, 0] \cup (3, \infty)$.

4. $\text{Domain} = \{x \leq -3\}$, $\text{range} = \{y \geq 0\}$.

5. This is the basic absolute value function shifted 3/2 units

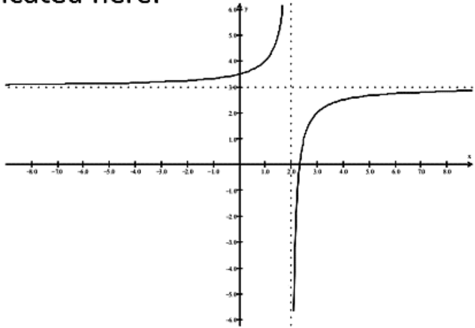
All Real Numbers and range is $\{y \geq -2\}$.

to the right and down two units. Domain is 6. Reflect and shift the general quadratic function as indicated here:

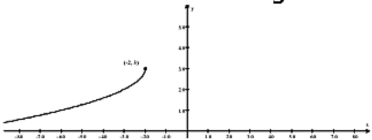


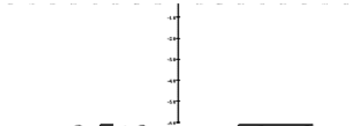
7. Reflect and shift the general rational function as

indicated here:



8. Reflect and shift the general radical function as indicated here:





9. $f \circ g = -3\sqrt{x} + 2, g \circ f = \sqrt{-3x+2}$.

10. $f \circ g = x, g \circ f = x$; any functions where $f \circ g = x, g \circ f = x$ are called inverses; in this problem f and g are inverses of one another. Note that the domain for $f \circ g$ is restricted to only positive numbers and zero.

1.3 Models and Data

Learning Objectives

A student will be able to:

- Fit data to linear models.
- Fit data to quadratic models.
- Fit data to trigonometric models.
- Fit data to exponential growth and decay models.

Introduction

In our last lesson we examined functions and learned how to classify and sketch functions. In this lesson we will use some classic functions to model data. The lesson will be a set of examples of each of the models. For each, we will make extensive use of the graphing calculator.

Let's do a quick review of how to model data on the graphing calculator.

Enter Data in Lists

Press **[STAT]** and then **[EDIT]** to access the lists, **L1 - L6**.

18 ***View a Scatter Plot***

Press **2nd [STAT PLOT]** and choose accordingly.

Then press **[WINDOW]** to set the limits of the axes.

Compute the Regression Equation

Press **[STAT]** then choose **[CALC]** to access the regression equation menu. Choose the appropriate regression equation (Linear, Quad, Cubic, Exponential, Sine).

Graph the Regression Equation Over Your Scatter Plot

Go to **Y**=> **[MENU]** and clear equations. Press **[VARS]**, then enter 5and **EQ** and press **[ENTER]** (This series of entries will copy the regression equation to your **Y =** screen.) Press **[GRAPH]** to view the regression equation over your scatter plot

Plotting and Regression in Excel

You can also do regression in an Excel spreadsheet. To start,

copy and paste the table of data into Excel. With the two columns highlighted, including the column headings, click on the **Chart** icon and select **XY scatter**. Accept the defaults until a graph appears. Select the graph, then click **Chart**, then **Add Trendline**. From the choices of trendlines choose **Linear**.

Now let’s begin our survey of the various modeling

situations.

Linear Models

For these kinds of situations, the data will be modeled by the classic linear equation $y = mx + b$. Our task will be to find appropriate values of m and b for given data.

Example 1:

It is said that the height of a person is equal to his or her wingspan (the measurement from fingertip to fingertip when your arms are stretched horizontally). If this is true, we should be able to take a table of measurements, graph the measurements in an $x - y$ coordinate system, and verify this relationship. What kind of graph would you expect to see? ***(Answer: You would expect to see the points on the line*** $y = x$.)

Suppose you measure the height and wingspans of nine of your classmates and gather the following data. Use your graphing calculator to see if the following measurements fit this linear model (the line $y = x$).

