

Timing Attacks on Implementations of Diffie-Hellman, RSA, DSS, and Other Systems

paul@cryptography.com

Abstract

Keywords

1 Introduction

2 Cryptanalysis of a Simple Modular Exponentiator

$$\begin{array}{ccccccc} y^x & n & n & & y & & R \\ & & & x & & & \\ y^x & n & & & y & y^{-n} & \\ & & & & & x & \end{array}$$

y

$$R = y^x \cdot n \cdot x \cdot w$$

```

Let s
For k upto w
  If k x is then
    Let  $R_k = s_k \cdot y \cdot n$ 
  Else
    Let  $R_k = s_k$ 
    Let  $s_k = R_k \cdot n$ 
  EndFor
Return  $R_w$ 

```

$$b \qquad b \qquad b$$

$$b \qquad b \qquad \text{For} \qquad s_b \qquad R_b \qquad s_b \cdot y \qquad n$$

$$R_b \qquad s_b \cdot y \qquad n \qquad s_b \qquad y$$

$$b \qquad R_b \qquad s_b \cdot y \qquad n \qquad R_b \qquad s_b \cdot y \qquad n$$

$$b \qquad s_b \qquad R_b \qquad n$$

3 Error Correction

$$b \qquad R_k \cdot b$$

error detection

4 The General Attack

$$T = T_j \cdot y_j \cdot x_b$$

$$P(x_b) = \prod_i^j F(T_i, t_{y_i, x_b})$$

$$y_i^x = \frac{t_{y_i, x_b}}{n}$$

$$T = t_{y, x_b} \cdot \frac{y}{T_i} \cdot t_{y_i, x_b} \cdot x_b \cdot F$$

$$T_i = t_{y_i, x_b}$$

$$s = x_b \cdot x'_b$$

$$x_b$$

$$\frac{\prod_i^j F(T_i, t_{y_i, x_b})}{\prod_i^j F(T_i, t_{y_i, x'_b})}$$

$$F$$

5 Simplifying the Attack

$$T = e \cdot \sum_i^w t_i$$

$$F$$

$$t_i$$

$$\begin{aligned}
& x_b \qquad T \qquad e \qquad \sum_i^w t_i \qquad \sum_i^b t_i \qquad e \qquad \sum_i^w t_i \qquad y \qquad x_b \\
& \qquad \qquad \qquad e \qquad w \qquad b \qquad t \qquad e \qquad \sum_i^w t_i \qquad c \qquad b \\
& \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad e \\
& w \qquad b \qquad c \qquad t \\
& \qquad \qquad \qquad t \\
& \qquad \qquad \qquad t \\
& \qquad \qquad \qquad b \qquad w \qquad j \\
& \qquad \qquad \qquad c \\
& \sum_i^b t_i \\
& \qquad \qquad \qquad t_i \\
& e \\
& P \sum_i^j \sqrt{w \ b} X_i \sqrt{b \ c} Y_i \sum_i^j \sqrt{w \ b} X_i \\
& P \sqrt{b \ c \ w \ b} \sum_i^j X_i Y_i \ b \ c \sum_i^j Y_i \\
& X \qquad Y \\
& \qquad \qquad \qquad \sum_i^j Y_i \ j \qquad \sum_i^j X_i Y_i \ j \\
& \qquad \qquad \qquad \sqrt{j} \\
& P \sqrt{b \ c \ w \ b} \sqrt{j} Z \ b \ c \ j \qquad P \ Z \ \frac{\sqrt{j \ b \ c}}{\sqrt{w \ b}} \\
& Z \\
& \qquad \qquad \qquad \infty \ \frac{\sqrt{\frac{j \ b \ c}{w \ b}}}{x} \qquad x \qquad j \\
& \qquad \qquad \qquad w
\end{aligned}$$

6 Experimental Results

$$\begin{aligned}
& TM \\
& \qquad \qquad \qquad a \qquad b \qquad n \\
& \qquad \qquad \qquad TM
\end{aligned}$$

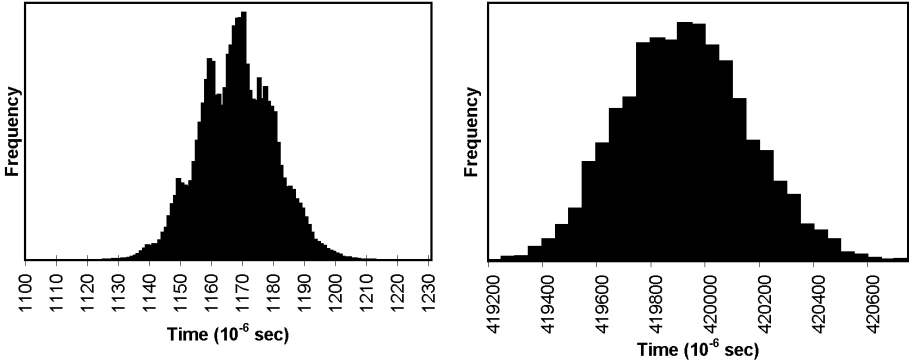
n

$y \qquad y \qquad n$

$\sqrt{\hspace{1cm}}$ s s
 s s

FIGURE 1: RSAREF Modular Multiplication Times

FIGURE 2: RSAREF Modular Exponentiation Times



j b c w

$\sqrt{\frac{j \ b \ c}{w \ b}}$

$\sqrt{\hspace{1cm}}$

y p p y
 p
 p y
 p p
 p
 p

8 Timing Cryptanalysis of DSS

r q x k s k $H\ m$ $x\ r$ q
 $H\ m$ $H\ m$ $x\ r$ q
 k q
 $x\ r$ q
 $H\ m$ $H\ m$ q
 $x\ r$
 r
 x
 x
 x
 $x\ r$ x x
 $x\ k$

9 Masking Timing Characteristics

$$v_i-v_f$$

$$w$$

$$provably$$

$$n$$

$$n$$

11 Further Work

$$f\sum_n\frac{\frac{56}{n}}{56}=\frac{56}{\frac{56}{n}}$$

$$p\quad q$$

12 Conclusions

13 Acknowledgements

References

- Proceedings of Crypto 82* *Advances in Cryptology*
tions on Information Theory *IEEE Transac*
On the Design and Security of Block Ciphers
- of Computation* *Mathematics*
Fast Software Encryption Second
International Workshop Leuven Belgium December 1994 Proceedings
- Communications of the ACM* **21**
- Fast Software Encryption Cambridge Security Workshop Cambridge*
U K December 1993 Proceedings
- r**
- Fast Software Encryption Second International Workshop Leuven*
Belgium December 1994 Proceedings