

Monotone likelihood ratio: The family of densities $\{f_\theta : \theta \in \Theta\}$ with $\Theta \subset \mathcal{R}$ is said to have monotone likelihood ratio in $Y(x)$ if, for any $\theta_1 < \theta_2$, $\theta_i \in \Theta$, $f_{\theta_2}(x)/f_{\theta_1}(x)$ is a nondecreasing function of $Y(x)$ for values x at which at least one of $f_{\theta_1}(x)$ and $f_{\theta_2}(x)$ is positive.

Optimal rule: An optimal rule (within a class of rules) is the rule has the smallest risk over all possible populations.

Pivotal quantity: A known Borel function R of (X, θ) is called a pivotal quantity if and only if the distribution of $R(X, \theta)$ does not depend on any unknown quantity.

Population: The distribution (or probability measure) of an observation from a random experiment is called the population.

Power of a test: The power of a test T is the expected value of T with respect to the true population.

Prior and posterior distribution: Let X be a sample from a population indexed by $\theta \in \Theta \subset \mathcal{R}^k$. A distribution defined on Θ that does

not depend on X is called a prior. When the population of X is considered as the conditional distribution of X given θ and the prior is considered as the distribution of θ , the conditional distribution of θ given X is called the posterior distribution of θ .

Probability and probability space: A measure P defined on a σ -field $\mathcal{F}$ on a set Ω is called a probability if and only if $P(\Omega) = 1$. The triple $(\Omega, \mathcal{F}, P)$ is called a probability space.

Probability density: Let $(\Omega, \mathcal{F}, P)$ be a probability space and ν be a σ -finite measure on $\mathcal{F}$. If $P \ll \nu$, then the Radon-Nikodym derivative of P with respect to ν is the probability density with respect to ν (and is called Lebesgue density if ν is the Lebesgue measure on $\mathcal{R}^k$).

Random sample: A sample $X = (X_1, \dots, X_n)$, where each X_j is a random d -vector with a fixed positive integer d , is called a random sample of size n from a population or distribution P if $X_1, \dots, X_n$ are independent and identically distributed as P .

Randomized decision rule: Let X be a sample with range $\mathcal{X}$, A be the action space, and $\mathcal{F}_A$ be a σ -field on A . A randomized decision rule is a function $\delta(x, C)$ on $\mathcal{X} \times \mathcal{F}_A$ such that, for every $C \in \mathcal{F}_A$, $\delta(X, C)$ is a Borel function and, for every $X \in \mathcal{X}$, $\delta(X, C)$ is a probability

measure on $\mathcal{F}_A$. A nonrandomized decision rule T can be viewed as a degenerate randomized decision rule δ , i.e., $\delta(X, \{a\}) = I_{\{a\}}(T(X))$ for any $a \in A$ and $X \in \mathcal{X}$.

Risk: The risk of a decision rule is the expectation (with respect to the true population) of the loss of the decision rule.

Sample: The observation from a population treated as a random element is called a sample.