Height (inches)	*Wingspan (inches)*
67	65
64	63
56	57
60	61
62	63
71	70

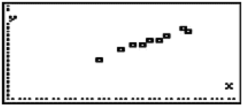
19 ________

Height (inches)	*Wingspan (inches)*
72	69
68	67
65	65

We observe that only one of the measurements has the condition that they are equal. Why aren't more of the measurements equal to each other? ***(Answer: The data do not always conform to exact specifications of the model. For example, measurements tend to be loosely documented so there may be an error arising in the way that measurements were taken.)***

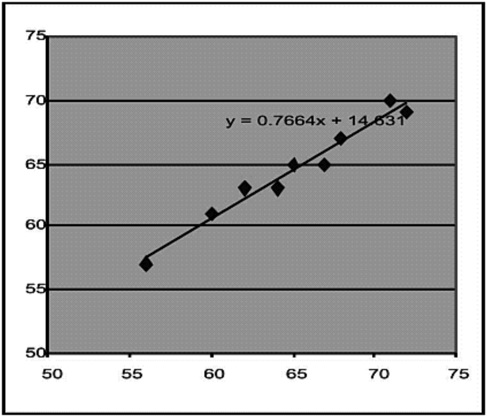
We enter the data in our calculator in **L1** and **L2**. We then view a scatter plot. (Caution: note that the data ranges exceed the viewing window range of $[-10, 10]$. Change the window ranges accordingly to include all of the data, say $[40, 80]$.)

Here is the scatter plot:



Now let us compute the regression equation. Since we expect the data to be linear, we will choose the **linear regression** option from the menu. We get the equation $y = .76x + 14$.

In general we will always wish to graph the regression equation over our data to see the goodness of fit. Doing so yields the following graph, which was drawn with Excel:



Since our calculator will also allow for a variety of non-linear functions to be used as models, we can therefore examine quite a few real life situations. We will first consider an example of quadratic modeling. ***Quadratic Models***

Example 2:

The following table lists the number of Food Stamp recipients (in millions) for each year after 1990.

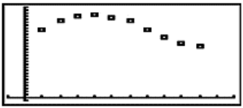
20

Years after 1990	*Participants*
1	22.6
2	25.4
3	27.0
4	27.5
5	26.6
6	2.55

7	22.5
8	19.8
9	18.2
10	17.2

We enter the data in our calculator in **L3** and **L4** (that enables us to save the last example's data). We then will view a scatter plot. Change the window ranges accordingly to include all of the data. Use $[-2, 10]$for x and $[-2, 30]$for y.

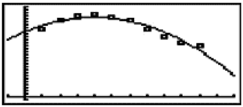
Here is the scatter plot:



Now let us compute the regression equation. Since our scatter plot suggests a quadratic model for the data, we will choose **Quadratic Regression** from the menu. We get the equation:

$$y = -0.30x^2 + 2.38x + 21.67.$$

Let's graph the equation over our data. We see the following graph:



Trigonometric Models

The following example shows how a trigonometric function can be used to model data. **Example 3:**

With the skyrocketing cost of gasoline, more people have looked to mass transit as an option for getting around. The following table uses data from the *American Public Transportation Association* to show the number of mass transit trips (in billions) between 1992 and 2000.

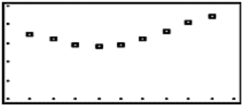
21

Year	*Trips (billions)*
1992	8.5
1993	8.2
1994	7.93
1995	7.8
1996	7.87

1997	8.23
1998	8.6
1999	9.08
2000	9.4

We enter the data in our calculator in **L5** and **L6**. We then will view a scatter plot. Change the window ranges accordingly to include all of the data. Use $[-2, 10]$ for both x and y ranges.

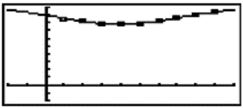
Here is the scatter plot:



Now let us compute the regression equation. Since our scatter plot suggests a sine model for the data, we will choose **Sine Regression** from the menu. We get the equation:

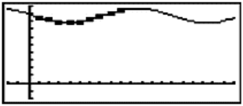
$$y = .9327 * \sin(.4681x + 2.8734) + 8.7358.$$

Let us graph the equation over our data. We see the following graph:



This example suggests that the sine over time t is a function that is used in a variety of modeling situations.

Caution: Although the fit to the data appears quite good, do we really expect the number of trips to continue to go up and down in the future? Probably not. Here is what the graph looks like when projected an additional ten years:



Exponential Models

22 Our last class of models involves exponential functions. Exponential models can be used to model growth and decay situations. Consider the following data about the declining

number of farms for the years 1980 - 2005.

