

- Monotone likelihood ratio:** The family of densities  $\{f_\theta : \theta \in \Theta\}$  with  $\Theta \subset \mathcal{R}$  is said to have monotone likelihood ratio in  $Y(x)$  if, for any  $\theta_1 < \theta_2$ ,  $\theta_i \in \Theta$ ,  $f_{\theta_2}(x)/f_{\theta_1}(x)$  is a nondecreasing function of  $Y(x)$  for values  $x$  at which at least one of  $f_{\theta_1}(x)$  and  $f_{\theta_2}(x)$  is positive.
- Optimal rule:** An optimal rule (within a class of rules) is the rule has the smallest risk over all possible populations.
- Pivotal quantity:** A known Borel function  $R$  of  $(X, \theta)$  is called a pivotal quantity if and only if the distribution of  $R(X, \theta)$  does not depend on any unknown quantity.
- Population:** The distribution (or probability measure) of an observation from a random experiment is called the population.
- Power of a test:** The power of a test  $T$  is the expected value of  $T$  with respect to the true population.
- Prior and posterior distribution:** Let  $X$  be a sample from a population indexed by  $\theta \in \Theta \subset \mathcal{R}^k$ . A distribution defined on  $\Theta$  that does not depend on  $X$  is called a prior. When the population of  $X$  is considered as the conditional distribution of  $X$  given  $\theta$  and the prior is considered as the distribution of  $\theta$ , the conditional distribution of  $\theta$  given  $X$  is called the posterior distribution of  $\theta$ .
- Probability and probability space:** A measure  $P$  defined on a  $\sigma$ -field  $\mathcal{F}$  on a set  $\Omega$  is called a probability if and only if  $P(\Omega) = 1$ . The triple  $(\Omega, \mathcal{F}, P)$  is called a probability space.
- Probability density:** Let  $(\Omega, \mathcal{F}, P)$  be a probability space and  $\nu$  be a  $\sigma$ -finite measure on  $\mathcal{F}$ . If  $P \ll \nu$ , then the Radon-Nikodym derivative of  $P$  with respect to  $\nu$  is the probability density with respect to  $\nu$  (and is called Lebesgue density if  $\nu$  is the Lebesgue measure on  $\mathcal{R}^k$ ).
- Random sample:** A sample  $X = (X_1, \dots, X_n)$ , where each  $X_j$  is a random  $d$ -vector with a fixed positive integer  $d$ , is called a random sample of size  $n$  from a population or distribution  $P$  if  $X_1, \dots, X_n$  are independent and identically distributed as  $P$ .
- Randomized decision rule:** Let  $X$  be a sample with range  $\mathcal{X}$ ,  $A$  be the action space, and  $\mathcal{F}_A$  be a  $\sigma$ -field on  $A$ . A randomized decision rule is a function  $\delta(x, C)$  on  $\mathcal{X} \times \mathcal{F}_A$  such that, for every  $C \in \mathcal{F}_A$ ,  $\delta(X, C)$  is a Borel function and, for every  $X \in \mathcal{X}$ ,  $\delta(X, C)$  is a probability measure on  $\mathcal{F}_A$ . A nonrandomized decision rule  $T$  can be viewed as a degenerate randomized decision rule  $\delta$ , i.e.,  $\delta(X, \{a\}) = I_{\{a\}}(T(X))$  for any  $a \in A$  and  $X \in \mathcal{X}$ .
- Risk:** The risk of a decision rule is the expectation (with respect to the true population) of the loss of the decision rule.
- Sample:** The observation from a population treated as a random element is called a sample.