Example 4:

The number of dairy farms has been declining over the past $20+$years. The following table charts the decline:

Year	*Farms (thousands)*
1980	334
1985	269
1990	193
1995	140
2000	105

2005	67

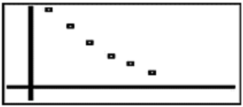
We enter the data in our calculator in **L5** (again entering the years as 1, 2, 3...) and **L6**. We then will view a scatter

choose

plot. Change the window ranges accordingly to include all the range $[-50, 350]$ with a scale of $25.$

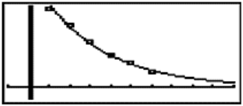
of the data. For the large $y-$values,

Here is the scatter plot:



Now let us compute the regression equation. Since our scatter plot suggests an exponential model for the data, we will choose **Exponential Regression** from the menu. We get the equation: $y = 490.6317 * .7266^x$

Let's graph the equation over our data. We see the following graph:



In the homework we will practice using our calculator extensively to model data.

Lesson Summary

1. Fit data to linear models.
2. Fit data to quadratic models.
3. Fit data to trigonometric models.
4. Fit data to exponential growth and decay models.

Review Questions

1. Consider the following table of measurements of circular objects:
    a. Make a scatter plot of the data.
    b. Based on your plot, which type of regression will you use?
    c. Find the line of best fit.
    d. Comment on the values of m and b in the equation.

Object	*Diameter (cm)*	*Circumference (cm)*
Glass	8.3	26.5
Flashlight	5.2	16.7

Aztec calendar	20.2	61.6
Tylenol bottle	3.4	11.6
Popcorn can	13	41.4
Salt shaker	6.3	20.1
Coffee canister	11.3	35.8
Cat food bucket	33.5	106.5
Dinner plate	27.3	85.6
Ritz cracker	4.9	15.5

2. Manatees are large, gentle sea creatures that live along the Florida coast. Many manatees are killed or injured by power boats. Here are data on powerboat registrations (in thousands) and the

number of manatees killed by boats in Florida from 1987 - 1997.

a. Make a scatter plot of the data.

b. Use linear regression to find the line of best fit.

c. Suppose in the year 2000, powerboat registrations increase to 700,000. Predict how many manatees will be killed. Assume a linear model and find the line of best fit.

Year	*Boats*	*Manatees killed*
1987	447	13
1988	460	21
1989	480	24

Year	Boats	Manatees killed
1990	497	16
1991	512	24
1992	513	21
1993	526	15
1994	557	33
1995	585	34
1996	614	34
1997	645	39

3. A passage in ***Gulliver's Travels*** states that the measurement of "Twice around the wrist is once around the neck." The table below contains the wrist and neck measurements of 10 people.

a. Make a scatter plot of the data.

b. Find the line of best fit and comment on the accuracy of the quote from the book.

c. Predict the distance around the neck of Gulliver if the distance around his wrist is found to be 52 cm.

Wrist (cm)	*Neck (cm)*
17.9	39.5
16	32.5

16.5	34.7
15.9	32
17	33.3
17.3	32.6
16.8	33
17.3	31.6
17.7	35
16.9	34

4. The following table gives women's average percentage of men's salaries for the same jobs for each 5-year period from 1960 - 2005.

a. Make a scatter plot of the data.

b. Based on your sketch, should you use a linear or quadratic model for the data?

c. Find a model for the data.

d. Can you explain why the data seems to dip at first and then grow? ______

Year	*Percentage*
1960	42
1965	36
1970	30
1975	37
1980	41
1985	42

1990	48

25

Year	Percentage
1995	55
2000	58
2005	60

5. Based on the model for the previous problem, when will women make as much as men? Is your answer a realistic prediction?

6. The average price of a gallon of gas for selected years from 1975 - 2008 is given in the following table:

   a. Make a scatter plot of the data.
   b. Based on your sketch, should you use a

linear, quadratic, or cubic model for the data?

c. Find a model for the data.

d. If gas continues to rise at this rate, predict the price of gas in the year 2012. ______

Year	Cost
1975	1
1976	1.75
1981	2
1985	2.57
1995	2.45
2005	2.75
2008	3.45

7. For the previous problem, use a linear model to analyze the situation. Does the linear method provide a better estimate for the predicted cost for the year 2011? Why or why not?

8. Suppose that you place $1,000 in a bank account where it grows exponentially at a rate of 12% continuously over the course of five years. The table below shows the amount of money you have at the end of each year.

a. Find the exponential model.

b. In what year will you triple your original amount?

Year	*Amount*
0	1000

1	1127.50
2	1271.24
3	1433.33
4	1616.07
5	1822.11
6	2054.43

9. Suppose that in the previous problem, you started with $3,000 but maintained the same interest rate.

a. Give a formula for the exponential model. (Hint: note the coefficient and exponent in the previous answer!)

b. How long will it take for the initial amount, $3,000, to triple? Explain your answer.

10. The following table gives the average daily temperature for Indianapolis, Indiana for each month of the year.

a. Construct a scatter plot of the data.

b. Find the sine model for the data.

Month	*Avg Temp (F)*
Jan	22
Feb	26.3
March	37.8
April	51

May	61.7
June	75.3
July	78.5
Aug	84.3
Sept	68.5
Oct	53.2
Nov	38.7
Dec	26.6

Review Answers

1.
   a. .
   b. Linear.

c. $y = 3.1334x + .3296.$

d. m is an estimate of π, and b should be zero but due to error in measurement it is not.

2.

a. .

b. $y = .120546x - 39.0465$

c. About 46 manatees will be killed in the year 2000. Note: there were actually 81 manatees killed in the year 2000.

3.

a. .

b. $y = 2.0131x - 0.2634$

c. 104.42 cm

4.

a. .

b. Quadratic

c. $y = .4848x^2 - 2.4545x + 39.7333.$

d. It might be because the first wave of women into the workforce tended to take whatever jobs they could find without regard for salary.

5. The data suggest that women will reach 100% in 2009; this is unrealistic based on current reports that women still lag far behind men in equal salaries for equal work.

6.

a. .

b. Cubic.

c. $y = .0277x^3 - 0.3497x^2 + 1.6203x - 0.3157.$

d. $12.15

7. Linear $y = 0.35x + 0.88$; Predicted cost in 2012 is $4.73; it is hard to say which model works best but it seems that the use of a cubic model may overestimate the cost in the short term.

8.

a. $A = 1000 * 2.7182^{.12t}$

b. the amount will triple early in Year 9.

9.

a. $A = 3000 * 2.7182^{.12t}$.

b. The amount will triple early in Year 9 as in the last problem because the exponential equations $3000 = 1000 * 2.7182^{.12t}$ and $9000 = 1000 * 2.7182^{.12t}$ both reduce to the same equation $3 = 2.7182^{.12t}$ and hence have the same solution.

10. $y = 30.07 * \sin(.5196x - 2.1503) + 51.46$.

<u>Day 1</u>

Determine whether the real number is rational or irrational.

1) 0.7

2) -3678

3) $\frac{3\pi}{2}$

4) $3\sqrt{2} - 1$

5) $4.341\overline{451}$　　6) $\frac{22}{7}$

7) $\sqrt[3]{64}$　　8)

$0.8177\overline{8177}$

28 9) $\sqrt[3]{60}$　　10) 2e

Determine whether the given value of *x* satisfies the inequality.

11) $5x-12>0$

$(a) x=3$　　$(b) x=-3$

12) $x+1>\frac{2x}{3}$

$(a)\, x = 0$ $(b)\, x = 4$

$(c)\, x = \frac{5}{2}$ $(d)\, x = \frac{3}{2}$

$(c)\, x = -4$ $(d)\, x = -3$

13) $0 < \frac{x-2}{4} < 2$

14)

$-1 < \frac{3-x}{2} \leq 1$

$(a)\, x = 4$ $(b)\, x = 10$

$(a)\, x = 0$ $(b)\, x = \sqrt{5}$

$(c)\, x = 0$ $(d)\, x = \frac{7}{2}$

$(c)\, x = 1$ $(d)\, x = 5$

29 **Day 2- Worksheet A & Worksheet B**

Calculus X

Name______________________________

Calculator Basics

Unit 1 – Worksheet A

Date______________________________

1) Display graphs of the following equations

Answers

in the standard viewing window:

$y = -0.15x^3 + 0.64x^2 - 2$

and

$y = 1.15x - 6$

To the right, sketch a graph of what you see.

2) Display a graph of the following equation

Answers

in the standard viewing window:

$y = 0.058x^3 + 0.3x^2 + 1$

Use the features of the trace cursor to estimate the coordinates of the *y*-intercept of the curve by positioning the free-moving cursor at the location where the curve crosses the *y*-axis.

To the right, record the coordinates of the *y*-intercept.

3) Display a graph of the following equation

Answers

in the standard viewing window:

$y = 0.72x^2 + 1.3x - 4$

Use the features of the trace cursor to estimate the coordinates of the x-intercepts of the curve by positioning the free-moving cursor at the location where the curve crosses the x-axis.

To the right, record the coordinates of the x-intercepts.

*Continue on next page

30 **4)** Display a graph of the following equation

Answers

in the standard viewing window:

$$y = 8\cos\left(\frac{x}{3.2} - 2\right)$$

Use the features of the trace cursor to estimate x-values where the corresponding y-value is 6.5.

To the right, record the x-values where the corresponding y-value is 6.5.

5) Two runners start from rest side by side and

Answers

race along a straight course. The total distance that the first runner (runner A) has traveled is given by the equation:

$$y = 0.092x^2$$

and the distance that second runner (runner B) has traveled is given by the equation:

$$y = 0.74x$$

where x is the time since the start of the race, in seconds, and y is the total distance traveled, in meters.

Estimate which runner is ahead seven seconds into the race and by what distance that runner leads. To the right, record your findings.

*Continue on next page

Name______________________________

Calculator Basics

Unit 1 – Worksheet B

Date_______________________________

6) Display a graph of the following equation

Answers

in a viewing window extending from –3 to 7 along the *x*-axis and from 2 to 9 along the *y*-axis:

$y = 0.18x^3 - 1.3x^2 + 1.8x + 6$

To the right, sketch a graph of what you see.

7) Display graphs of the following equations

Answers

in the standard viewing window:

$$y = \frac{8.1}{x}$$

and

$$y = -0.42x + 8$$

Zoom in on the graph by applying the Zoom In instruction. Center the zoom on the intersection point between the line and the curve.

Use the features of the trace cursor to estimate the coordinates of the intersection point by positioning the free-moving cursor at the location where the line crosses the curve.

To the right, record the coordinates of the intersection point.

8) Display a graph of the following equation

Answers

in the standard viewing window:

$y = 8\cos(2x)$

Zoom in on the first two complete oscillations which are to the right of the y-axis by executing the ZBox instruction.

To the right, sketch a graph of what you see.

*Continue on next page **9)** Display graphs

of the following equations in the standard viewing window:

Answers

$$y = 0.28x^2 + 3.1x + 7$$

and

$$y = 0.56x + 8$$

Zoom in on the section of the graph that includes the two intersection points by applying the ZBox instruction.

Use the features of the trace cursor to estimate the coordinates of the intersection points by positioning the free-moving cursor at the location where the line crosses the curve.

To the right, record the coordinates of the intersection points.

10) A certain machine part elongates under tension

Answers

(when it is pulled on) in a manner that approximately follows the equation:

$$y = 0.0156x^2$$

where x is the force in Newtons and y is the increase in the length of the part in meters. The relationship only holds true for forces raging form 0 to 6.5 Newtons. Beyond 6.5 Newtons the part deforms and breaks.

Graph the curve of the relationship between tension and elongation and adjust the viewing window so that it only displays the range for which this relationship is true.

Use the graph to determine how far the part can be elongated before it deforms and breaks. To the right, record your findings.

33 **Day 3**

Solve the inequality and sketch the graph of the solution on the real number line.

1) $x - 5 \geq 7$

2) $2x > 3$

3) $4x + 1 < 2x$

4) $2x + 7 < 3$

5) $2x - 1 \geq 0$

6) $3x + 1 \geq 2x + 2$

7) $4 - 2x < 3x - 1$

8) $x - 4 \leq 2x + 1$

9) $-4 < 2x - 3 < 4$

10) $0 \leq x + 3 < 5$

11) $\frac{3}{4} > x + 1 > \frac{1}{4}$

12) $-1 < -\frac{x}{3} < 1$

13) $\frac{x}{2} + \frac{x}{3} > 5$

14) $\frac{x}{2}-\frac{x}{3}>5$

15) $x^2 \le 3-2x$

16) $x^2 - x \le 0$

17) $x^2 + x - 1 \le 5$

18) $2x^2 + 1 < 9x - 3$

*Continue on next page

34 19) P dollars

is invested at a (simple) interest rate of r. After t years, the balance in the account is given by

$A = P + Prt$, where the interest rate r is expressed in decimal form. In order for an investment of \$1000 to grow ***more than*** \$1250 in 2 years, what must the interest rate be?

20) A doughnut shop at a shopping mall sells a dozen doughnuts for \$2.95. Beyond the fixed costs (for rent, utilities, and insurance) of \$150 per day, it costs \$1.45 for enough materials (flour, sugar, etc.) and labor to produce each dozen doughnuts. If the daily profit *varies between* \$50 and \$200, between what levels (in dozens) do the daily sales vary?

21) The revenue for selling x units of product is $R = 115.95x$, and the cost of producing x units is $C = 95x + 750$. In order to obtain a profit, the revenue must be *greater than* the cost. For what values of x will this product return a profit?

22) A utility company has a fleet of vans for which the annual operating cost per van is $C = 0.32m + 2300$, where m is the number of miles traveled by a van in a year. What number of miles will yield an annual

operating cost per van that is *less than* $10,000?

23) A square region is to have an area of *at least* 500 square meters. What must the length of the sides of the region be?

24) An isosceles right triangle is to have an area of *at least* 32 square feet. What must the length of each side be?

<u>Day 4 – Worksheet 1</u>

Calculus X **Name**

Graphing Calculator ______________________________

Solving Inequalities

Date______________________________

Worksheet 1

Inequalities in one variable of the form $Y > 0$ or $Y < 0$ can be graphed with a graphing calculator and analyzed with the zoom feature.

1) Graph each of the following polynomials in the standard viewing window. Give the number of real zeros of each polynomial. How do the zeros help you determine the solution to an inequality of the form $Y < 0$ or $Y > 0$?

Y	Number of Real Zeros
$x^3 - 3x + 1$	
$2x^2 - x + 4$	
$x^3 - x^2 + x + 1$	

2)

Set the screen dimensions at $-5 \le x \le 5$ and $-5 \le y \le 5$.

Graph $Y = x^3 - x + 1$. For which interval(s) is $Y > 0$? For which interval(s) is $Y < 0$?

3) Graph each of the following polynomials in the standard viewing window and complete the chart.

Y	Approximate Zero(s)	Interval(s) $Y > 0$	Interval(s) $Y < 0$
$(x+3)(x-2)$			
x^4-3x^3+3x-2			
$x^3-2(x+1)$			

*Continue on next page

4) Use the information in Exercise 3 to solve each inequality.

a. $(x+3)(x-2)<0$

b. $x^4-3x^3+3x-2>0$

c. $x^3-2(x+1)<0$

5)

Graph the equation $Y=\frac{(x-1)(x+2)^2}{x+4}$. Adjust the dimensions of the screen to view a complete graph (the complete graph cannot be seen in the standard viewing window).

What are the zeros?

Solve the inequality $\frac{(x-1)(x+2)^2}{x+4}<0$.

6) Graph $Y_1=-x^4+x^3+4x-2$ and $Y_2=x^4+x^2-1$ on the same set of axes.

For what values of x is $Y_1=Y_2$? For what values of x is $Y_1>Y_2$?

37 **Day 5**

For each of the following find (a) the <u>directed distance</u>

from *a* to *b*, (b) the directed distance from *b* to *a*, and (c) the distance between *a* and *b*.

1) $a = 126,\ b = 75$

2) $a = 9.34,\ b = -5.65$

Find the midpoint of the given interval.

3) [7, 21]

4) [-6.85, 9.35]

Solve the inequality and sketch the graph of the solution on the real number line.

5) $|x| < 5$

6) $\left|\frac{x}{2}\right| > 3$

7) $|x + 2| < 5$

8) $\left|\frac{x-3}{2}\right| \geq 5$

9) $|10-x|>4$

10) $|9-2x|$ <1

11) $|x-a|\leq b$

*Continue on next page

38 **Use absolute values to describe the given interval (or pair of intervals) of x-values.** 12) [-2, 2]

13) (-∞, -2) U (2, ∞)

14) [2, 6]

15) (-∞, 0) U (4, ∞)

16) All numbers *less than* two units from 4.

17) y is *at*

most two units from a. 18) The heights h of two-thirds of the members of a certain population satisfy the inequality

$\left|\frac{h-68.5}{2.7}\right| \le 1$, where h is measured in inches.

Determine the interval on the real number line in which these heights lie.

19) The estimated daily production x at a refinery is given by $|x-200,000| \le 125,000$, where x is measured in barrels of oil. Determine the high and low production levels.

39 **Day 6**

Evaluate the expression for the indicated value of x.

1) $-3x^3$, *when* $x=2$

2) $4x^{-3}$,

$when \;\; x = 2$

3) $\dfrac{1+x^{-1}}{x^{-1}}, \quad when \;\; x = 2$

4) $3x^2 - 4x^3, \quad when \;\; x = -2$

5) $6x^0 - (6x)^0, \quad when \;\; x = 10$

6) $\sqrt[3]{x^2},$

$when \;\; x = 27$

7) $x^{-1/2}, \quad when \;\; x = 4$

8) $x^{-2/5},$

$when \;\; x = -32$

9) $500x^{60}, \quad when \;\; x = 1.01$

10) $\sqrt[3]{x},$

$when \;\; x = -154$

Simplify each expression.

11) $5x^4\left(x^2\right)$

12)

$6y^2\left(2y^4\right)^2$

13) $10\left(x^2\right)^2$

14) $\dfrac{7x^2}{x^{-3}}$

15) $\dfrac{12\left(x+y\right)^3}{9(x+y)}$

16) $\dfrac{3x\sqrt{x}}{x^{1/2}}$

17) $\left(\dfrac{\sqrt{2}\sqrt{x^3}}{\sqrt{x}}\right)^4$

*Continue on next page

40 **Simplify each radical.**

18) $(a)\ \sqrt{8}$ $\qquad (b)\sqrt{18}$

19) $(a)\ \sqrt[3]{16x^5}$ $\qquad (b)\sqrt[4]{32x^4z^5}$,

20) $(a)\ \sqrt{75x^2y^{-4}}$ $\qquad (b)\sqrt{5(x-y)^3}$

41 <u>**Day 7 – Problems & Worksheet 2**</u>

Find the domain of the given expression.

1) $\sqrt{x-1}$ $\qquad$ 2) $\sqrt{5-2x}$

3) $\sqrt{x^2+3}$ $\qquad$ 4) $\sqrt[5]{1-x}$

5) $\dfrac{1}{\sqrt[3]{x-1}}$ $\qquad$ 6) $\dfrac{1}{\sqrt{x+4}}$

7) $\dfrac{1}{\sqrt[4]{2x-6}}$

8) $\dfrac{\sqrt{x-1}}{x+1}$

9) $\sqrt{x-1}+\sqrt{5-x}$

10) $\dfrac{1}{\sqrt{2x+3}}+\sqrt{6-4x}$

Continue on and do Worksheet 2 which is on the next page.

Unit 1 – Worksheet 2

Simplify each of the following expressions by factoring.

1) $5x^3 - 15x^7$

2) $3x^{1/2} + 9x^{3/2}$

3) $8x^{-1/2} + 12x^{3/2}$

4) $(x-2)^{1/2} + 3(x-2)^{3/2}$

5) $(6x+5)^{-1/2} - 4(6x+5)^{1/2}$

6) $(x-3)(x+4)^{1/2} - 5(x-3)^3(x+4)^{3/2}$

7) $(2x-5)^{-1/2}(3x+1)^{3/2} + (2x-5)^{1/2}(3x+1)^{3/2}$

8) $2(x-7)^{1/2}(3x-2)^{5/2} + 5(x-7)^{3/2}(3x-2)^{3/2}$

Day 8

Factor each of the below quadratic expressions.

1) $x^2 - 4x + 4$

2) x^2

$+10x + 25$

3) $4x^2 + 4x + 1$

4)

$9x^2 - 12x + 4$

5) $x^2 + x - 2$

6)

$2x^2 - x - 1$

7) $3x^2 - 5x + 2$

8)

$x^2 - xy - 2y^2$

9) $x^2 - 4xy + 4y^2$

10)

$a^2b^2 - 2abc + c^2$

Completely factor each polynomial expression.

11) $81-y^4$ 12)

x^4-16

13) x^3-8 14)

y^3-64

15) x^3+64 16)

z^3+125

17) x^3-27 18)

$(x-a)^3+b^3$

19) x^3-4x^2-x+4 20)

x^3-x^2-x+1

21) $2x^3-3x^2+4x-6$ 22)

$x^3-5x^2-5x+25$

23) $2x^3-4x^2-x+2$ 24)

$x^3 - 7x^2 - 4x + 28$

44 **Day 11**

Use the Quadratic Formula to find all real zeros of each expression.

1) $6x^2 - x - 1$ 2) $8x^2 - 2x$

-1

3) $4x^2 - 12x + 9$

4) $9x^2 + 12x + 4$

5) $y^2 + 4y + 1$ 6) $x^2 + 6x$

-1

Use synthetic division to complete the indicated factorization.

7) $x^3 + 8 = (x + 2)(\qquad\qquad)$ 8)

$2x^3 - x^2 - 2x + 1 = (x-1)($ $)$ **Use the Rational Zero Theorem and find all real solutions of each equation.**

9) $x^3 - x^2 - x + 1 = 0$

10) $x^3 - 6x^2 + 11x - 6 = 0$

11) $4x^3 - 4x^2 - x + 1 = 0$

12) $x^3 - 3x^2 - 3x - 4 = 0$

45 **Day 12-Worksheet C**

Calculus X

Name______________________________

Calculator Basics

Unit 1 – Worksheet C

Date_______________________________

1) Display a graph of the following equation

Answers

in the standard viewing window:

$y = 2.1x^2 - 3.5x - 1$

Use the minimum feature to find the coordinates of its minimum.

To the right, record the coordinates of the minimum.

2) Display a graph of the following equation

Answers

in the standard viewing window:

$y = -0.9x^2 + 2.1x - 6$

Use the maximum feature to find the coordinates of its maximum.

To the right, record the coordinates of the maximum.

3) Display a graph of the following equation

Answers

in the standard viewing window:

$y = -0.156x^2 + 2x + 2$

Use the maximum feature to find the coordinates of its maximum.

To the right, record the coordinates of the maximum.

*Continue on next page

46 **4)** The power output of a particular engine varies

Answers

with its speed according to the equation:

$$y = -0.133x^2 + 1.119x$$

where y is the output in hundreds of horsepower and x is the speed of the engine in thousands of rpm's. Determine the maximum power output of the engine in horsepower and the engine speed at which that occurs (in rpm's).

To the right, record your findings.

5) The annual cost per employee of running a

Answers

Particular enterprise varies depending on the number of employees. The relationship can be roughly approximated by the equation:

$$y = 0.1x^2 - 0.744x + 4.02$$

where y is the cost per employee in tens of thousands of dollars and x is the number of employees in hundreds. Determine the lowest cost per employee and number of employees when that cost is achieved.

To the right, record your findings.

 Day 13

Perform the indicated operations and simplify your answer.

1) $\frac{5}{x-1}+\frac{x}{x-1}$

2) $\frac{2x}{x^2+2}-\frac{1-3x}{x^2+2}$

3) $\frac{4}{x}-\frac{3}{x^2}$

4) $\frac{2}{x+2}-\frac{1}{x-2}$

5) $\frac{5}{x-3}+\frac{3}{3-x}$

6) $\frac{A}{x-6}+\frac{B}{x+3}$

7) $-\frac{1}{x}+\frac{2}{x^2+1}$

8) $\frac{-x}{(x+1)^{3/2}}+\frac{2}{(x+1)^{1/2}}$

9) $\frac{2-t}{2\sqrt{1+t}}-\sqrt{1+t}$

10) $\frac{1}{x^2-x-2}-\frac{x}{x^2-5x+6}$

Day 14

Rationalize the numerator or denominator and simplify.

1) $\frac{3}{\sqrt{27}}$

2) $\frac{5}{\sqrt{10}}$

3) $\frac{\sqrt{2}}{3}$

4) $\frac{\sqrt{26}}{2}$

5) $\frac{x}{\sqrt{x-4}}$

6) $\frac{4y}{\sqrt{y+8}}$

7) $\frac{\sqrt{y^3}}{6y}$

8) $\frac{x\sqrt{x^2+4}}{3}$

9) $\frac{49(x-3)}{\sqrt{x^2-9}}$

10) $\frac{10(x+2)}{\sqrt{x^2-x-6}}$

11) $\frac{5}{\sqrt{14}-2}$

$\overline{6+\sqrt{10}}$

12) 13

13) $\frac{2x}{5-\sqrt{3}}$

$\overline{\sqrt{2}+\sqrt{3}}$

14) x

15) $\frac{1}{\sqrt{6}+\sqrt{5}}$

16) $\frac{\sqrt{15}+3}{12}$

17) $\frac{\sqrt{3}-\sqrt{2}}{x}$